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| line |  | Text |  |
| 1 | T | What are the data? | Soliciting |
| 2 | S | The radius of the base of the cone, the altitude of the cone, the rate at which the water is flowing into the vessel and the depth of water at a certain moment | Responding |
| 3 | T | Correct. The statement of the problem seems to suggest that you should disregard, provisionally, the numerical values, work with the letters, express the unknown in terms of a,b,r,y and only finally, after having obtained the expression of the unknown letters, substitute the numerical values. I would follow this suggestion. Now, what is the unknown? | Responding  Structuring |
| 4 | S | The rate at which the surface is rising when the depth of the water is y | Responding |
| 5 | T | What is that? Could you say it in other terms? | Soliciting |
| 6 | S | The rate at which the depth of the water is increasing | Responding |
| 7 | T | What is that? Could you restate it still differently? | Soliciting |
| 8 | S | The rate of change of the depth of the water | Responding |
| 9 | T | That is right, the rate of change of y. But what is the rate of change? Go back to the definition. | Reacting  Soliciting |
| 10 | S | The derivative is the rate of change of a function | Responding |
| 11 | T | Correct. Now is y a function? As we said before, we disregard the numerical value of y. Can you imagine that y changes? | Reacting  Soliciting  Structuring |
| 12 | S | Yes, y, the depth of the water, increases as the time goes by. | Responding |
| 13 | T | Thus, y is a function of what? | Soliciting |
| 14 | S | Of the time t | Responding |
| 15 | T | Good. Introduce suitable notation. How would you write the ‘rate of change of y’ in mathematical symbols? | Reacting  Structuring  Soliciting |
| 16 | S |  | Responding |
| 17 | T | Good. Thus, this is your unknown. You have to express it in terms of a,b,r,y. By the way one of these data is a ‘rate’. Which one? | Reacting  Structuring  Soliciting |
| 18 | S | r is the rate at which water is flowing into the vessel | Responding |
| 19 | T | What is that? Could you say it in other terms? | Reacting  Soliciting |
| 20 | S | r is the rate of change of the volume of the water in the vessel | Responding |
| 21 | T | What is that? Could you restate it still differently? How would you write it in suitable notation? | Reacting  Soliciting  (structuring) |
| 22 | S |  | Responding |
| 23 | T | What is V? | Soliciting |
| 24 | S | The volume of the water in the vessel at the time t | Responding |
| 25 | T | Good. Thus, you have to express in terms of a,b,, y. How will you do it? | Reacting  Structuring  Soliciting |
| 26 | T | If you cannot solve the proposed problem try to solve first some related problem. If you do not see yet the connection between and the data, try to bring in some simpler connection that could serve as a stepping stone | Structuring |
| 27 | T | Do you not see that there are other connections? For instance, are y and V independent of each other? | Structuring  Soliciting |
| 28 | S | No. When y increases, V must increase too. | Responding |
| 29 | T | Thus, there is a connection. What is the connection? | Reacting  Soliciting |
| 30 | S | Well, V is the volume of a cone of which the altitude is y. But I do not know yet the radius of the base. | Responding  Reacting |
| 31 | T | You may consider it, nevertheless. Call it something, say x | Structuring |
| 32 | S |  | Responding |
| 33 | T | Correct. Now, what about x? Is it independent of y? | Reacting  Structuring  Soliciting |
| 34 | S | No. When the depth of the water, y, increases the radius of the free surface, x, increases too. | Responding |
| 35 | T | Thus, there is a connection. What is the connection? | Reacting  Soliciting |
| 36 | S | Of course, similar triangles. | Reacting  Responding |
| 37 | T | One more connection, you see. I would not miss profiting from it. Do not forget, you wished to know the connection between V and y | Structuring |
| 38 | S | I have | Responding |
| 39 | T | Very good. This looks like a stepping stone, does it not? But you should not forget your goal. What is the unknown? | Reacting  Structuring  Soliciting |
| 40 | S | Well, . | Responding |
| 41 | T | You have to find a connection between , and other quantities. And here you have one between y, V and other quantities. What to do? | Structuring |
| 42 | S | Differentiate! Of course!  Here it is. | Responding  Reacting |
| 43 | T | Fine! And what about the numerical values? | Reacting  Soliciting |
| 44 | S | If then | Res ponding |

|  |  |  |  |
| --- | --- | --- | --- |
| line |  | Text |  |
| 1 | T | “Here it is. First, I just want you to watch this very brief video.” | Structuring |
| 2 |  |  |  |
| 3 | T | “Would you go ahead and write down the first question that comes to your mind, if any? No question? That’s perfectly fine.” | Soliciting |
| 4 |  |  |  |
| 5 | T | “Would you introduce yourself to your neighbor and share your question? See if it’s the same question, or a different question.” | Soliciting |
| 6 |  |  |  |
| 7 | T | I’m really curious what questions are out there. Just toss one out. Who else finds that question interesting?” | Soliciting |
| 8 |  |  |  |
| 9 | T | “I like that you coined a vocabulary term there for us. ‘Layers.'” | Reacting |
| 10 |  |  |  |
| 11 | T | “I would love to get to all these questions but given limited time we’ll start with these ones up here.” | Structuring |
| 12 | S |  |  |
| 13 | T | * “I want you to write down on a piece of paper your best, gut-level guess for how many coins there are. I’m curious who can guess the closest.” | Soliciting |
| 14 | S |  |  |
| 15 | T | “Would you also write down a number you know is too high – there couldn’t possibly be that many pennies – and a number you know is too low – there couldn’t possibly be that few pennies. Share them with your neighbor.” | Soliciting |
| 16 | S |  |  |
| 17 | T | “I’m very curious in here who has our highest guess. “ | Soliciting |
| 18 |  |  |  |
| 19 | T | “What’s our lowest guess in here? | Soliciting |
| 20 |  |  |  |
| 21 | T | “What information do you need from me? What information will be necessary here?” | Structuring |
| 22 |  |  |  |
| 23 | T | “I want to go ahead and capitalize ‘stack’ here. Does everybody know what stack means? Tell me how stacks and layers are related.” | Soliciting |
| 24 |  |  |  |
| 25 | T | “Are all the stacks the same?” | Soliciting |
| 26 | T | “Did you use all the same coins?” | Soliciting |
| 27 | T | * “I’m gonna add a question to the list here: ‘Why 13?'” | Structuring |
| 28 |  |  |  |
| 29 | T | “How many on the base layer do you think? | Soliciting |
| 30 |  |  |  |
| 31 | T | “So what’s on the next level up? 38 by 38? 39 by 39? What am I looking for if it’s 38 by 38?” | Structuring |
| 32 |  |  |  |
| 33 | T | “That’s everything you said you needed. You asked for this info because you had some kind of fuzzy plan in your head. Might not have been a perfect plan. But you had some need for this information. So I want to see you put that information into play somehow.” | Structuring |
| 35 | T | “This guy wants to make a pyramid out of a billion pennies. And I’m curious how big that would be. Help me with that if you’re completely finished here. Or tackle some of the other questions we had up there earlier.” | Structuring |
| 36 | S |  |  |
| 37 | T | “Is that number in between your high and low from earlier? Does it fit in the range of possible numbers for you? If it didn’t we should go back and ask ourselves ‘do we trust the mathematics here?'” |  |
| 38 |  |  |  |
| 39 | T | “I’m going to show you the answer here.” |  |
| 40 |  |  |  |
| 41 |  | “Who guessed closest to that? Margaret or Eddie. Let’s all give one clap to Eddie.” |  |
| 42 |  |  |  |
| 43 |  | “Who got the closest guess overall? Who is closer? 250,000 or 300,000? One clap for these two.” |  |
| 44 |  |  |  |
|  |  | “Let’s look at other questions we had back here.” |  |
|  |  | “How could we figure out how long it would take?” |  |