



The physical modelling of a sitar

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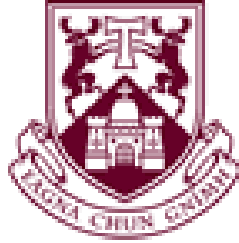
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O L L S C O I L L U I M N I G H

UNIVERSITY OF LIMERICK

MASTER OF SCIENCE IN MUSIC TECHNOLOGY

The Physical Modelling of a Sitar

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DECLARATION

Title: The Physical Modelling of a Sitar

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This dissertation is presented in partial fulfillment of the requirements for the degree of Master of Science in Music Technology 2010. It is entirely my own work and has not been submitted to any other university or higher education institution, or for any other academic award in this university. When use has been made of work of other people it has been fully acknowledged and fully referenced.

Signature:

Date:

Dedicated to Nora 'Bunny' Ronan.

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Abstract

There has been very little research done with regard to the physical modelling synthesis of the Indian classical instrument the Sitar. This dissertation intends on expanding on what little research has been done on the subject and attempts to model the instrument with modern modelling techniques such as bi-directional digital waveguides, fractional delay filtering and sympathetic vibrations.

It also presents a new and unique implementation of a dynamically changing delay line for a non-linear system such as the sitar string that has not been attempted before. It does this by making use of the Karplus-Strong algorithm to control the dynamic delay line. The Karplus-Strong was chosen because of how naturally it represents the decay of a string. This dissertation also attempts to model the sympathetic strings and the resonator of the sitar.

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Chapter 1

Introduction

Musical ideas are prisoners, more than one might believe, of musical devices.

Pierre Schaeffer

1.1 Motivation

The physical modelling of musical instruments is an interesting topic that has been around for quite some time now. It is effectively the term used for the computational models of acoustic-mechanical instruments (Karjalainen et al 1993). These models consist of normally simplified laws of physics that govern sound production. These physical laws can be used to describe say the plucking of a string or the beating of a drum. What is so exciting about this topic is that all the algorithms and methods that are being used for modelling are derived from natural physical phenomena. It forces computer scien-

tists to apply this natural phenomena to different data structures and logical procedures and see it work on a very fundamental level.

This dissertation proposes a physical model of the Indian classical instrument the sitar. The reason this instrument was chosen is because of the lack of existing physical models for it. There are numerous articles and papers with regard to the classical western guitar but unfortunately very little in-depth research has been done with regard to the sitar. This may be because of the sitars complex structure as a sitar normally has six or seven main playable strings and twenty or so strings that arent played but are there to vibrate sympathetically. It also has a non-linear bridge structure, which is what gives it its very distinct characteristic buzzing timbre that you would normally associate with the sitar. As you can see already there is a lot more to consider when it comes to approximating a model for this instrument as opposed to the six string classical western guitar.

1.2 Sitar Physical Model

The physical modelling approach used in this dissertation is that of digital waveguides. Digital waveguide models consist of digital delay lines and digital filters. Together these delay lines and digital filters can be understood to propagate and filter sampled travelling-wave solutions to the wave equation (Smith 2010). The wave equation being a very important second order partial differential equation that describes the propagation of waves with speed v . Originally the idea was to use the Karplus-Strong algorithm for the main and sympathetic string synthesis because of its low computational costs, but on further research, investigation and testing it was decided to use the bi-directional digital waveguide approach instead. The Karplus-Strong algorithm was reserved for

another modelling approach. The bi-directional digital waveguide approach is a much more realistic model of how a one-dimensional strings vibrates as it takes into account two acoustic waves travelling in opposite directions. It is known that the vibration of an ideal string can be described as the sum of two travelling waves going in opposite directions (D'Alembert 1747).

Another modelling approach being used, that is unique to this particular model is with regard to the non-linear bridge structure or jawari as it is officially called. The jawari because of its design, requires that there be a dynamically changing delay line. The amount of delay length modulation that occurs in this delay line is all relative to how much energy is in the plucked string.

The length of the string changes more rapidly at the attack portion of the signal, gradually becoming less random and settling into a more periodic pattern as the energy dissipates through the termini. This particular problem of non-linearity was solved using the Karplus-Strong algorithm and a feedback loop from the main sitar string itself. This technique is explained with more clarity further on in this document.

This model also makes use of fractional delay filtering. Fractional delay filtering is a modelling technique that allows for the accurate cancellation and dampening of musical tones (Lehtonen et al 2008). Normally delay lines in these particular types of models could only be of an integer sample length causing the physically modelled instrument to be slightly out of tune, but by using fractional delay filtering this can be avoided.

The other modelling techniques used in this particular model are all techniques that have been used for the modelling of the western classical guitar but they have been adapted to the sitar. These techniques include the sympathetic resonance of strings, comb filtering,

all-pass filtering and body resonance filtering.

1.3 Implementation

The entire modelling process has been done with the visual programming language MaxMSP. MaxMSP was chosen for its ease of use and the fact that it has many built in digital signal processing objects that are required for the modelling process. Fortunately, there was no need to install any MaxMSP externals for the model to be completed. MaxMSP also allows the model to be realised in real-time as opposed to having to compute the models expected outcome each time in a software program like Matlab. This is gave a great advantage when it came to the testing and analysis stage. As it was very easy to go back and make whatever slight changes were needed, and immediately hear and see the result.

Originally the proposed idea was for this model to be realised in C++, developed as an Audio Unit and distributed freely on the Internet but due to time constraints this was not possible.

1.4 Dissertation Overview

1.4.1 Literature Review

The first part of this dissertation is the literature review. It is here that the fundamentals of acoustics, wave motion, modelling techniques, digital waveguides and the structure

of the sitar are covered. They are presented in the order that they need to be understood.

Firstly the sitar will be examined. Giving a brief history of the instrument and then looking at the actual physical parts of the instrument. It will show how these all work together to give the sitar its unique timbre. It is also here that it shall be demonstrated why the sitar is such a particularly difficult instrument to model and how the jawari is the key to successful physical model. The next two subjects to be introduced are acoustics and wave motion. Here the fundamental ideas of these physical phenomena are presented on a basic level and shown how they are applied to a vibrating string.

Then finally different modelling techniques that are in use are explained as well as how the Karplus-Strong algorithm and digital waveguides were developed. It will also show the reader why the bi-directional digital waveguide technique was chosen and explain why other techniques would have been unsuitable.

1.4.2 Physical Model

It is here that the bulk of all the work done to have the instrument realised as a physical model will be shown. It will go into detail as to how the sympathetic strings were implemented and how the jawari were implemented as well as all the other implementations. Each of these particular implementations will be discussed in detail and it will be explained why each of these particular techniques were selected. It will also explain what particular components were crucial to achieving the characteristic buzzing timbre of the sitar.

1.4.3 Results and Analysis

Finally, spectral analysis of the sitar model will be performed and compared to the spectral analysis of a real sitar. This will be done a number of times to show how the sitar sound improved as the model was tweaked and played under different conditions. The efficiency of the model will also be discussed, this is with respect to CPU power and how much it uses. Furthermore, implementation issues will be discussed as there was a number of these to do with MaxMSP

Chapter 2

Background

In this section a brief history and taxonomy of physical modelling synthesis is presented to clarify to the reader the origins and the time line of the different approaches to modelling as they were discovered.

2.1 A Brief History

The first use of physically-based models to synthesize sound was by John Kelly and Carol Lochbaum (Kelly and Lochbaum 1962). They implemented a simplified model of the human vocal tract as a one-dimensional acoustic tube of varying cross-section. This being the most widely heard example of physical modelling for many years due to its use in Stanley Kubricks 2001: A Space Odyssey.

Most of the early work on physical modelling of musical instruments was focused on vibrating strings. This was due to them being computationally efficient to calculate. It

was Pierre Ruiz in 1970 that was the first person to synthesize a musical instrument using a physical model. It was then Ruiz and Lejaren Hiller that discovered the crucial fact that the quality of a vibrating string sound was mainly defined by the way the string loses energy (Hiller and Ruiz 1971a, 1971b). There were also approaches similar to those of Ruiz and Lejaren Hiller published by McIntyre and Woodhouse that would describe theoretical results to a realistically lossy vibrating string equation (McIntyre and Woodhouse 1979).

These techniques were then to be followed by the Karplus-Strong algorithm (Karplus and Strong 1983). The Karplus-Strong algorithm was discovered as a very simple computational technique that arose from work being conducted on wavetable synthesis. It works by feeding a burst of white noise into a feedback loop of length L samples. On each loop the white noise is filtered over and over again by a simple averaging filter. The frequency dependent decay of the white noise that was created for the first time on a computer sounded very string like. What made this algorithm so successful was that the realistic string timbres that could be produced with great ease were very computationally efficient. This was very relevant at the time as processing power would have been limited by modern day standards.

Seemingly this technique had nothing to do with physics and it wasnt until David Jaffe and Julius O. Smith did further work with it and showed a clearer understanding of it in relation to the physics of a plucked string (Smith 1983; Jaffe and Smith 1983). It was after this that Julius Smith introduced the theory of digital waveguides and generalised the underlying ideas of the Karplus-Strong algorithm (Smith 1987). Karjalainen says that digital waveguides are physically relevant abstractions yet computationally efficient models, not only for plucked strings but also for a variety of one-, two-, and

three-dimensional acoustic systems (Karjalainen et al 1998). These digital waveguides proved to be an efficient model of many linear and physical systems such as strings and acoustic tubes. One of the advantages of these waveguides over analytical methods was the ability to introduce non-linearity into models, just like those that would have to be considered when modelling the sitar (Smith 1987). This enabled researchers to produce a variety of different realistic instrumental sounds. To this day digital waveguides are still an important modern research topic with respect to the field of physical modelling. They are still used extensively in many commercial synthesis systems whether it is hardware or software. The first commercially available systems to include digital waveguides were at the beginning of the 1990s. These were Bontempi-Farfisas MARS in 1992 and then this was followed by Yamahas VL1 in 1993 (Fig. 2.1) .



Figure 2.1: Yamaha LV1.

2.2 A Taxonomy

Below in (Fig. 2.2) is a taxonomy of the different types of physical modelling synthesis techniques that can be used. Only DWGs (Digital Waveguides) will be covered in this dissertation. This figure has been given so the reader can see where this technique is derived from and how it relates to other techniques.

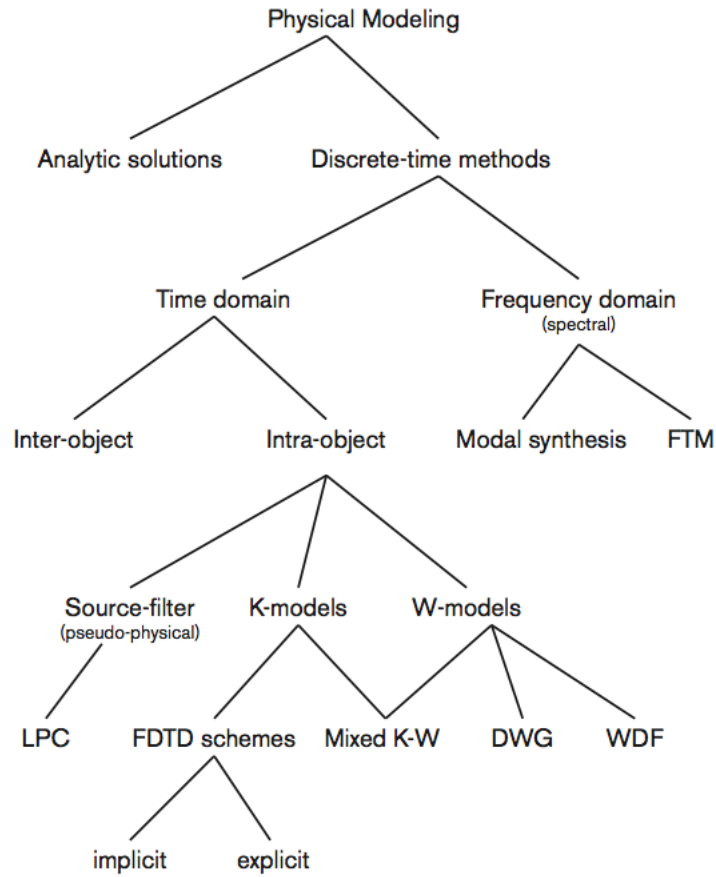


Figure 2.2: Physical Modelling Synthesis Taxonomy

2.3 The Sitar

2.3.1 Introduction

The Sitar is an ancient Indian string instrument that features heavily in Indian classical music. This is usually accompanied by a Tambura, a similar drone type instrument that is used to set the tonic of the piece being performed. The origin of the sitar can be dated back as far as the Middle Ages and is usually found in the northern part of India. It does not feature at all in southern Indian classical music.



Figure 2.3: The Indian classical instrument the Sitar (Courtney 2010).

The sitar became popular in the western world through the music of Pandit Ravi Shankar during the 1950s and George Harrison of the Beatles in the 1960s (Park 2008). It is known for its unique timbral quality, which is attributed to its sympathetic strings, the construction of its bridge, long hollow neck and its resonating chamber. It is usually played by balancing the instrument between the players left foot and right knee. This position then allows the players hands to move freely around the instrument neck without having to support its weight.

2.3.2 Mechanics

The sitar has a very unique and distinguishable body. On the neck of the instrument all the frets are moveable, allowing for fine-tuning and the use of micro tones. There are normally around 14 of those depending on the type of sitar. They are also suspended off the neck allowing the sympathetic strings to run underneath and resonate freely.

Normally the sitar has about 21 strings, most of these being sympathetic. These sympathetic strings are also known as tarb. These strings are never really ever touched as they are just meant to vibrate sympathetically. Although, some times you may hear a player strum all of these at once for effect. Along with the sympathetic strings you have the main six or seven strings. Three of these, called chikaari, provide the drone while the rest are used to play the melody (Courtney 2010).



Figure 2.4: Badaa goraa and Chota goraa of a Sitar (Courtney 2010).

The most important parts of the sitar are the two bridges. There is the large bridge called the badaa goraa for holding the drone and melody strings in place and then there is the smaller bridge for the sympathetic strings called the chota goraa. These bridges are collectively known as jawari and are normally made of camel bone. The shape of

the jawari are like slopes and it is the way the string interacts with these slopes when plucked that give the sitar its particular timbre.

Initially, when the sitar string is plucked is, there is a shortening and lengthening of the string relative that is relative to the slope which leads to the string generating overtones. This particular process is explained in better detail further on in this dissertation.

The resonator of the sitar is called the Kadu. These are very delicate and are normally just made of a gourd. On some sitars there are two resonators, the other one being at the top of the neck. They gourds may also sometimes have strings inside of them that are there to resonate sympathetically.

2.3.3 Bridge Structure

As mentioned before it is the sitars sloped bridge construction and its relationship to the strings that give it its specific buzzing sound. What happens specifically is a type of nonlinear distortion occurs when the string is plucked due to the interaction between the camel bone and the string. This nonlinear distortion gives rise to the production of additional overtones, which are somewhat similar to what happens when amplitude clipping occurs. Due to the square wave like properties forced upon by the clipping it creates odd harmonics that are not present in the original signal (Park 2008).

In the previous section it was mentioned that a shortening and lengthening of the string relative to the jawari occurs. This also lends to the buzzing timbre and also affects the pitch of the string ever so slightly since its length is changing. With regard to the actual model this requires that there be a dynamically changing delay line. How this was implemented is explained further on in the documentation.



Figure 2.5: Shortening and lengthening of string due to the shape of the bridge.

Further to the point of how the nonlinear distortion occurs due to the friction between the string and jawari, the transverse waves that are travelling along the string interact with the jawari just before they reach the point of termination. Normally in a stringed instrument with a typical style bridge such as that of a western classical guitar this termination point is where the waves usually flip over and travel in the opposite direction. However, in the sitar, before this happens, the larger amplitude transverse waves in the string interact with the jawari earlier than the smaller ones, altering the string's shape and causing it to bulge. The transverse waves are not terminated at this point but interact with the jawari, unlike the smaller ones, which mostly reflect. This greatly increases the higher partial content at large wave amplitudes but obviously not as much at smaller wave amplitudes. This interaction between the string and jawari reduces gain substantially, as the energy is transferred to the louder, higher partials. The imprecise termination point of the sitar is akin to the fretless electric bass.

Chapter 3

Simple Physical Model

3.1 What is a model?

Model-building is a fundamental human activity. For our purposes, a model can be defined as any form of computation that predicts the behavior of a physical object or phenomenon based on its initial state and any “input” forces. Our first successful models occurred in our heads (Hawkins 2004). It is effectively constructing a simplified abstract view of what normally may be a very complex system. Gaining an understanding of a complex natural system such as a musical instrument is usually accomplished by combining or building upon simpler and more basic models. If say we were to look at a guitar. The guitar is comprised of many mechanical parts such as the strings, resonator and the bridge. Each of these parts are the building blocks to the overall complex model. For virtual musical instruments and audio effects, the model replaces the real thing allowing us have a deeper understanding of how it works (Smith 2010).

3.2 Basic Vibrating String Model

The basic model of a vibrating string is based on Newtonian principles. The vibrations in the string are transverse waves or in this case transverse acoustic waves. To derive the equations governing small transverse vibrations of an elastic string, which is stretched to length L you have to make simplifying assumptions in order that the resulting equation does not become too complex.

First of all place the string along the x - *axis*, stretch it to length L , and fix it at the ends $x = 0$ and $x = L$. The string is then distorted at some instant, say $t = 0$, it is then released and allowed to vibrate. The problem is to determine the vibrations of the string, that is, to find its deflection at $u(x, t)$ at any point x and where $t > 0$.

In order to do this the following can be assumed.

1. The mass of the string per unit length is a constant. The string is perfectly elastic and doesn't offer any resistance to bending.
2. The tension caused by stretching the string before fixing it at the end points is so large that the action of the gravitational force on the string can be neglected.
3. The motion of the string is a small transverse vibration in a vertical plane, that is, each particle of the string moves strictly vertically.

These assumptions are made so that the solutions to the one-dimensional wave equation $u(x, t)$ that are obtained will reasonably well describe the small vibrations of the physical string. These assumptions give us the partial differential equation(PDE) for the one-dimensional wave equation as follows:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad c^2 = \frac{T}{\rho} \quad (3.1)$$

Where T is the string tension in the string and ρ is the linear mass density of the string. The derivation of this equation is beyond the scope of this dissertation but can be found in any applied mathematics textbook (Kreyszig 1999). This PDE is the starting point for both digital waveguide models and finite difference schemes.

3.3 Mass-Spring System

This section discusses the principles behind the motions of a basic mechanical system, a mass on an elastic spring. The string of a musical instrument is a mass-spring system.

If you are to take an ordinary spring and suspend it vertically from a support and then at the other end attach a body of mass m . Here you are to assume that m is so large you can disregard the mass of the spring. Then pull the body down a certain distance and then release it. You will notice that it undergoes a motion. This motion is governed by Newtons second law:

$$Mass \times Acceleration = my'' = Force \quad (3.2)$$

Where $Force$ is the resultant of all the forces acting on the body. Here, $y'' = \frac{d^2 y}{dt^2}$, where $y(t)$ is the displacement of the body and t is time. At first the string is unstretched, but then when the body is attached, the body stretches the string by an amount s_0 . This

causes and upward force F_0 in the spring. It has been experimentally shown that this restoring force F_0 is relative to stretch, say,

$$F_0 = -ks_0 \quad (3.3)$$

This is known as Hookes law. k is called the spring constant. Where the larger the value for k , the more stiff the spring is, hence giving a smaller s_0 . s_0 being the amount of displacement.

The extension of s_0 is such that F_0 balances the weight $W = mg$. Consequently $F_0 + W = -ks_0 + mg = 0$. These forces do not affect the motion. The entire system is at rest, this is what is called the static equilibrium of the system. The position of the body at the static equilibrium position is $y = 0$. We measure the displacement of the body from the static equilibrium position as $y(t)$. The main point is that F_0 is the restoring force. It has the tendency to restore the system back to its static equilibrium position $y = 0$.

With this understanding of a how a mass-spring system works, it brings us on to damped and undamped mass-spring systems. Every system has damping otherwise it would just keep moving forever. It would be like if a string was plucked and it kept vibrating forever. Although, to explain the next point we are going to look at an undamped system first.

Let's take for an example an iron weight on the end of a spring. In this situation F_1 is the only force in (3.2) causing the motion. Hence, making $my'' = -ky$ from (3.2). This means that the model for the mass-spring system without damping becomes:

$$my'' + ky = 0 \quad (3.4)$$

By finding the complex roots of this equation we get the general solution

$$y(t) = A \cos \omega_0 t + B \sin \omega_0 t \quad \omega_0 = \sqrt{\frac{k}{m}} \quad (3.5)$$

The corresponding motion to this equation is called a harmonic oscillation. These harmonic oscillations are similar to the waves that occur when a string is plucked. When the string is plucked or in this case when the iron weight is displaced the spring makes these harmonic oscillations. By applying the addition formula for \cos , this equation can be written as

$$y(t) = C \cos(\omega_0 t - g) \quad (3.6)$$

And, since the period of the trigonometric function (3.6) is $\frac{2\pi}{\omega_0}$, the body executes at $\frac{\omega_0}{2\pi}$ cycles per second. This quantity is called the frequency of the oscillation and is measured in Hertz.

In the case where the system has been damped which is more likely to be the situation. We connect the mass to a dashpot to demonstrate its properties. By looking at the equation governing the system we can derive three different cases. The damped system equation being

$$my'' + cy' + ky = 0 \quad (3.7)$$

Where $-cy' = F_2$, this being the force imposed by the dashpot. The three different cases are, overdamping, critical damping and underdamping. It is the roots of equation (3.7) that determine this.

Case 1: In the overdamping case the body does not oscillate since the damping takes the energy from the the system and there is no external force that keeps the motion going. The equation (3.7) has distinct real roots λ_1, λ_2 in this case

Case 2: The critical case marks the border between the non-oscillatory motions and oscillations; this explains its name "critical case". It has to do with the fact equation (3.7) has a real double root.

Case 3: Underdamping is the most interesting case. Underdamping occurs when the roots of the equation are complex conjugate roots. Underdamping would be similar to the case in most strings on an instrument. When the string is initially plucked it settles into a periodic behaviour corresponding to a harmonic oscillation.

These three cases are illustrated in Fig. (3.1)

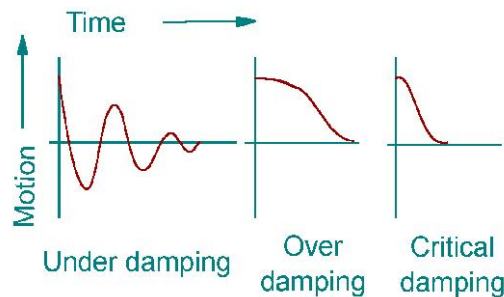


Figure 3.1: The three cases of damping

There is a particular modeling technique based solely on this mass-spring paradigm as mentioned before (Hiller and Ruiz 1979). As can be seen it requires the precise

description of all the physical characteristics of the vibrating objects and furthermore it requires that you stipulate the boundary conditions for the PDE of the one-dimensional wave equation. It also requires the physical description of the excitation mechanism. The difference equations that were presented earlier are the equations that are then used to compute what the resulting sound output will be (Bianchini and Cipriani 2008).

3.4 D'Alemberts Solution of the Wave Equation

With D'Alemberts travelling wave solution it can be shown that the vibration of an ideal string can be described as the sum of two travelling waves going in opposite directions using the wave equation. We will start with the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad c^2 = \frac{T}{\rho} \quad (3.8)$$

If we are to denote the right travelling waves and the left travelling waves by the following equations:

$$v = x + ct, \quad z = x - ct \quad (3.9)$$

Then u becomes a function of v and z . The derivatives of the wave equation in (3.8) can now be expressed in terms of the derivatives with respect to v and z by the use of the chain rule. This becomes

$$u_x = u_v v_x + u_z z_x = u_v + u_z \quad (3.10)$$

We now apply the chain rule to the right side of the equation giving us

$$u_{xx} = (u_v + u_z)_x = (u_v + u_z)_v v_x + (u_v + u_z)_z z_x = u_{vv} + 2u_{vz} + u_{zz} \quad (3.11)$$

Now we transform the other derivative in (3.8) giving

$$u_{tt} = c^2(u_{vv}2u_{vz} + u_{zz}) \quad (3.12)$$

By inserting the two results into (3.8) we get

$$u_{vz} \equiv \frac{\partial^2 u}{\partial z \partial v} = 0 \quad (3.13)$$

This resulting equation can now be solved by two successive integrations with respect to z .

$$\frac{\partial u}{\partial v} = h(v) \quad (3.14)$$

where $h(v)$ is an arbitrary function of v . Integrating this with respect to v gives

$$u = \int h(v)dv + \psi(z) \quad (3.15)$$

where $\psi(z)$ is an arbitrary function of z . Since the integral is a function of v , say, $\phi(v)$, the solution u is of the form $u = \phi(v) + \psi(z)$. Then because of (3.4) we get

$$u(x, t) = \phi(x + ct) + \psi(x - ct) \quad (3.16)$$

This is known as D'Alemberts solution of the wave equation. The traveling-wave solution of the wave equation was first published by d'Alembert in 1747 (D'Alembert 1747)(Kreyszieg 1999). The bi-directional digital waveguide is based on this very principle and will be discussed further on the dissertation.

3.5 Sampled Traveling-Wave Solution

In order to use the traveling wave solution in the "digital domain" it is necessary that you sample the traveling-wave amplitudes at intervals of T seconds. The continuous traveling-wave solution to the wave equation given in (3.16) can be sampled to give

$$y(nT, mX) = \phi(nT - \frac{mX}{c}) + \psi(nT + \frac{mX}{c}) \quad (\text{set } X = cT) \quad (3.17)$$

$$= \phi(nT - mT) + \psi(nT + mT) \quad (3.18)$$

$$\triangleq y^+(n - m) + y^-(n + m) \quad (3.19)$$

where $x = cT$ denotes the spatial sampling interval in meters, T denotes the time sampling interval in seconds, and y^+ and y^- are defined for notational convenience (Smith 2010) .

Chapter 4

Digital Waveguides

In this chapter the theory of digital waveguides is presented and explained. A lot of what is presented in this chapter has already been touched on in chapter two. Here the Karplus-Strong algorithm and the extended version of it will be explained in detail. This chapter will also introduce the bi-directional digital waveguide, this is the modelling technique that is central to the modelling of the sitar strings.

4.1 Karplus-Strong algorithm

The Karplus-Strong algorithm was discovered by two men around 1980. Their names being Alan Karplus and Kevin Strong. The paper on this algorithm was published in 1983. It was Alex Strong in December of 1978 that conceived its simplest modification and called it the Plucked-String algorithm. How it works is by simply averaging two successive samples (Karplus and Strong 1983). This can be written mathematically

as

$$Y_t = \frac{1}{2}(Y_{t-p} + Y_{t-p-1}) \quad (4.1)$$

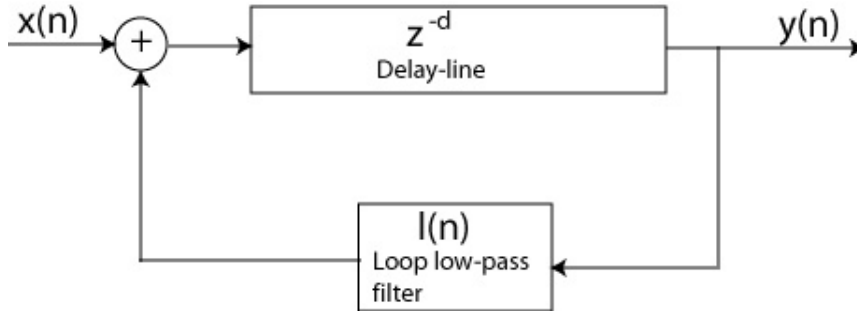


Figure 4.1: Karplus-Strong Algorithm

It was discovered that this averaging process produced a slow decay of whatever waveform was being computed by it. This algorithm produced a pitch of period $p + \frac{1}{2}$ samples and sounded similar to the decay of a plucked string. What was so remarkable about this algorithm was that there was no multiplication required. Making it extremely computationally efficient. Back then they did not have anywhere near the same microprocessing power that we have now days so this would have been fast and easy to implement considering the limitations at the time (Karplus and Strong 1983).

Strong says the naturalness of the sound derives largely from differing decay rates for the different harmonics. No matter what initial spectrum a tone has, it decays to an almost pure sine wave, eventually decaying to a constant value (silence) (Karplus and Strong 1983).

The actual excitation of the algorithm requires that a noise burst be fed into the system. How Strong originally did this was by feeding the algorithm with a wavetable filled with random values. The use of a different random wavetable every time had the advantage of giving each repetition of the same pitch a slightly different harmonic structure. This

gave each note its own character sort of like a real instrument. Normally what what would be used to excite the system would be a burst of pink or white noise (Karplus and Strong 1983).

Once the noise burst is fed into the system it is immediately output and then fed back into a delay line of L samples long. The output of this delay line is then fed into the averaging filter as described already. This is normally a first order low pass filter. Also, the gain of the filter must always be less than 1 or else the signal will never decay and could make the system unstable. The output of the averaging filter is then output and at the same time sent back into the delay line. This process keeps repeating until the signal is averaged out to silence (Karplus and Strong 1983).

The length L in samples of the delay line determines the fundamental pitch of the note being played. L is determined by the equation $L = F_s/F_1$ where F_s is the sampling frequency. The overall effect of the algorithm is quite realistic and very similar to a plucked string sound considering it is such a simplistic procedure. It may not have a natural sounding guitar string tone but there are different extensions that can be applied to help this, which will be discussed next. Alan Karplus conceived a simple variation of the algorithm for drum timbres. Since we are only interested in strings, this will not be discussed.

4.2 Karplus-Strong Extended

Around the same time that the paper about the original Karplus-Strong algorithm was published, David A. Jaffe and Julius O. Smith published a paper with regard to different extensions to the original algorithm. The need to implement these extensions came

from the musical needs that arose out of the composition of May All Your Children Be Acrobats (1981) and Silicon Valley Breakdown (1982) both by David Jaffe (Jaffe and Smith, 1983).

One of the first modifications made was with regard to the tuning. The fact that the delay line length L had to be an integer caused tuning problems. The tuning problems occurred at high frequencies. The fundamental frequency $f_1 = \frac{f_s}{(N+\frac{1}{2})}$, this meant that the pitches were rounded off. This was barely noticeable for low pitches (large N) but as the pitch increased it becomes more and more off sounding (Jaffe and Smith, 1983).

The solution to this problem was fractional delay filtering. It can be shown experimentally that by using a fractional delay filter there is a more accurate cancellation and dampening of musical tone partials (Lehtonen et al 2008). What was needed was the introduction of a filter into the feedback loop and that would delay the signal slightly with out altering the loop gain. The filter that was introduced was an all-pass filter. It ensured there was no change to the gain of the signal. The equation for this filter and its transfer function is as follows

$$Y_n = CX_n + X_{n-1} - C_{y_{n-1}} \quad (4.2)$$

$$H_{c(z)} \triangleq \frac{(C + z^{-1})}{1 + Cz^{-1}} \quad (4.3)$$

The only thing that the all-pass filter affected was the phase of the signal (Jaffe and Smith, 1983).

Another problem with the algorithm was with regard decay-time, the difference between the decay times for a low pitch and a high pitch were drastically different. The ability to control decay time is very important if you want to have a realistic realisation of a plucked string. Consequently, Jaffe and Smith found methods that could be used to control decay time. One of the methods was to introduce a loss factor ρ . Where equation (4.1) becomes:

$$Y_n = X_n + \rho \frac{Y_{n-N} + Y_{n-(N+1)}}{2} \quad (4.4)$$

Where $|\rho| \leq 1$ if the string is to be stable. Essentially what decay shortening does is produce a damped version of the Karplus-Strong algorithm. Where low-pitched notes are comparable to low notes on real strings. Another technique that was employed was decay stretching. This was done by changing the feedback average (H_a) to a two-point weighted average. This reduces the amount of energy loss at high frequencies. For the greatest control it is said both the uniform loss method and two-point-averaging method should be used together (Jaffe and Smith, 1983).

Dynamics was another issue that was dealt with. Where the output of the system was directly related to the noise burst being input into the system. What enabled this to work was, since the strings that were plucked hard had more energy in the higher partials than the strings plucked lightly, a one-pole low pass filter could be used to attenuate these higher partials before they were fed into the system. This allowed the user to be able to set if the string was to sound muted when it was plucked or alternatively sound like an open string. All that the user had to do was adjust the cut-off point of the one-pole low pass filter and you could get varying excitation timbres (Jaffe and Smith, 1983).

Some of the other extensions had to do with pick position and pick direction. Pick

position involved implementing a comb filter just after the noise burst. Depending on the delay length of the comb filter you can pick the string at different positions allowing you to suppress certain harmonics. Pick direction can then also be controlled by lowpass filtering the noise burst before it is fed into the delay line or by using a rich harmonic spectrum as opposed to a noise burst. Another way to affect the noise burst is to change the duration of the noise burst.

In order to model sympathetic string vibration, Jaffe and Smith sent a small percentage of the string output from a plucked string to another string that had been tuned to a different pitch. Since the sympathetic string was tuned to a different pitch all the partials of the plucked string that did not coincide with the sympathetic string would have been attenuated (Jaffe and Smith, 1983). There will be a further discussion with regard to sympathetic strings further on in this dissertation, as it is central to the sitar model. It can be seen here that through these extensions it can make the very basic algorithm much more expressive and realistic sounding. Normally the Karplus-Strong algorithm, although very similar to a plucked string, does have a very artificial sound.

4.3 Bi-directional Digital Waveguides

A bi-directional digital waveguide is essentially a bi-directional delay line at some wave impedance. This is also considered a lossless digital waveguide. Wave impedance is basically the ratio between the force of a wave to the velocity of a wave. For linear time invariant systems, impedance may vary with angular frequency (ω) such that

$$R(\omega) = \frac{F(\omega)}{V(\omega)} = \frac{Force(\omega)}{Velocity(\omega)} \quad \omega = 2\pi f \quad (4.5)$$

How the bi-directional waveguide works is that each delay line contains a sampled acoustic travelling wave (Smith 2010).

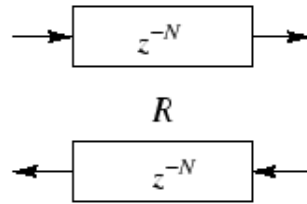


Figure 4.2: A digital waveguide (Smith 2010).

Since it is a bi-directional waveguide, this means that there is a sampled acoustic wave travelling from left to right and right to left in each of the delay lines. This models d'Alemberts travelling wave solution whereby it can be shown that the vibration of an ideal string can be described as the sum of two travelling waves going in opposite direction (d'Alembert 1747).

The type of bi-directional digital waveguide that we will be dealing with in this dissertation is one with rigid terminations. If we terminate a length L ideal string at $x = 0$ and $x = L$, we then have the boundary conditions

$$y(t, 0) = 0 \quad y(t, L) = 0 \quad (4.6)$$

How this system works is, the excitation is fed into the system at the arbitrary point ζ in Fig. (4.3). The acoustic travelling waves proceed to travel around the bi-directional waveguide being delayed by $\frac{N}{2}$ samples by the delay lines. It can be seen in the diagram

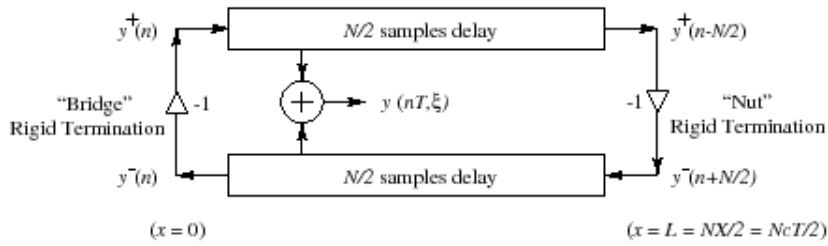


Figure 4.3: Digital waveguide model of a rigidly terminated ideal string (Smith 2010).

that there are the two termination points as mentioned before. These would normally be the nut and bridge of say a guitar. The reader may also notice the -1 at each of these termination points. The -1 is there to invert the phase of the acoustic wave. Just like how an acoustic wave would flip over and change direction in the real physical world if it were to meet the termination point.

This is a far more realistic simulation of a travelling acoustic wave than the single delay line technique formulated by Karplus and Strong. The example that has been discussed here is for only a one-dimensional waveguide. This technique can be extended to two and three dimensional waveguides and be used to model drum skins using digital waveguide meshes.

A number of the different extensions that were discussed in the previous section can be applied to the bi-directional digital waveguide such as fractional delay filtering and excitation position. Matti Karjalainen et al have looked at the possibilities of this in another paper. They employed two different models, one where the bi-directional digital waveguide had a bridge output and the other where it had a pick-up output (Karjalainen et al 1998). The sitar model demonstrated in this dissertation was loosely based around this.

Chapter 5

String and Instrument Modelling Techniques

This chapter looks at some of the modelling techniques that have been developed in recent years. Some are relevant to the sitar while others are not. They are discussed because they would have been considered when it came to figuring out how to model the different parts of the sitar.

5.1 Sympathetic String Vibrations

In nearly all stringed musical instruments where there are adjoining strings, sympathetic vibrations can occur. Whereby if one string is excited, some of the other strings may also be excited via the body through resonance. This phenomenon is known as sympathetic vibration and is defined in the acoustic dictionary as "resonant or near-resonant response of a mechanical or acoustical system excited by energy from an adjoining system in

steady-state vibration” (Morefy 2001) (Carou et al 2005).

At the bridge of say a western classical guitar, all the strings induce the movement of the top plate. Since this top plate is moving and all the strings are attached to the same plate, this means that all the strings are affected. Nakaerts says that strings cannot be seen as independent entities but must be seen as larger, coupled system (Nakaerts 2001).

A simpler approach to sympathetic vibrations was taken in the Karplus-Strong extensions paper. They basically took the approach of sending a small percentage of the main plucked string to another string tuned differently (Jaffe and Smith 1989).

Sympathetic vibrations are essential to the modelling process if you are to have a natural sounding model since sympathetic vibrations exist in nearly all stringed instruments. It is particularly important to the sitar since it has a number of strings that are only supposed to vibrate sympathetically.

With regard to the sitar a rather simplistic but effective approach was taken, this will be discussed further on in the dissertation.

5.2 String Coupling Effects

In natural string instruments several coupling mechanisms exist. In a real string, there are two orthogonal planes of transverse waves, which are directly coupled together. There are also longitudinal waves, which are related to string tension.

To realise the two orthogonal planes, you have to consider the transverse vibrations in the horizontal and vertical planes of polarisation. Smith says that no vibrating string in musical acoustics is truly rigidly terminated, because such a string would produce

no sound through the body of the instrument. Termination results in coupling of the horizontal and vertical planes of vibration. In typical acoustic stringed instruments, nearly all of this coupling takes place at the bridge of the instrument (Smith 2010).

Since that in real instruments the horizontal and vertical waves react differently with the bridge, for example on the guitar the string is restricted more in the horizontal plane of vibration as opposed to the vertical plane, this requires that when modelling the two planes, damping parameters will need to be different for each plane. This means that the string will decay faster in the horizontal plane and have an effect on the tone of string. What happens is a two stage amplitude envelope is created because of the unequal rate of decay between the two planes. Smith says that the initial fast decay gives a strong onset to the note, while the slower late decay provides a long lasting sustain—two normally opposing but desirable features (Smith 2010).

5.3 Commuted Synthesis

Commuted synthesis is a modelling technique that is used to model the resonator of an instrument. Typically the energy from a plucked string is transmitted to the bridge and then to some resonating acoustic structure. Typically this resonating structure or resonator imposes its own frequency response on the sound being radiated and works like a very large filter. One of the approaches that is normally taken to model the resonator is to figure out the body resonances of the instrument in question and then use a band-pass filter bank to apply them. This can be computationally expensive so this is why the commuted technique is used.

This technique only works for linear time-invariant systems, the idea is to commute the

string and the resonator. The excitation method is convolved with the impulse response of the instrument being modelled. This is the basic idea behind commuted synthesis, and it greatly reduces the complexity of stringed instrument implementations, since the body filter is replaced by an inexpensive lookup table (Smith, 2010).

However, due to the non-linear nature of the sitar string, commuted synthesis could not be used. This technique was included in the dissertation as it was one of the main resonator modelling techniques that was being considered when researching the sitar model.

Chapter 6

Sitar Model

6.1 Introduction

The sitar model as mentioned before was developed and tested in MaxMSP. The entire patch consists of three main parts. There is the poly~ abstraction of the main strings, the sub patch for the sympathetic strings and then there is a bank of digital filters being used as a means of modelling the resonator. All these components fit together to model the sitar. How the patch is controlled either by an external MIDI device or by the kslider object in MaxMSP.

The most important part of the patch is the poly~ abstraction. It is within this abstraction the main strings are modelled and it also gives the patch its seven note polyphony. Since this is the most important part of the model, it is the first part that will be discussed in detail.

6.2 Main Strings - poly~ abstraction

It is this part of the patch contains the main digital waveguides, the excitation mechanism and different objects to make sure the poly~ functions correctly. These can be seen in Fig. (7.2).

The first group of objects in the patch working from left to right are there to receive the pitch and velocity to be used in the waveguide sub patches, there is also a thispoly~ object to decide whether that instance of poly~ is busy or not.

The next group of objects in Fig. (7.2) are there to excite the strings. There is a linear ramp generator there to create a pink noise envelope. Pink noise is used because all the frequencies present are of equal amplitude and also because of its random nature, meaning that no two excitations will be the same. The original idea was to use a recording of a sitar impulse response and use the commuted waveguide synthesis technique but as explained before due to the sitars non-linear model this would not have been effective. There is also a comb filter setup just before the excitation is sent to the strings. This comb filter is there so that the user can adjust the pick position with the slider in the main patch. This a Karplus-Strong extended algorithm concept as discussed earlier in this document. The slider can be seen in Fig. (7.1).

The next group of objects in Fig. (7.2) are the bi-directional digital waveguide sub patches. The reason there are two of these is because of string coupling. One of these is the string vibrating in the horizontal plane and the other is vibrating in the vertical plane. This gives the string a more realistic sound. Normally if just one waveguide is used it sounds very static. These are both then summed together to give the overall string sound. They are also scaled since they are being summed together. The contents

of the digital waveguide sub patches will be discussed further on in this section.

After the waveguides have been summed there is a group of objects to test to see if the gain of the strings is less than 0.001 and if so, it mutes the `poly~` instance it is in and set its status to being not busy. This was implemented to make `poly~` more effective.

6.3 Bi-directional Digital Waveguide sub-patches

This is the most important part of the whole patch and can be seen in Fig. (7.3). This particular sub-patch is broken in to two parts. On the left side you have the bi-directional digital waveguide of the string and then on the right you have a Karplus-Strong algorithm implementation. It is this KS algorithm that is fundamental in giving the string its non-linear distortion and its characteristic buzzing timbre. First of all the bi-directional digital waveguide part of the sub-patch will be explained and then the KS algorithm implementation will be tied in.

6.3.1 Bi-directional Digital Waveguide

The bi-directional digital waveguide that has been implemented in this patch also uses some of the Karplus-Strong extended algorithm concepts. It makes use of the tuning all-pass filter, dynamic-level lowpass filter and string damping lowpass filter as well as a few implementations that were necessary for the string to sound like a sitar string.

As soon as the string is excited it is passed through a one-pole lowpass filter, this is the dynamic-level filter. The value for this filter is controlled from the main patch and there is one for each dimension of the string. This filter controls the timbre of the string each

time it is plucked. It is used to make the string sound as if it has been muted, if this is the desired effect.

After the one-pole filter the excitation enters the bi-directional digital waveguide. It can be seen in Fig. (7.3) that there are four `tapin~` and `tapout~` objects. These objects are effectively the delay lines. These are responsible for the pitch of the string. If you were to unwind the waveguide and have the two `*~ -1` multipliers as your termination points of the string, you will see that each delay line is effectively divided in two by the two `tapin~ tapout~` pairs. The reason for this being that excitation of the string has to be at least in the centre of the delay line and be fed into the circuit at the same position on each direction of the delay line. This makes sense, since if you were to pluck a string in a real physical system, you can only do so at one position at any given time.

If you look at the delay sub-patch within the patch in Fig. (7.3) you will see that this is the mechanism that controls the delay time for `tapin~ tapout~`. How this works is that the MIDI value received is converted into the frequency of the note being played. Since frequency is measured in Hertz and Hertz means cycles per second, the frequency value is divided into 1000 to give the delay time in milliseconds. The reader may also notice that this is then fed into a `mstosamps~` object, one is subtracted from it and then a `sampstoms~` object is used to convert back again. The `mstosamps~` and `sampstoms~` are used simply just to convert from milliseconds to samples. The reason why 1 is subtracted is because the creators of MaxMSP have designed the `tapin~ tapout~` to have a minimum delay of one vector size and this needs to be compensated for.

Once the excitation is in the waveguide it moves through it just like a transverse wave would in a real physical system. The `*~ -1` multipliers are there to reverse the phase of the wave every time it passes through them. The exact same way a wave flips over when

it reaches its termination point in a real physical system. This is why the two ~ -1 objects are considered the termination points.

The string damping dials on the main patch control the damping lowpass filters featured in the delay loop, these can be seen in Fig. (7.1). The velocity of the note being played is mapped to the MIDI values 100-127 and then these are converted to frequency values for the lowpass filters. This is how the string damping mechanism works.

The string damping is then followed by a clip \sim so as to normalise the signal going through the digital waveguide. This is just in case the model becomes unstable. This is then followed by a multiplier; the multiplier is used to set the rate at which the strings decay. The velocity of the note being played is mapped to the values of each of multipliers. It works on the principle that the larger the velocity the longer it will take for the strings to decay.

The last object left to discuss in the digital waveguide is the all-pass filter. This is the most important part of the waveguide as it is the part that gives the delay line a fractional delay and also dynamically changes the delay length giving the sitar its characteristic timbre. The middle inlet for the all-pass object is what controls the delay time of the filter. Two different processes modulate this value. The main one being the Karplus-Strong algorithm that is to the right of the bi-directional digital waveguide and the other is by a sub-patch called delayallpass. Within the delayallpass sub-patch you have a mechanism to create a slight vibrato, this is to create the overall beating effect between the strings. The amount of beating that occurs is relative to the velocity of the note being played. At any one time that a string is being played all the other strings that can be active through the poly \sim object are receiving a very slight signal, which is being modulated by the delayallpass sub-patch. This is to help model sympathetic resonance

between the main strings.

6.3.2 Karplus-Strong Algorithm

The Karplus-Strong algorithm in this case is not being used to generate sound but as a way to control the delay length of the bi-directional waveguide dynamically. This was implemented because it was felt that the best way to control the decay rate of the dynamically changing delay length of the bi-directional waveguide was by using something similar to the waveguide. The KS algorithm was chosen because it is inexpensive and it would naturally compliment it. The KS algorithm receives the same excitation and pitch values as the bi-directional waveguide so that decay rates of the two are somewhat similar.

The velocity of the note being played also affects how much the KS algorithm modulates the decay rate of the dynamically changing delay length of the bi-directional waveguide. It can be seen in the patch that the receive object known as `thisbridgelength` controls this. This takes the velocity of each note being played and maps it to suitable bridge modulation parameters. How the velocity of the note affects the dynamic delay length is modelled on how it works for a real sitar. The output of the KS algorithm is then fed into a sub-patch that smoothes out the changes in delay length. If this was not implemented it would drastically affect how the sitar sounded due to the sudden changes in delay length and cause glitches in the audio output.

Some of the output of the bi-directional digital waveguide is also fed back into the KS algorithm. This keeps the energy in the KS algorithm relative to the energy in the bi-directional delay line.

The rate of change of the dynamic delay length changes more randomly in the attack portion of the signal, over time it becomes less random and then settles into a more periodic pattern as the strings waveform itself becomes more periodic. Eventually this approaches zero. It could not be found anywhere in all of the literature reviewed or on the Internet, this approach to non-linear distortion being implemented and is unique to this attempt at physically modelling the sitar. It is hoped that this approach to the modelling of this type of non-linear distortion is considered for other instruments.

6.4 Sympathetic Strings (Tarafdar)

The sympathetic strings of the sitar are on a separate bridge to the main strings. The bridge has the same shape as the main bridge, so the sympathetic strings were implemented in a similar way to the main ones. However there are a few differences, one of them being that each of the individual sympathetic strings can be tuned to whatever note the user desires with respect to the western musical scale. The other difference being that there is no string coupling, this due to the limitations of the CPU.

How the sympathetic string sub-patch works is, that all the energy that comes from the main strings is scaled and fed into each of the individual sympathetic strings. It is scaled due to the amount of energy that would be lost in the energy travelling from one bridge to another. This amount of scaling was determined by trial and error. Also, the damping on each of the strings is higher than on the main strings, this due to that fact that these strings arent being plucked but are only resonating with respect to the main strings. The sympathetic strings in the sub patch are by default tuned to what they would typically be tuned to in Indian classical music. Although this does vary greatly with respect to

the raga being played. Finally, the output of all the strings is summed and scaled again. Having these strings adds greater depth to the sound of the model. The instrument sounds very dry when they are turned off.

6.5 Resonator (Kaddu)

Originally the plan was to use commuted synthesis to model the resonator as mentioned already. On further investigation and research it was determined that this technique is unsuitable for a sitar due to its non-linear model. This technique only works for linear time invariant systems. Commuted synthesis is where you take an impulse response recording of the resonator of the instrument being modelled and convolve this recording with the excitation mechanism in the model. This is the basic idea behind commuted synthesis, and it greatly reduces the complexity of stringed instrument implementations, since the body filter is replaced by an inexpensive lookup table (Smith 1993).

Instead the implementation used in this model is a bank of bandpass filters set to different frequencies and Q values. Unfortunately an actual sitar was not obtainable at the time that this implementation was being developed so an analysis of the actual body resonances of a sitar was not performed. The resonances used were similar to those of a Martin D-28 guitar (Fletcher and Rossing, 2005). The exact resonances in Fletcher and Rossings book weren't used, they were used mainly as a guideline, and a lot of trial and error was involved in getting it to sound correct. There is a massive contrast between the sound of the sitar with the resonator and it not having it. It was the `fffb~` object that was used for the filter bank. The `fffb~` object is a MaxMSP implementation of a bank of bandpass filter objects. It is much more efficient to use this instead of a group

of reson~ objects.

6.6 Conclusion

As the reader can see, the approach that was used to model the sitar was to develop each part separately and then tie them all together at the end. It can also be seen that it is the dynamic delay line that is very important in giving the sitar its characteristic timbre. This is not to say that the resonator and sympathetic strings are not as important. As mentioned previously, without these the instrument would sound very artificial and not have any natural sounding qualities to it. It is also hoped that the unique approach to the dynamic delay line that was implemented has made the model more natural sounding.

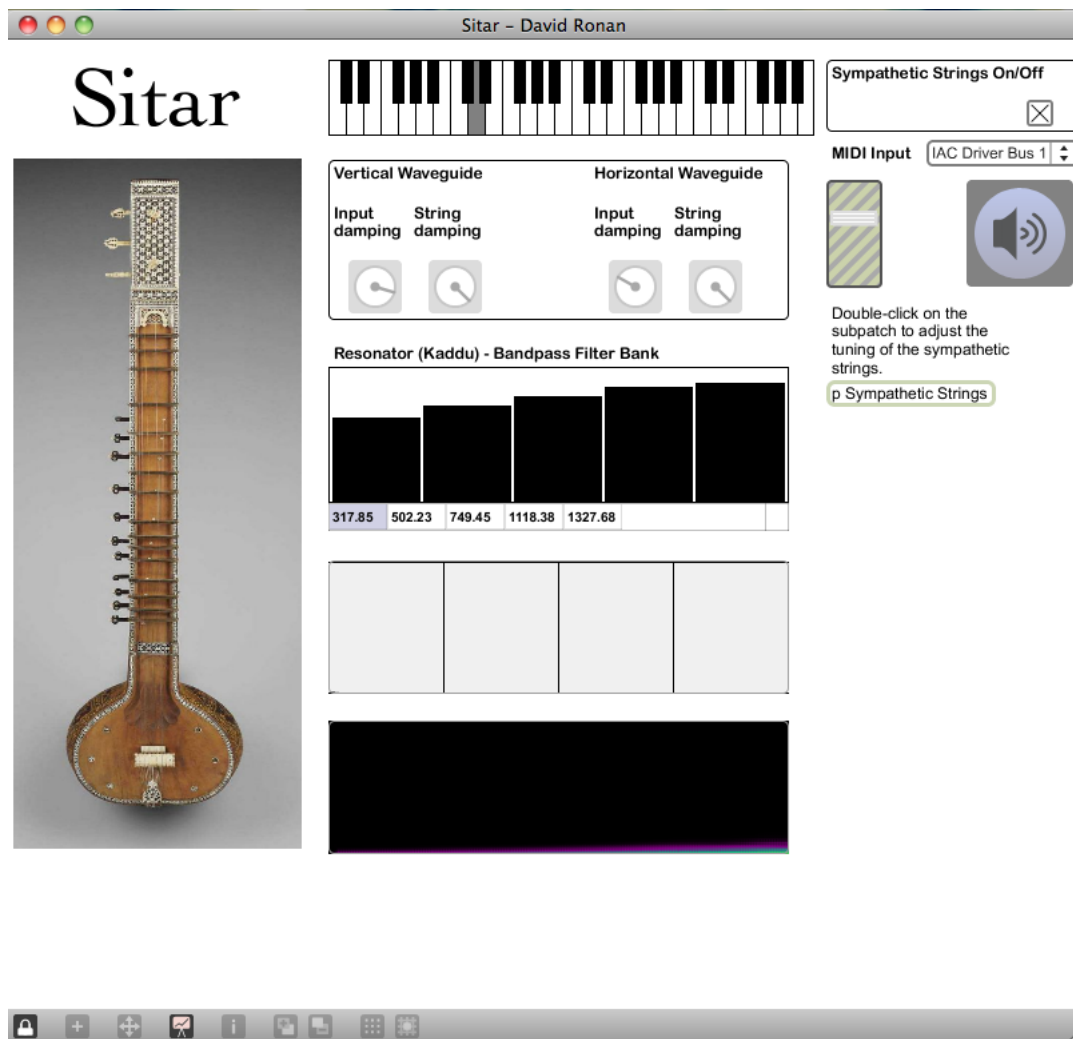


Figure 6.1: Main patch screenshot.

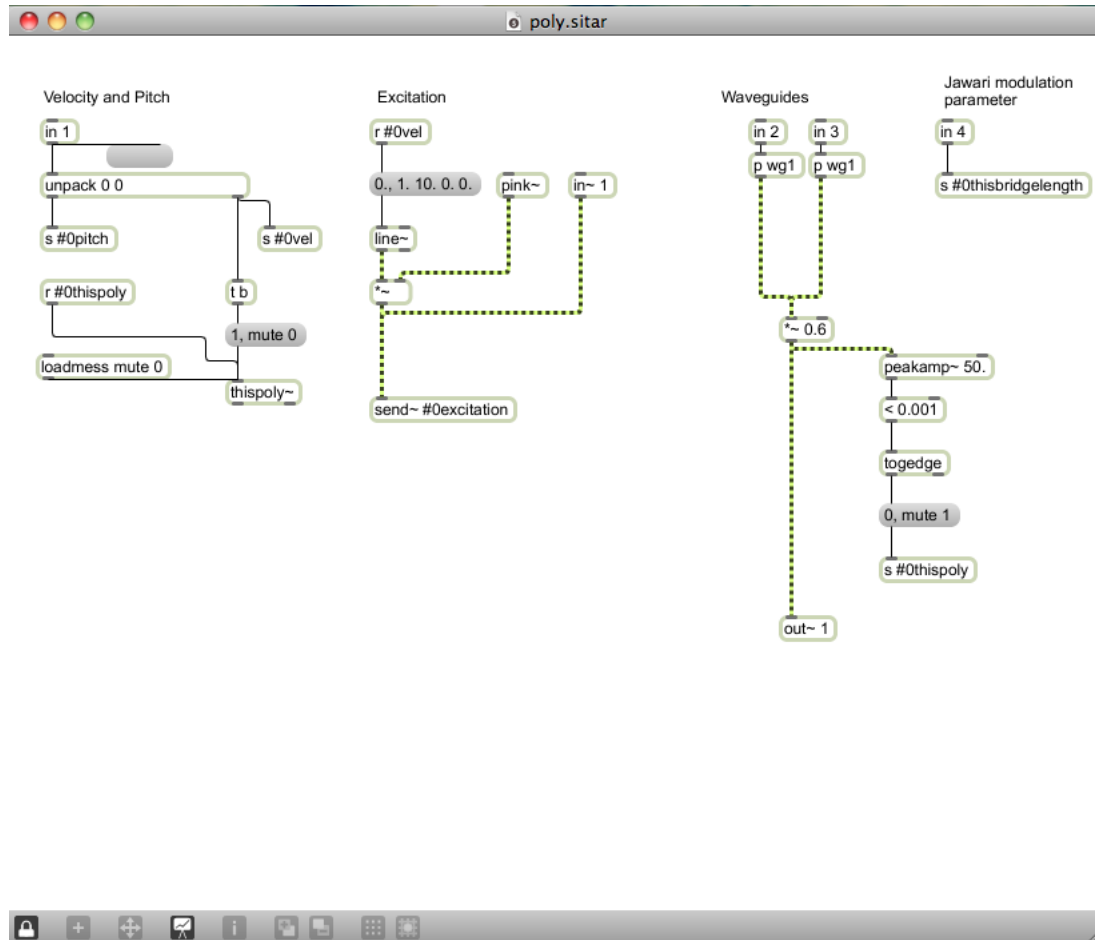


Figure 6.2: Poly~ screenshot.

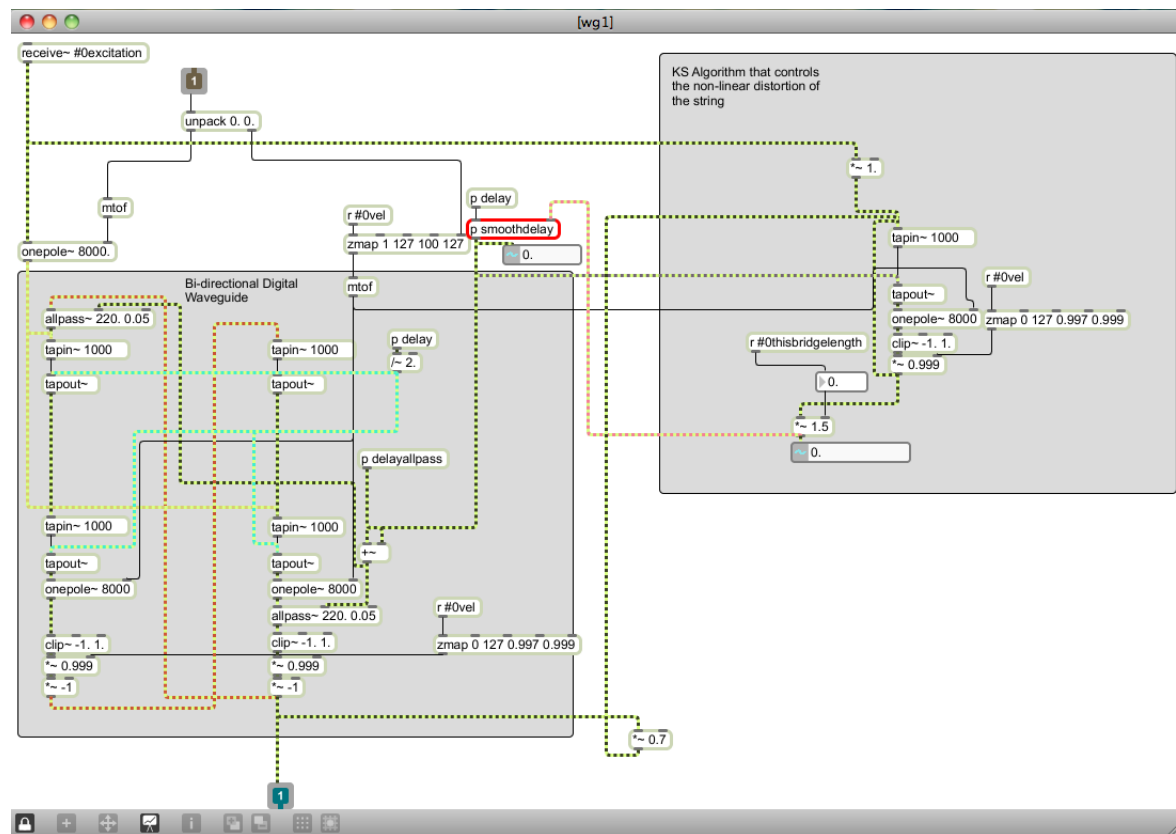


Figure 6.3: Bi-directional Digital Waveguide screenshot.

Chapter 7

Results and Analysis of Sitar Model

How the analysis was approached was by taking the recording of a real sitar playing the note F4, then this note was played a few different times on the modelled sitar and recorded. Audacity was then used to perform spectral analysis on each of the recordings. The type of analysis done was Fourier analysis with a Hanning window and a window size of 512.

Since every note played by the sitar is going to be different every time due to the random nature of every pink noise burst excitation not every modelled sitar pluck analysis will be the same. Once the first recording was made, analysed and then compared a second recording was made with a number of adjustments, which will be discussed in a moment. The results are as follows:

We see in the first analysis Fig. (7.1), the modelled sitar is very close to the real sitar up until roughly 10,000 Hz. We see the fundamental is close and a lot of the partials are the same but the modelled sitar is lacking a lot of energy in the higher partials. This

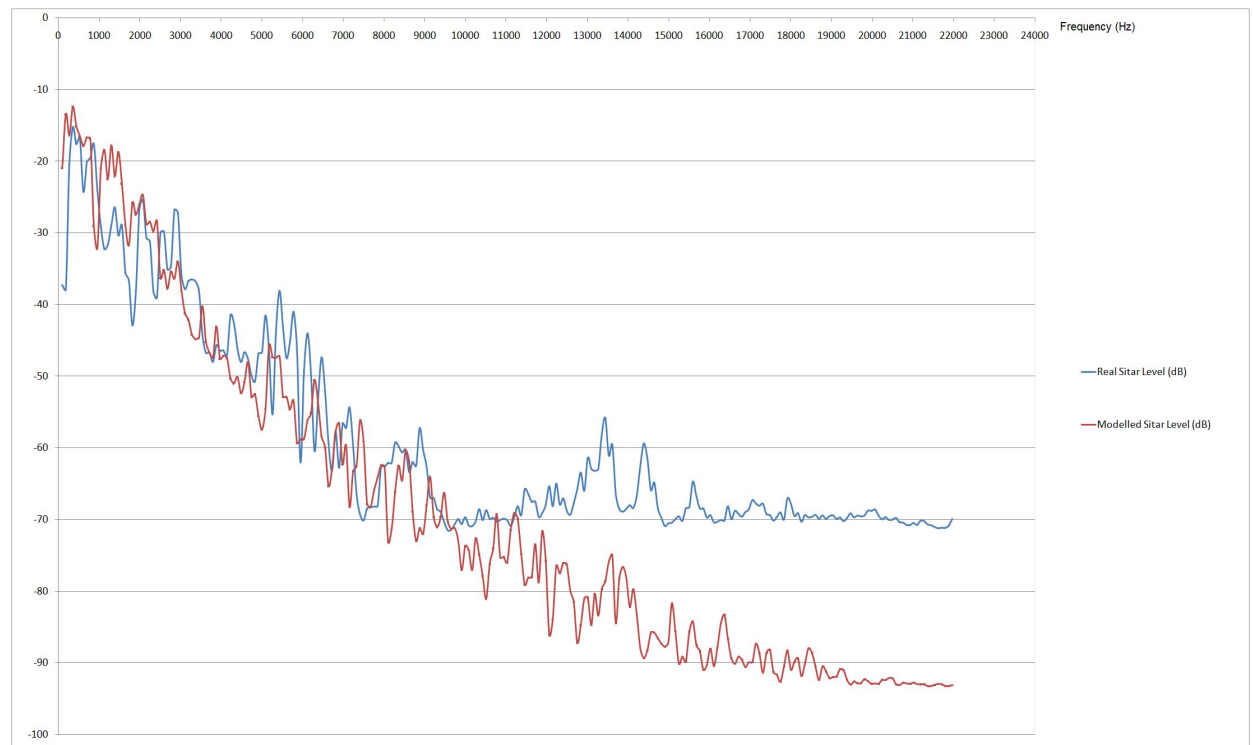


Figure 7.1: First Spectral Analysis. Red = Modelled Sitar, Blue = Real Sitar

could either be due to an incorrect tuning of the body resonances or it could be due to a lack of energy being supplied to sympathetic strings. Before the second recording was made a few adjustments were made to the sitar model. One of the body resonances was slightly changed and also the amount energy being sent to the sympathetic strings was increased slightly. It can be seen straight away in Fig. (7.2) that there is a difference. It seems very similar to the real sitar up until 12,500 Hz and then it begins to taper off, but at the same time the difference between the upper partials isn't as severe. This could possibly be due to the tuning of the sitar's sympathetic strings.

A third recording was made but this time the sitar's sympathetic strings were tuned up a whole octave. It can be seen in Fig. (7.2) the results were a lot more satisfactory. There wasn't as big a difference between the higher partials.

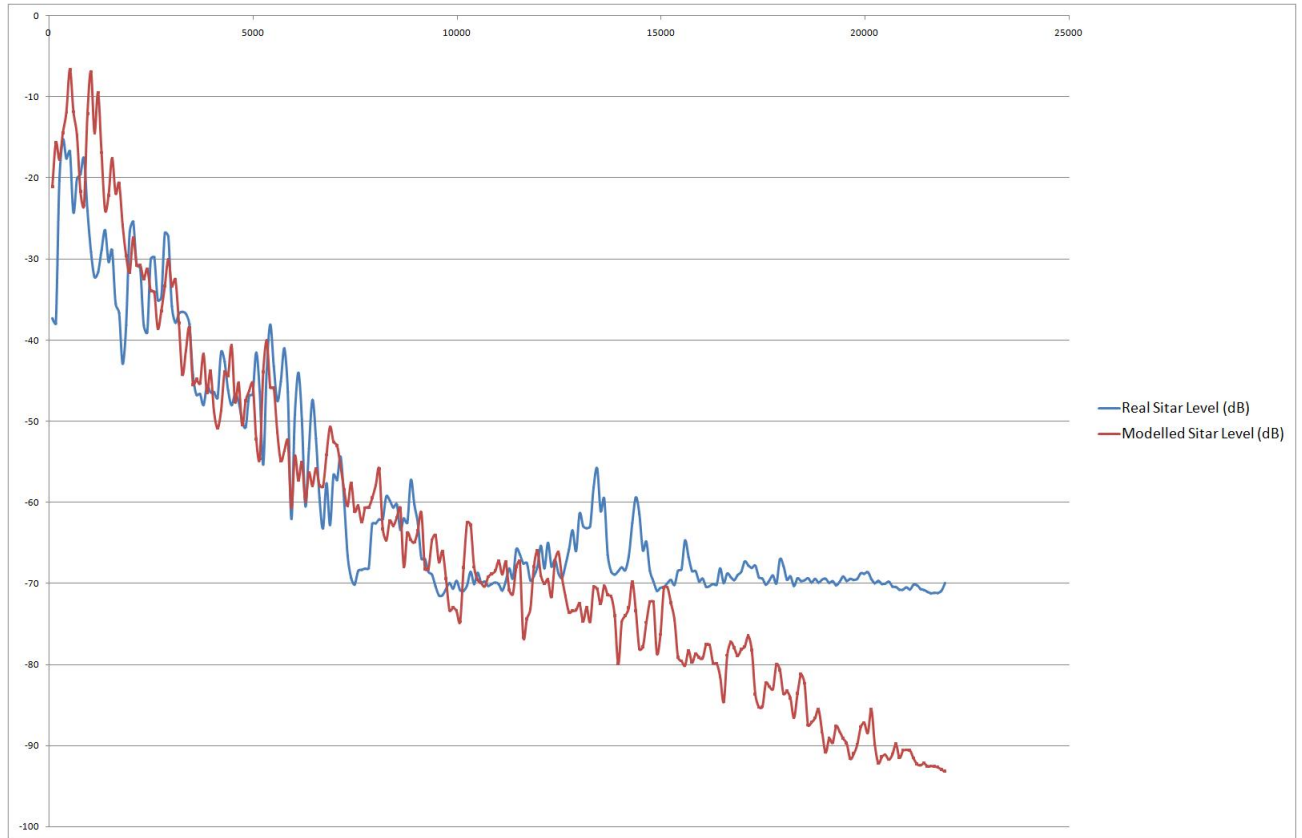


Figure 7.2: Second Spectral Analysis. Red = Modelled Sitar, Blue = Real Sitar

7.1 Efficiency

The model was tested on a MacBook Pro with a 2.26 Ghz Intel Core 2 Duo processor. It was found that the most amount of CPU power that was used was 62%. Considering the amount of different strings that were modelled this is very efficient. In the model you have a seven note polyphony poly~ abstraction and thirteen sympathetic strings all being used at the same time.

Although, when it was tested using string coupling in three different dimensions it would max out the CPU and distortion occurred. The third dimension being the longitudinal

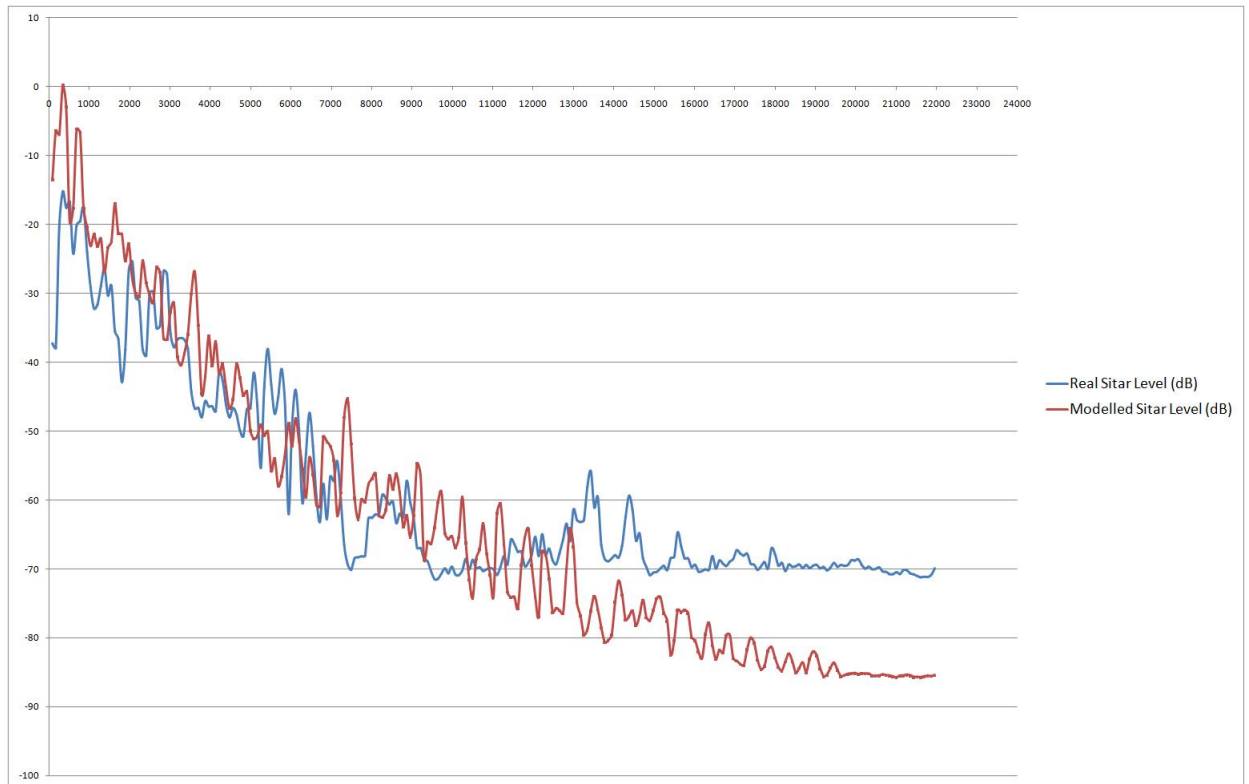


Figure 7.3: Third Spectral Analysis. Red = Modelled Sitar, Blue = Real Sitar

dimension. This third dimension could have been used to create a more realistic tone. The model could have been made even more efficient if the Karplus-Strong algorithm was used for the sympathetic strings although there may have been a loss in the quality of sound.

7.2 Implementation Issues

All of the implementation issues to do with the model had to do with MaxMSP. A lot of them were in relation to the CPU. It would have been more efficient to have implemented this model in C++ as MaxMSP has a lot of its own processes running when you are

using the patch, but time constraints would not allow this.

It was also originally planned to use a guitar with MIDI pickups as the interface for the model but due to the instability of the delay lines in MaxMSP this wasnt feasible. The ability to bend the strings of the MIDI guitar would have been a nice touch to the sitar and would have made it more expressive since there is a lot of string bending in real sitar playing.

The other MaxMSP issue that was encountered had to do with sigvs (Signal Vector Size). The higher the sigvs, the more accurate the high notes would sound. The reader may notice that when using the patch, the notes at the higher end of the kslider sound slightly out of tune. This is due to the fact that the sigvs could only be set to 8. If it is set any smaller it causes the audio to distort due to the CPU being overloaded.

Chapter 8

Conclusion

The goal of this dissertation was to physically model an instrument that hasnt really been developed that much in the physical modelling sense. As mentioned before a lot of the research with regard to physical modelling has been focused on the western classical guitar. The reason why the sitar may have been over looked so much is maybe because of its complex design. As there were a lot more factors to be taken into consideration when it came to modelling this particular instrument.

When this dissertation was originally started it was assumed that the modelling process would be relatively simple and the implementation would take a lot less time than predicted. The reason why the modelling process took so long was because of the non-linear bridge structure. It took a lot of testing and re-evaluation of parameters before the desirable sitar tone was achieved.

During the course of the development of the sitar model, as mentioned before, a new and unique modelling approach was taken with regard to the sitars non-linear bridge struc-

ture. The dynamically changing delay line that had to be implemented as a result of the bridge shape was controlled by the Karplus-Strong algorithm. The Karplus-Strong algorithm being chosen to control this parameter because of how computationally efficient it is and how likened it is to how a real string decays. Immediately after this was implemented the difference in how realistic the sitars timbre became was noticeable. The author believes that this modelling approach warrants further investigation as it has never been implemented before and is a new and innovative approach to this kind of modelling problem.

By looking at the spectral analysis of the sitar versus the real sitar it could be said the model was quite successful. Although, there are still a few bugs in the model, one is to do with regard the tuning particularly at the higher pitches. It would also be nice to implement the ability to pitch bend the notes. This was attempted but it was unsuccessful as it MaxMSP kept distorting.

In future research it would also be very interesting to model the effect of amplitude limitations for the strings at the frets since the sitar has such unique frets. This similar to what to the non-linear distortion that occurs in the slapbass technique.

It would also be interesting to see what the model sounds like if there was a string coupling effect applied to the sympathetic strings. It was due to CPU limitations that this couldn't be achieved.

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