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# MULTI-AGENT SIMULATION TO SUPPORT WATER DISTRIBUTION NETWORK PARTITIONING

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## KEYWORDS

Water distribution network, multi-agent systems, graph theory, simulation, district metered area (DMA)

## ABSTRACT

In managing water supply, engineers often need to divide a water distribution network (WDN) into smaller clusters. Commonly, they work with District Meter Areas (DMA), a discrete part of the system in which the quantities of water entering and leaving the area are metered. The division of a WDN into a collection of DMAs can be considered a graph partitioning problem which is NP-Hard. Additionally, this problem is constrained by the physical nature of the WDN including the geographic location of the elements in the network, the hydraulic features of the network, the topography of the area, the demand patterns of the consumers, and other factors. This research shows how to solve this factoring problem by using a two-step algorithm. It uses a k-means graph clustering algorithm to partition the network geographically into a predefined number of clusters. It then applies a multi-agent system negotiation mechanism to adjust graph nodes on the boundary of all clusters to account for the hydrological constraints. Despite the fact that we do not consider all the necessary hydraulic factors, the application of our method on a case study shows promising results.

## INTRODUCTION

Water distribution network (WDN) is the infrastructure that supplies drinking water to homes and businesses. It is a complex system composed of some sources and thousands of consumption nodes which are interconnected through thousands of links. Sources include reservoirs and tanks while links consist of pipes, pumps and valves. Consumption nodes are called junctions in civil engineering literature.

### Characteristics of Water Distribution Networks

A WDN has special characteristics typical of an ultra large cyber physical systems (CPS) (Lee 2008; Rajkumar et al. 2010). It is a large distributed system, including multiple parties with different and sometimes conflicting goals, different information, and partial view of the whole system (multiple sources, reservoirs, tanks, pumps, pipes, valves,

and different consumers with different needs and behaviors). It is also volatile in terms of both supply and demand. The behavior of the parties may change dynamically, resulting in unpredictable behavior patterns in the whole system. Customers and the way they use water determine how a WDN needs to behave. Water consumption varies over time, both long-term (seasonal) and short-term (daily), and from place to place (Walski et al. 2003).

The complexity of a WDN is amplified by several issues (Fernández 2011): leakages, pipe bursts, fires, floods, and unpredictable weather conditions which highly affect the quantity and quality of supply and even demand. Reports of leakages typically amount to 35% on average and even up to 65% of total supplied volume of water in some areas (Kingdom & Liemberger 2006; Babovic & Keijzer 2002).

As a result, management of a WDN, maintaining it and ensuring the quantity and continuity of supply to customers is a complicated task. Partitioning a network into smaller sub networks is a good strategy to manage this complexity which is advised by International Water Association (IWA) (Morrison et al. 2007). This practice is used in many cities around the world to control and operate WDNs (Empaling 2007; Macdonald & Yates 2005; Fernández 2011), and provides enhancement of management of water distribution systems through the “divide and conquer” strategy.

### District Metered Area (DMA)

The concept of DMA management was first introduced to the UK water industry in the early 1980's in Report 26 Leakage Control Policy & Practice (UK Water Authorities Association 1980). In that report, a DMA is defined as a discrete area of a distribution system usually created by closure of valves or complete disconnection of the pipes in which the quantities of water entering and leaving the area are metered (Morrison et al. 2007). DMA is also known as pressure zone, discrete hydraulic sector, or leakage district (Burrows et al. 2000).

The benefits of partitioning a WDN into a collection of MDAs include: providing different pressure levels (pressure zones), better rehabilitation and work planning, enhanced leakage and burst detection and management, improved demand management, improved sensor placing, and augmented contamination spread control, to name a few (Fernández 2011). Furthermore, different water customers

have different needs in terms of quality and quantity and exhibit different usage behaviors (domestic / industrial / gardening, private / public etc.). These justify managing water networks by dividing them into smaller partitions.

However, partitioning a WDN into DMAs is an NP-hard problem (Gomes, Marques, et al. 2012; Gomes, Sá Marques, et al. 2012; Fernández 2011; Herrera et al. 2012). The partitioning should consider natural situations of the region such as rivers, roads, railways, highways, and different geographic features, population density and its distribution. International Water Association (IWA) published a guideline (Morrison et al. 2007) about DMA management. According to this guideline, the factors that should be taken into account when designing a DMA include: size (geographical area and number customer connections), elevation (variation in ground level), pressure requirements, number of valves to be closed, number of meters to be installed, and infrastructural conditions.

Despite numerous benefits of deploying DMAs, the literature contains little information on DMA design (Gomes, Marques, et al. 2012).

In this work, we propose a heuristic solution to the NP-hard problem of partitioning a WDN into a collection of DMAs using multi-agent systems paradigm.

### Multi-Agent Systems

A Multi-Agent System (MAS) can be defined as a loosely coupled network of autonomous problem solvers (also called agents) that interact to solve common problems that are outside the individual competencies or knowledge of each of them. These agents can be heterogeneous in their nature. The characteristics of MASs are that (1) each agent has imperfect information or capabilities for solving the problem and, therefore, has a limited and partial perspective; (2) there is no global control; (3) data are decentralized; and (4) computation is asynchronous. Multi-agent systems are ideal for problems that have multiple problem solving methods, multiple perspectives and/or multiple problem solving entities (Sycara 1998).

### Why Multi-Agent Systems

The study of the characteristics of water distribution network and multi-agent systems shows a good matching between the two. As we discussed, a WDN is a large distributed system, comprising of multiple parties with different goals, actions, and information, and partial view of the whole system. It is dynamic in terms of both supply and demand. In a WDN, behavior of the parties may change dynamically, resulting in unpredictable behavior patterns in the whole system. Another characteristic of a WDN is that parties can form organizations and coalitions.

Macal and North (Macal & North 2009) discuss why and when multi-agent modeling is useful, specially:

- When the nature of the problem seems to be composed of agents, in other words agents are the natural representation of the problem.
- When behaviors of agents is important for us, and agents adapt and modify their behaviors.
- When agents can create organizations, and learning and adaptation at the organization level is important for us.

The capabilities of MAS paradigm seems to be ideally suitable for solving the WDN problems.

As we can see, the multi-agent systems paradigm suits the issues and challenges in water distribution networks.

In the rest of the paper we explain our method to solve the problem of partitioning a WDN into a collection of DMAs. This paper is organized as follows: In the next section, we introduce our methodology and proposed solution. Afterwards, we state how we did simulations and implemented our algorithm. Later, we demonstrate the appropriateness of our proposal through a case study. Finally, we conclude and discuss this work's limitations and our future directions.

### METHODOLOGY

According to the IWA guidelines (Morrison et al. 2007), for large water networks, it is advised to first divide the network into sectors of suitable sizes, through a comprehensive distribution mains map. This stage employs local information of the network, accessible hydraulic data (pressure and flow), current boundaries, natural structures such as railways, rivers, major roads, and the topography of the city, so the area is divided into prospective large pressure zones where applicable. Using mathematical hydraulic network models is advised in more complex networks to help identifying hydraulic balance points. It is not necessary to have equal sector sizes; however, to support flexibility of the supply it is advised not to have trunk mains in the sectors if possible.

The next step will be the division of the sectors into DMAs, which is the focus of our work. The first stage is really important and the output of the process must be revised by skilled hydraulic experts to ensure the best arrangement. Accordingly, the input of our process is a reviewed network of water distribution sector, which is a part of a WDN.

Figure 1 illustrates the proposed algorithms. We start with construction and calibration of the hydraulic simulation model. We use EPANET 2 (Rossman 2000) to model the WDN, which is a public-domain software developed by the US Environmental Protection Agency (EPA) and is capable of hydraulic simulation and analysis of a WDN. Then we import the information from the EPANET 2 model into a graph composing of the sources (reservoirs and tanks) and sinks (junctions) as nodes, and pipes, pumps, and valves as links, in our MAS simulation framework.

Then we use k-means graph clustering (Hartigan & Wong 1979) to partition the network geographically into a user-defined number of clusters. The default number of clusters is the number of sources in the network. This is heuristically a good starting point to start partitioning a WDN into some DMAs, according to the IWA guidelines (Morrison et al. 2007) as mentioned earlier.

The K-means clustering is an unsupervised learning method for discovery of clusters and cluster centers in a set of unlabeled data. It starts with a desired number of cluster centers, say K, which is an input to the algorithm, and iteratively moves the centers to lessen the overall inside cluster variance. Supposing an initial set of centers (K), the K-means algorithm repeat the two steps (Hastie et al. 2009):

- for each center identifies the subgroup of training points that is closer to it than any other center (its cluster);

- The means of each property for the data points in each cluster are calculated, resulting in a mean vector, and this mean vector turns out to be the new center for that cluster.

These two steps are repeated until convergence or a predefined number of iterations. Usually the preliminary centers are K arbitrarily selected observations of the training data.

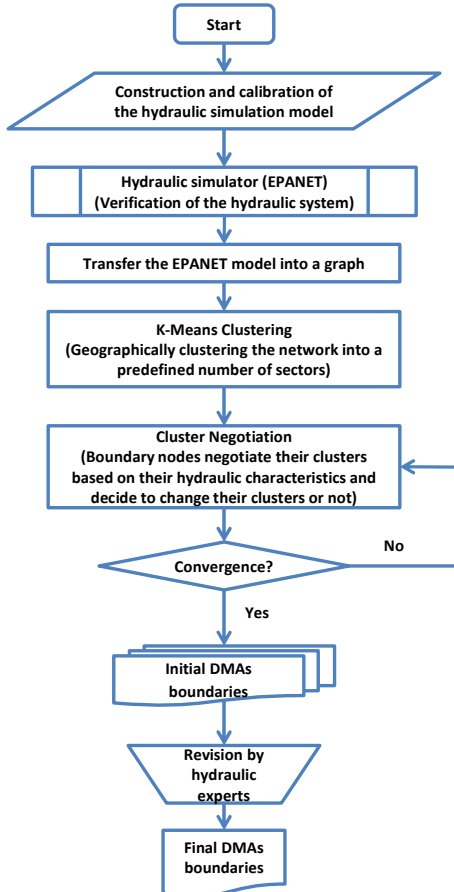


Figure 1: The Proposed Algorithm to Partition a WDN into some DMAs

Unfortunately, finding the optimal solution to the k-means clustering problem is an NP-hard problem (even for  $k=2$ ), however, a variety of heuristic algorithms are generally used to make the computation time smoother (Fernández 2011).

The output of this clustering process is a list of lists of nodes representing the different clusters, while each node can only be part of one cluster. Since the k-means clustering algorithm has some sort of randomness in it, results may be different for different runs.

Then we use another heuristic negotiation algorithm for the boundary nodes to negotiate their corresponding clusters based on hydraulic characteristics of the network. We consider the difference of the elevation of the boundary nodes with the neighboring clusters versus their corresponding clusters. If its elevation is closer to the other cluster than its current one, it will join that cluster and the negotiation will start again for the new network arrangement. This negotiation will happen for all of the boundary nodes in

a random sequence so these results could also be different for different runs. Convergence (stopping condition) is assessed based on the minimum number of nodes which change their clusters, and the minimum number of links that are in the boundaries so must be closed using valves or remain but must be equipped with meters.

The output of the process will be a set of proposed DMAs for a sector which are subject to review by hydraulic experts to decide the best DMA configuration. Our work is to support and facilitate DMA design, not to fully automate it.

## AGENT-BASED SIMULATIONS

For MAS simulation we use NetLogo (Wilensky 1999). NetLogo is a multi-agent programmable modeling environment. It is particularly suitable for modeling complex systems which change over time. Using this tool, we can give commands to a numerous of agents which work independently and autonomously. In NetLogo, the world is composed of agents, which are creatures that can follow commands. Two types of agents are important for our modeling purpose: turtles and links. Turtles are agents that may move around in the world. Links are agents that connect two turtles (Anon 2013). It is possible to create different breeds of agents for different types of phenomena in the modeling problem.

We have created three different breeds of turtles for sources (including reservoirs and tanks) and consumption nodes (junctions). Pipes are simulated using a breed of undirected links and pumps and valves are simulated using breeds of directed links.

We firstly transform an EPANET 2 model into a series of files which are suitable to be imported into NetLogo. For this purpose, we have developed a specific transformation tool using Python programming language which we called it EpanetExport. We have published this tool as an open source software (It is accessible on <https://github.com/saeed-hajebi/EpanetExport>).

Then we create and setup our world and its different breeds of turtles and links in NetLogo, and assign the network hydraulic and GIS information of the EPANET 2 model to the turtles' and links' variables.

In the next step, we do the clustering of the network into a user specified number of clusters using the k-means graph clustering algorithm. As mentioned, the default number of clusters is the number of sources in the network. In our settings, we use 0.01 as convergence threshold and 500 as the maximum number of iterations for the k-means algorithm, and the clustering will stop whichever comes first. Finally, we have a negotiation procedure which implements the negotiation algorithm in the Figure 2. In the algorithm, we ask the boundary nodes to negotiate their clusters based on their elevations. For a boundary node, if the difference of its elevation from the average elevation of the corresponding cluster is greater than the difference of its elevation from the average elevation of the other cluster, it will leave the current cluster and join the other one. The negotiation will continue until the number of changes decreases to a threshold which can be determined by the user. We have published the NetLogo model as well (<https://github.com/saeed-hajebi/MultiAgentWaterMngt>).

```

Negotiate:
while [total-num-of-changes >= threshold-num-of-changes] [
ask neighboring links [
ask one-of the end nodes [
keep-or-change-cluster
assign total-num-of-changes to total number of changes in the
negotiation process
]
]
]
end
keep-or-change-cluster:
if (elevation - mean-elevation-my-cluster > elevation - mean-
elevation-neighbor-cluster) [
leave the current cluster and join the other one
calculate total number of changes in the negotiation process
]
end

```

Figure 2: The Proposed Heuristic Negotiation Algorithm

**CASE STUDY**

We use the WDN for the city of Novato, California as a case study. This WDN is the most complex example included in the EPANET 2 (Rossman 2000) which covers an area of about 150 km<sup>2</sup>. It is a dual-source network, composed of 92 junctions, 2 reservoirs, 3 tanks, which are interconnected thorough 117 pipes and 2 pumps. Figure 3 illustrate the layout of this network in the EPANET 2 water distribution simulation software.

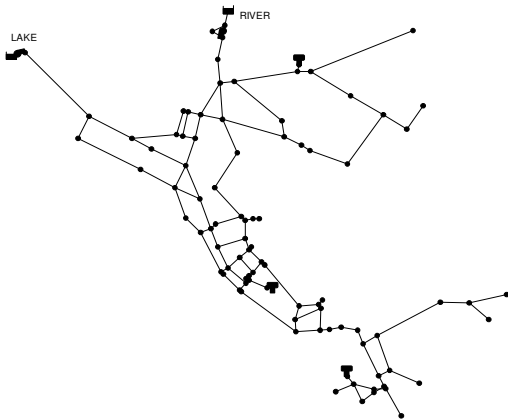


Figure 3- The layout of Net3 in EPANET 2

We firstly transfer the EPANET 2 network model into NetLogo and create the world in it. Then we do the geographical clustering using the k-means graph clustering algorithm. Figure 4 shows the result of this process. The details which are show in Table 1, designate that the network is partitioned into 3 sectors (the number of the partitions is specified by the user) of 34, 30, and 28 junctions respectively. Sector 1 will be detached from sector 2 using 4 valves, and we need 2 valves for detachment of sector 2 from 3. Totally, we need to deploy 6 valves to create the DMAs. The nexy step in our heuristic algorithm is the multi-agent negotiation. The results of the negotiation process is illustrated in Figure 5 and explained in Table 2 in more details, clarifying that after boundary nodes finished their negotiations, the new sectors configurations will be 3 sectors of sizes 38, 24, and 30, with 5 required valves for the boundaries. In the new arrangement, the number of valves is

decreased by one, which can reduce the costs of network changes. As we mentioned earlier, it is not necessary to have equal sector sizes.

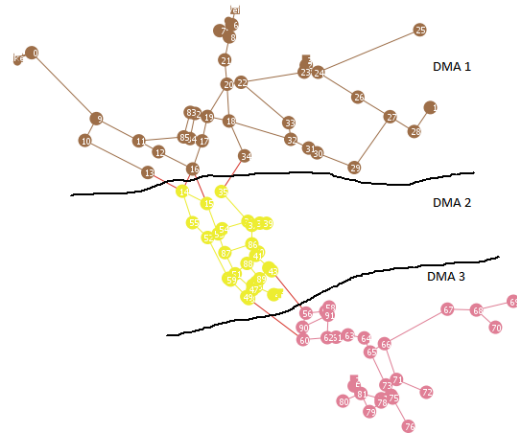


Figure 4- The Network is Clustered Geographically into 3 DMAs, before Negotiation

Table 1- Results of clustering the network into 3 DMAs, before Negotiation

Sector	Nodes#	Pipes#	Avg elevation	Valves#
Sector 1	34	43	24.92	4
Sector 2	30	41	12.31	4+2
Sector 3	28	33	14.9	2
Total valves #				6

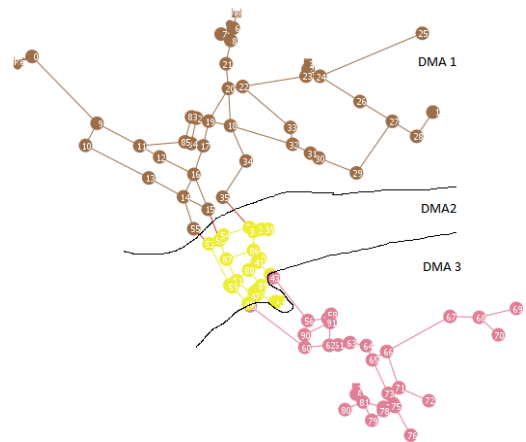


Figure 5- The network is clustered geographically into 3 DMAs, after Negotiation

Table 2- Results of clustering the network into 3 DMAs, after Negotiation

Sector	Nodes#	Pipes#	Avg elevation	Valves#
Sector 1	38	43	23.37	3
Sector 2	24	41	13.18	3+2
Sector 3	30	33	14.3	2
Total valves #				5

## RELATED WORK

We can divide the related work into two parts: the first part is about DMA design, and the second part is about using MAS approach to solve WDN problems. As mentioned in the introduction, the literature contains little information on DMA design. Tzatchkov et al. (Tzatchkov et al. 2008) presents algorithms for partitioning large networks, based on graph theory, which find the number of independent sectors and the corresponding nodes for each sector, among others. Di Nardo and Di Natale (Di Nardo & Di Natale 2011) propose a design support methodology based on graph theory to identify the location of flow meters and of boundary valves required to describe DMAs. Sempewo et al. (Sempewo et al. 2009) present a water distribution zone segregation method that exploits the similarity of graph theoretic and graph partitioning principles which are used in distributed computing, to recommend best zoning structures based on consistent length, demand or flow inside zones. Perelman and Ostfeld (Perelman & Ostfeld 2011) developed a tool based on graph theory which splits the network into clusters based on the flow directions in pipes. More recently, Gomes et al. (Gomes, Sá Marques, et al. 2012) propose a methodology to ascertain the best entry points at DMAs, and the location and settings of the required valves. As for applying MAS paradigm to solve WDN problems, Giannetti et al. (Giannetti et al. 2005) proposes an intelligent agent system for controlling an urban water network, which is capable of founding the desired water necessities, working limitations, and evaluation criteria to recommend an optimal control arrangement. Cao et al. (Cao et al. 2007) present a genetic algorithm to optimize water networks. Izquierdo et al. (2009) (Izquierdo et al. 2009), Izquierdo et al. (2011) (Izquierdo et al. 2011), and more recently Herrera et al. (Herrera et al. 2011) use a multi-agent approach to divide a WDN into DMAs. This series of work is the most related work to ours, however, our approaches is different from theirs. They start from a source node, which is not necessary, according to the IWA guidelines (Morrison et al. 2007), and expand a DMA by negotiation. On the other hand, we start from clustering the network geographically, which is advised by the IWA guidelines, and negotiate on the boundary nodes for the best hydraulic arrangement.

## CONCLUSION

A water distribution network (WDN) is a highly complex system composing of thousands of nodes, links and other elements. Partitioning a WDN into smaller sectors facilitates its management. In civil engineering domain, they call this sub networks as district metered areas (DMAs). Partitioning a WDN into some DMAs is a good strategy to facilitate management of such a complex system with a variety of benefits; however it is an NP-hard task. In this work we propose a multi-agent system approach to solve this problem and support partitioning a WDN into a collection of DMAs. Our heuristic algorithm uses k-mean graph clustering to partition the network geographically and another heuristic for the boundary nodes to negotiate their corresponding clusters based on hydraulic characteristics of the network. The application of our method on a case study shows promising results.

It should be noted that due to the inherent randomness in the k-means clustering algorithm and our heuristic negotiation algorithm, the results for different runs may differ. The algorithm should be run a couple of times and chooses the best configuration.

Nonetheless, our work has some limitations. We take only the elevation as the hydraulic feature for negotiation. We plan to embrace another network and hydraulic characteristics of the water distribution network such as pipe sizes, water flow, pressure requirement, and demand patterns as other negotiation criteria in our future work.

Despite the limitations, our work shows that the adoption of multi-agent systems paradigm facilitates clustering a WDN into DMAs.

## ACKNOWLEDGEMENT

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