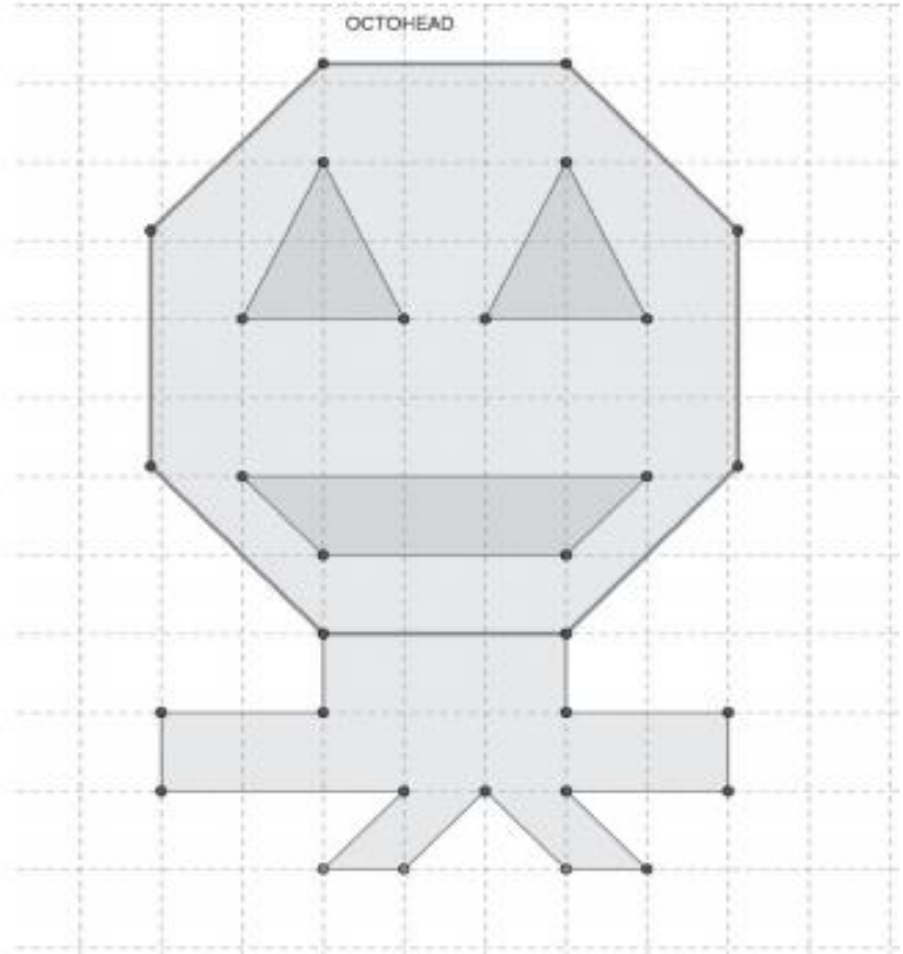
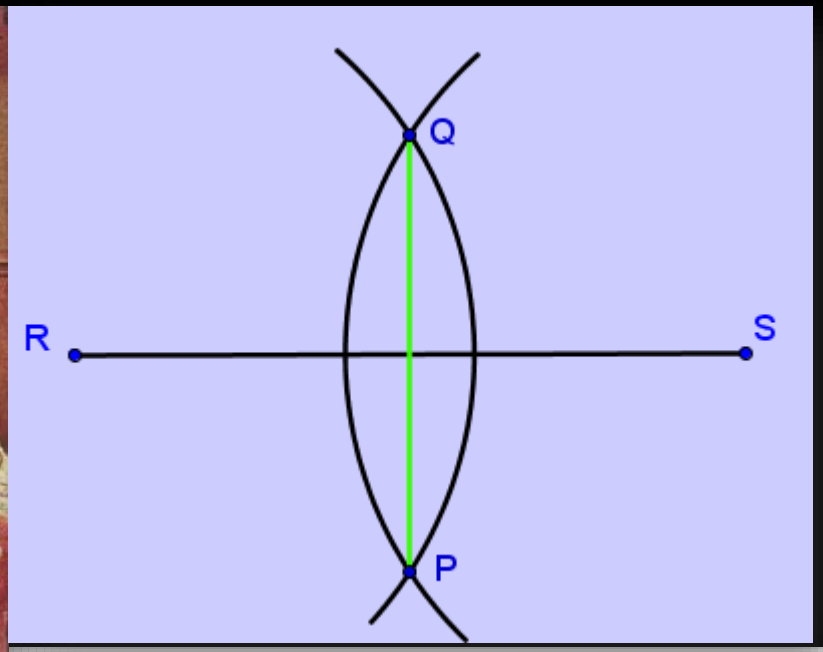
Proportional Reasoning: 'twice as big'



Literacy & Numeracy for Geometry Teaching in Ireland
Workshop 2

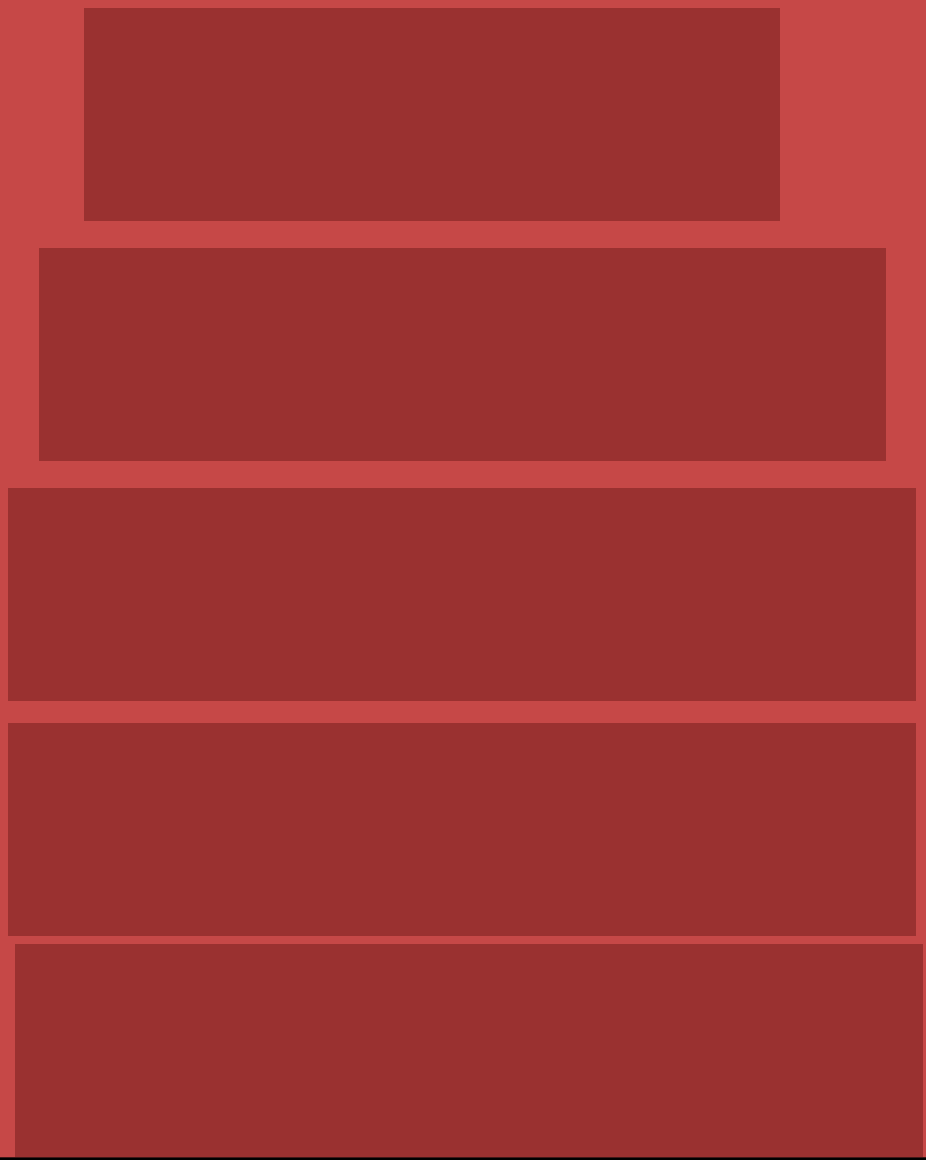


Bernie O'Donoghue

Learning Intentions

- Recap
- MQI: Enacting L&N for mathematics teaching
- L&N in Geometry Definitions
- MQI: Enacting L&N for Ratio and Similarity theorems in Geometry
- Problem Solving – knowledge first

Mapping Numeracy to Mathematics



Think and communicate quantitatively

Make sense of data

Have spatial awareness

Understand patterns and sequences

Solve problems

Mapping Problem Solving to Literacy and Numeracy

communicate mathematics verbally and in written form

explain findings

justify conclusions

explore patterns and formulate conjectures

apply their knowledge and skills to solve problems in familiar and unfamiliar contexts

analyse information presented verbally and translate it into mathematical form

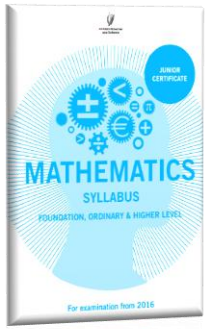
devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

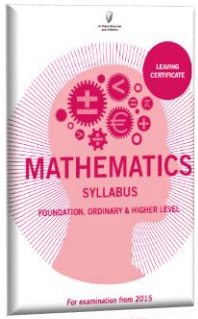
Mapping JC/LC Objectives to Numeracy Model

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, justification and communication
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

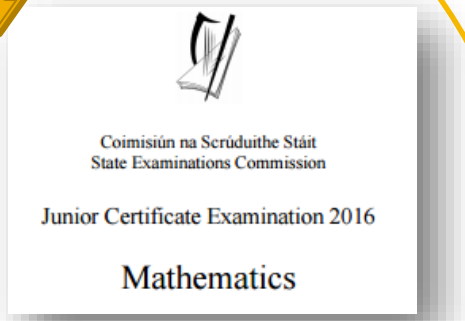
Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings.
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.

Improve literacy and numeracy standards in our young people in the mathematics classroom





MQI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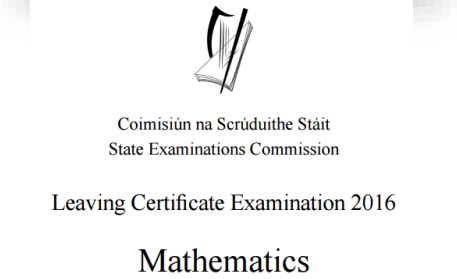
State Examinations Commission

Junior Certificate Examination 2016

Mathematics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State Examinations Commission

Leaving Certificate Examination 2016

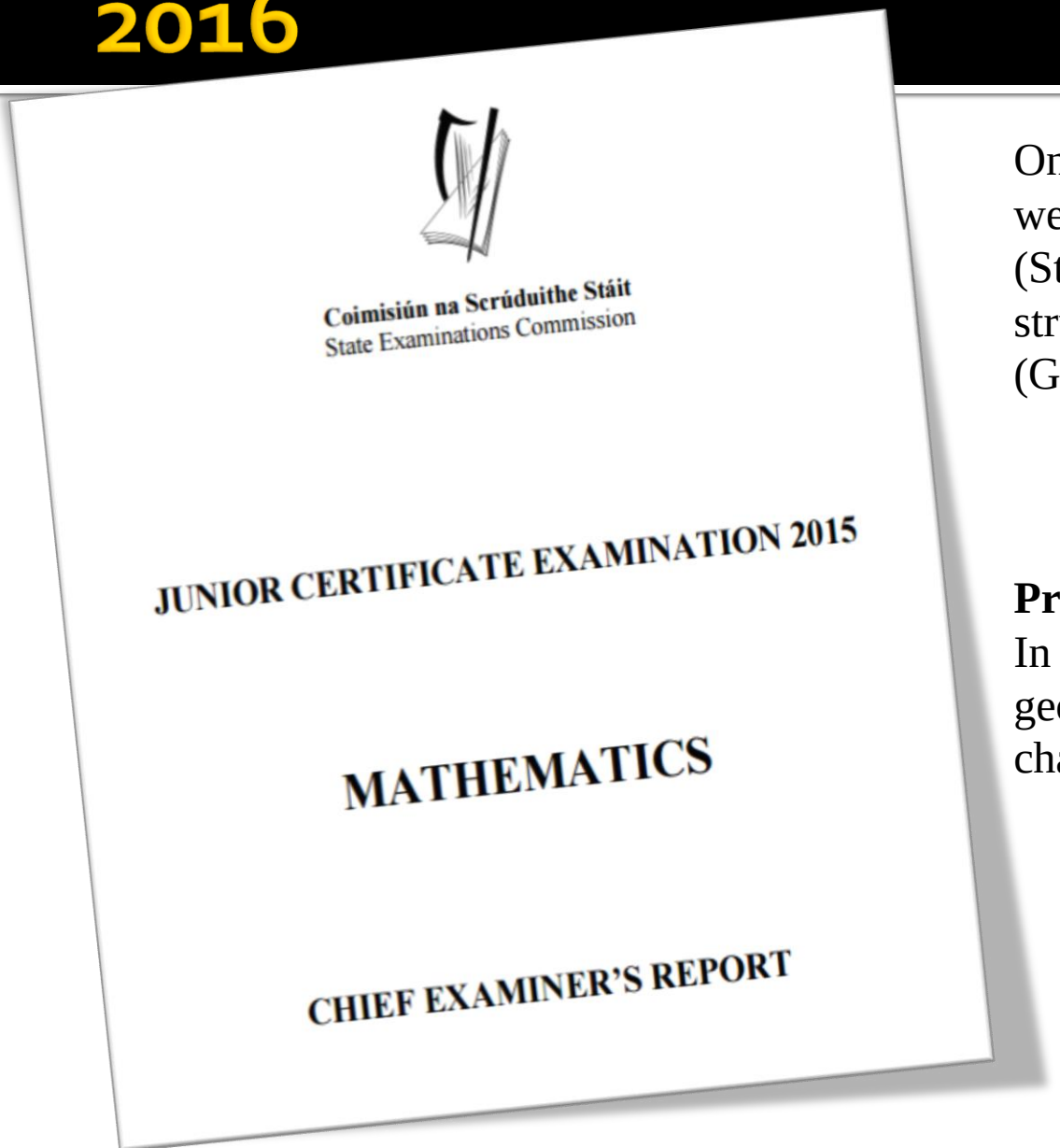
Mathematics

Mathematical Quality of Instruction

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language
- Mathematical Sense Making
- Explanations
- Multiple Procedures or Solution Methods
- Remediation of Student Errors and Difficulties
- Teacher uses student mathematical contributions

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language

Chief Examiner's Report published 2016


Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION 2015

MATHEMATICS

CHIEF EXAMINER'S REPORT

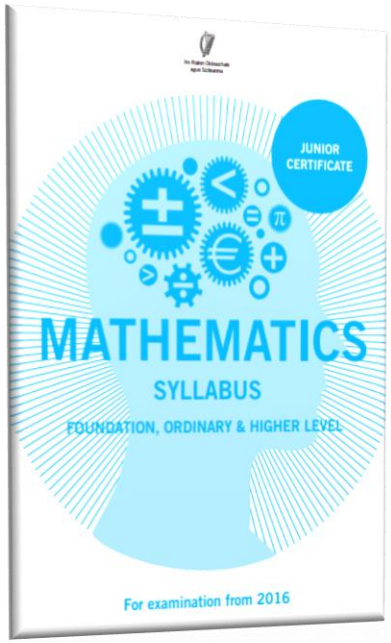
On Paper 2, Candidates performed very well in questions involving Strand 1 (Statistics and Probability), but struggled with topics from Strand 2 (Geometry and Trigonometry) p.13

Procedural Fluency
In Paper 2, candidates continue to find geometry and trigonometry challenging, p.21

Adaptive Reasoning

- As with other syllabus objectives, candidates struggled with adaptive reasoning relating to geometry. Candidates had difficulty proving a geometric cut, *viz*. that if the diagonal of a parallelogram bisects the angle, then the four sides of the parallelogram must be equal in length (Paper 2, Question 11(b)). P.29

Geometry and Trigonometry learner outcomes

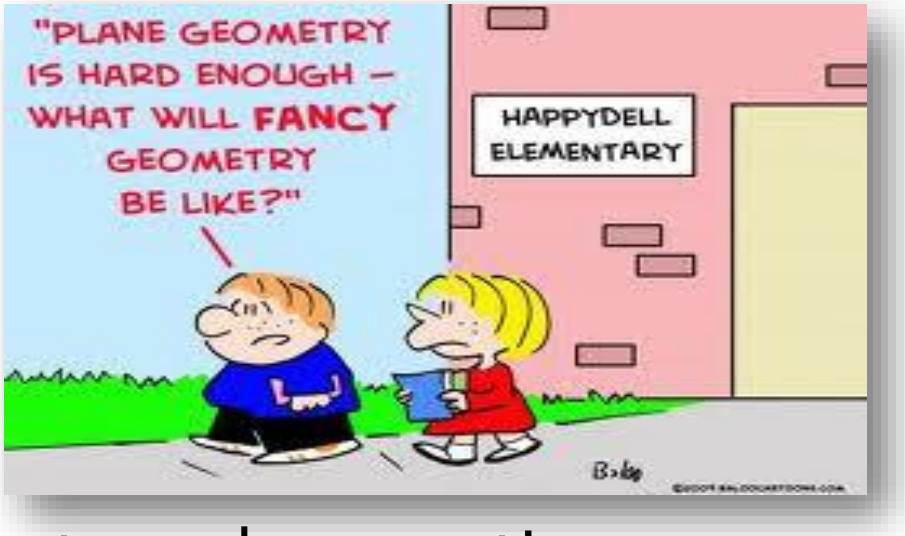


- recall basic facts related to geometry and trigonometry
- construct a variety of geometric shapes and establish their specific properties or characteristics
- solve geometrical problems and in some cases present logical proofs
- interpret information presented in graphical and pictorial form analyse and process information presented in unfamiliar contexts
- select appropriate formulae and techniques to solve problems.

Plane Geometry: 2 dimensions

Plane Geometry is about:

planus = latin for flat, level; geometry=measure of earth or land)



- (1) objects and properties
- (2) Proof

Quiz on Conceptual/Mathematical knowledge

- http://www.gradeamathhelp.com/support-files/free-geometry-worksheets-unit1.pdf

Quiz: Conceptual/Mathematical Knowledge

. The most basic figure in geometry: .

Quiz: Conceptual/Mathematical Knowledge

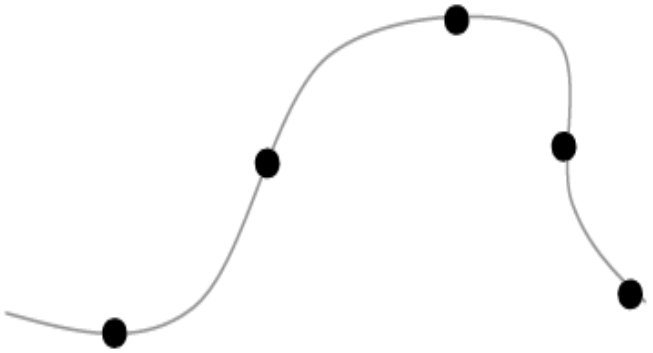
. The most basic figure in geometry: • Point

a. It is represented by a dot, but it really has no Size/length/width

b. Points are named with Capital letters! Example: •

c. **Every** geometric figure is made up of points!

d. Two different types of arrangements of points (on a piece of paper).

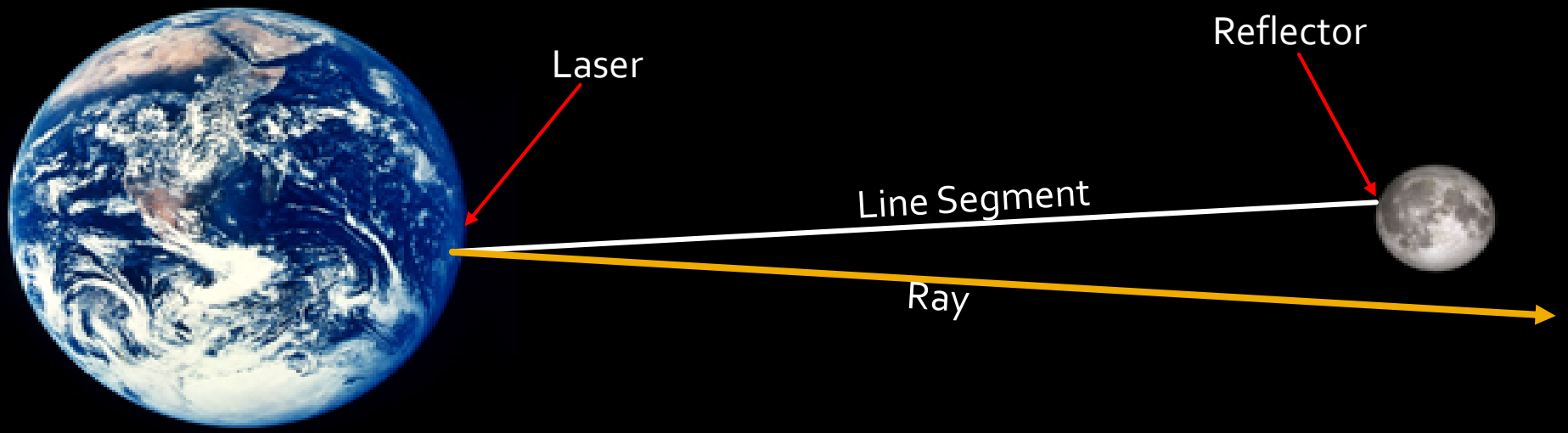


e. A group of points that "line up" are called Collinear points.

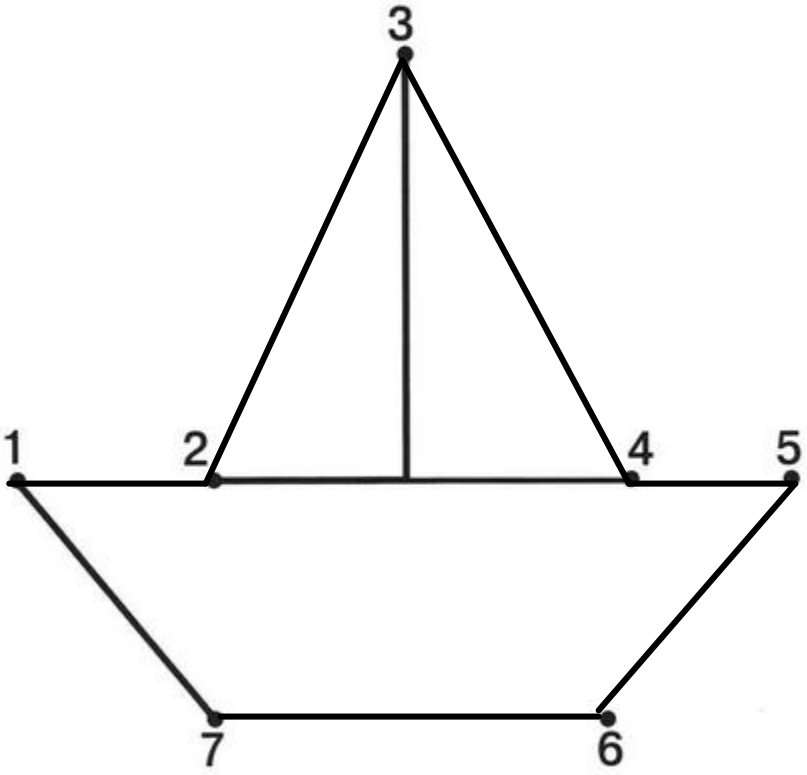
Definition: Line Segment

Definition 1. The line **segment** $[AB]$ is the part of the line AB between A and B (including the endpoints). The point A divides the line AB into two pieces, called **rays**. The point A lies between all points of one ray and all

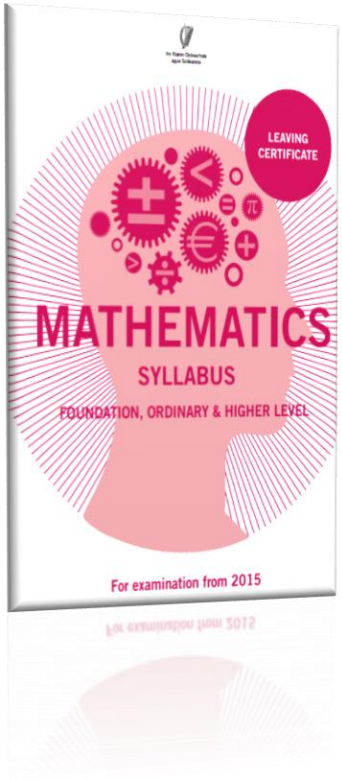
Context: Line Segment



Literacy
Line Segment & Polygons



Leaving Certificate



- The synthetic geometry covered at Leaving Certificate is a continuation of that studied at junior cycle. It is based on the Geometry for Post-primary School Mathematics, including terms, definitions, axioms, propositions, theorems, converses and corollaries.

Literacy: Geometrical Terms

- **Definition**

An accurate description of a particular term

- **Axiom**

Statement we accept as true without proof

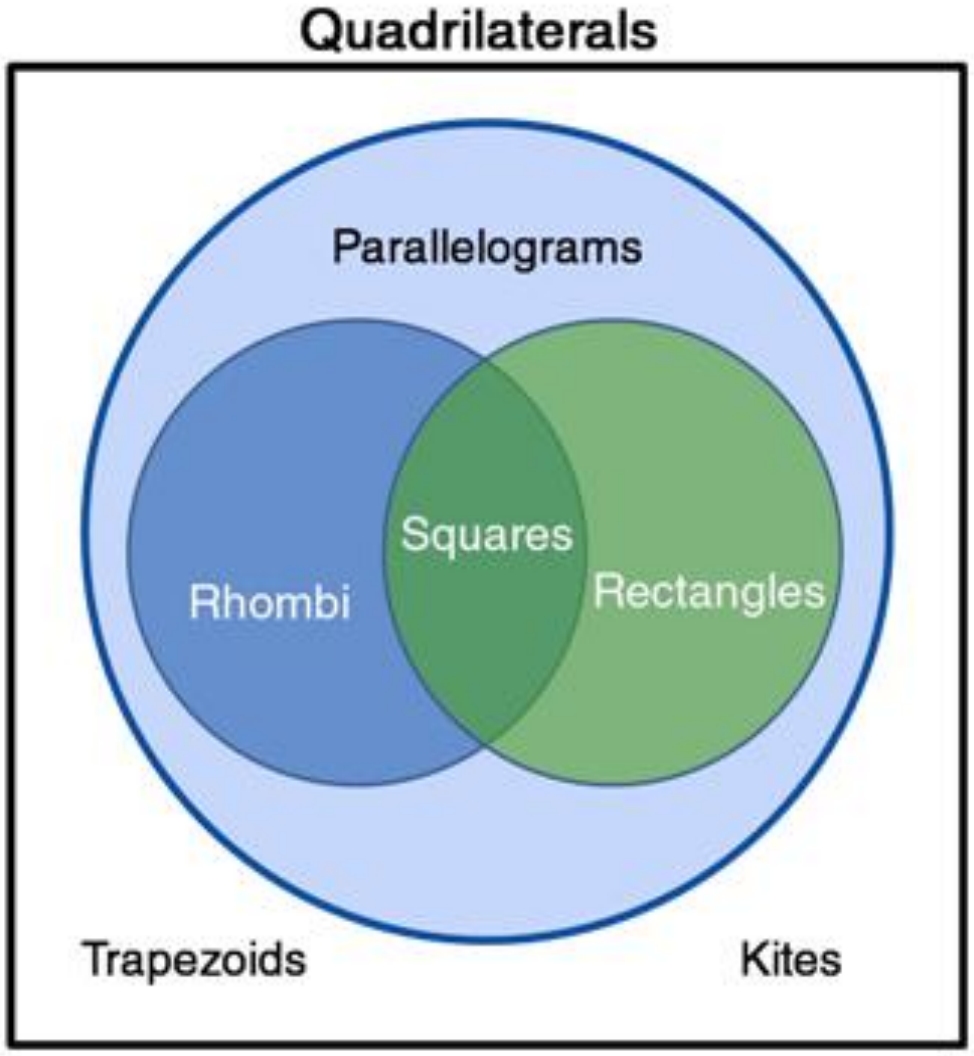
- **Proposition**

A useful statement that could be proved but where proof is not necessary

- **Theorem**

A statement we prove by logical argument

Classifying Quadrilaterals



Definitions

Rectangle

- A quadrilateral having right angles at all four vertices

Rhombus

- A quadrilateral having all four sides equal

Parallelogram

- Is a quadrilateral for which both pairs of opposite sides are parallel

MQI: Mathematical Language

$l_1, l_2, l_3,$ are parallel lines that make intercepts of equal length on the transversal k.

Theorem 11

A walk in Melbourne

How far is it from Elisabeth St to Russell St?

Patterns and Generalisations Theorem/Construction Sheet

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	
1	Bisector of an angle, using only compass and straight edge.	
2	Perpendicular bisector of a segment, using only compass and straight edge.	
3	Line perpendicular to a given line l, passing through a given point not on l.	
4	Line perpendicular to a given line l, passing through a given point on l.	
5	Line parallel to given line, through a given point.	
6	Division of a line segment into 2 or 3 equal segments without measuring it.	
7	Division of a line segment into any number of equal segments, without measuring it.	
8	Line segment of a given length on a given ray.	
9	Angle of a given number of degrees with a given ray as one arm.	
10	Triangle, given lengths of 3 sides.	
11	Triangle, given SAS data.	
12	Triangle, given ASA data	
13	Right-angled triangle, given length of hypotenuse and one other side	
14	Right-angled triangle, given one side and one of the acute angles.	
15	Rectangle given side lengths.	
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.	
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.	
18	Angle of 60° without using a protractor or set square.	
19	Tangent to a given circle at a given point on it.	
20	Parallelogram, given the length of the sides and the measure of the angles.	
21	Centroid of a triangle.	
22	Orthocentre of a triangle.	

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL only ** proof required for LCHL only ♦ These results are required as background knowledge for constructions and/or applications of trigonometry.	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	✓	✓	✓	✓♦	✓	✓
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	✓	✓	✓	✓♦	✓	✓
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	✓	✓	✓	✓♦	✓	✓
1	Vertically opposite angles are equal in measure.	✓	✓	✓		✓	✓
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	✓	✓	✓		✓	✓
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	✓	✓	✓	✓♦	✓	✓
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	✓	✓	✓	✓♦	✓	✓
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.	✓	✓	✓		✓	✓
4*	The angles in any triangle add to 180^0.	✓	✓	✓	✓♦	✓	✓
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	✓	✓	✓		✓	✓
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	✓	✓	✓		✓	✓
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					✓	✓
8	Two sides of a triangle are together greater than the third.					✓	✓
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		✓	✓		✓	✓
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			✓			✓
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		✓	✓		✓	✓

Project Maths Tionscadal Mata Development Team

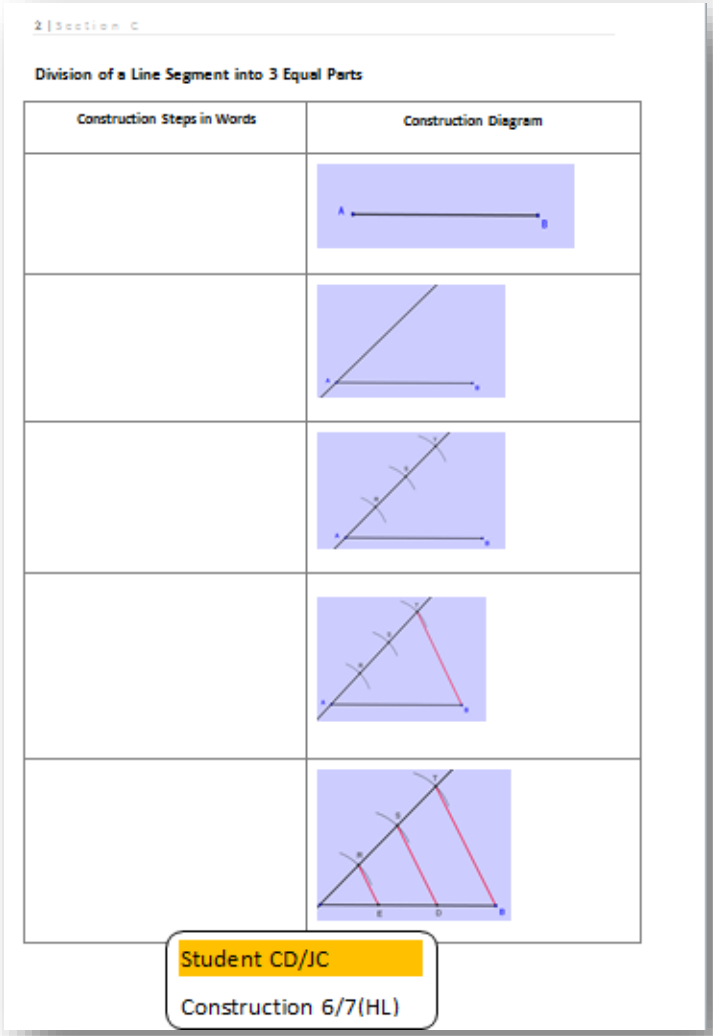
Geometry Quiz

Read the Rules of the Game

Tricky Triangles	Theorems	Symmetry	Coordinate Geometry	Trigonometry	Super Bonus Round
10 Points	10 Points	10 Points	10 Points	10 Points	100 A
20 Points	20 Points	20 Points	20 Points	20 Points	100 B
30 Points	30 Points	30 Points	30 Points	30 Points	100 C
40 Points	40 Points	40 Points	40 Points	40 Points	100 D
50 Points	50 Points	50 Points	50 Points	50 Points	100 E

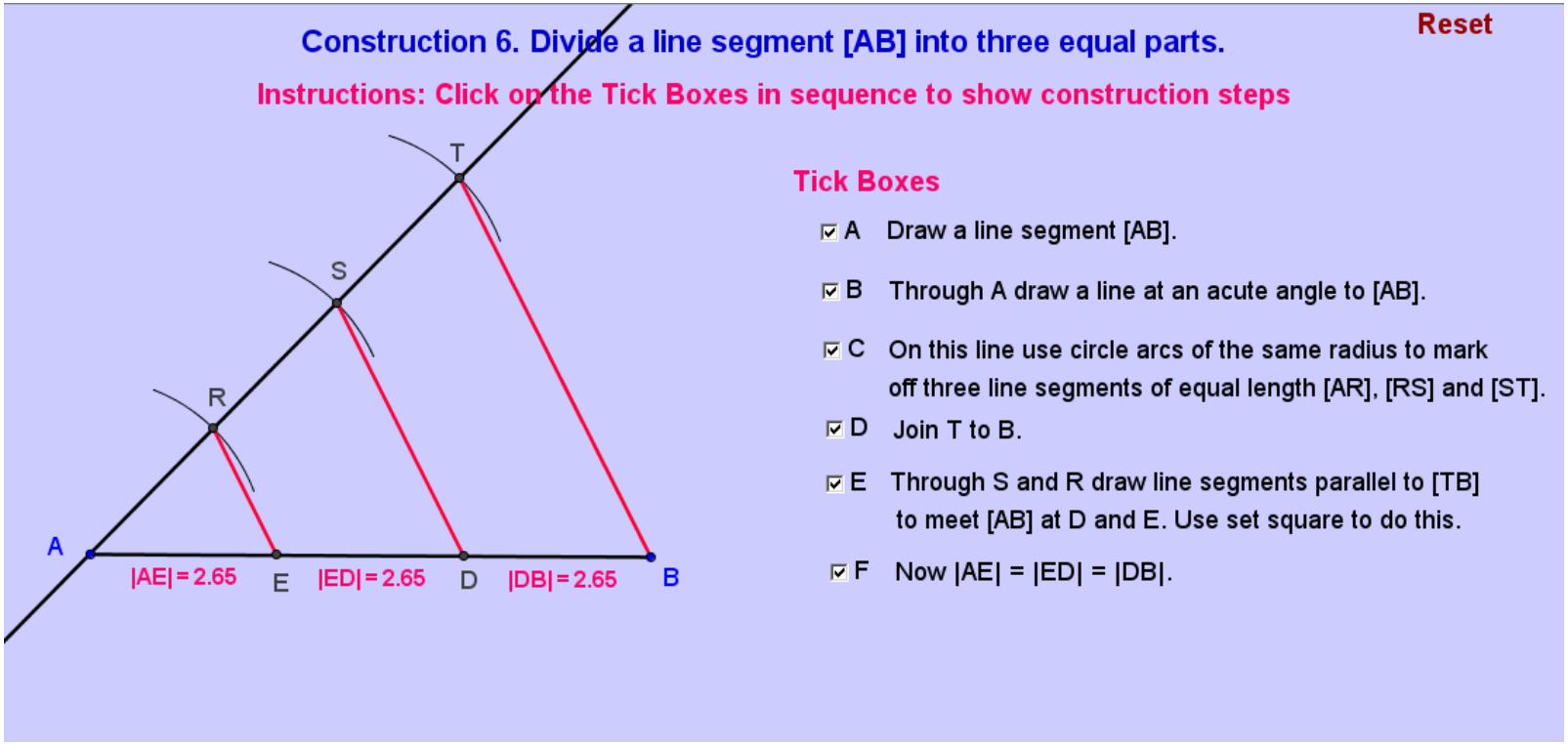
MQI: Linking Representations

- Line Segment
- $[AB]$
- Line
- Circular Arcs/Radius length
- Parallel

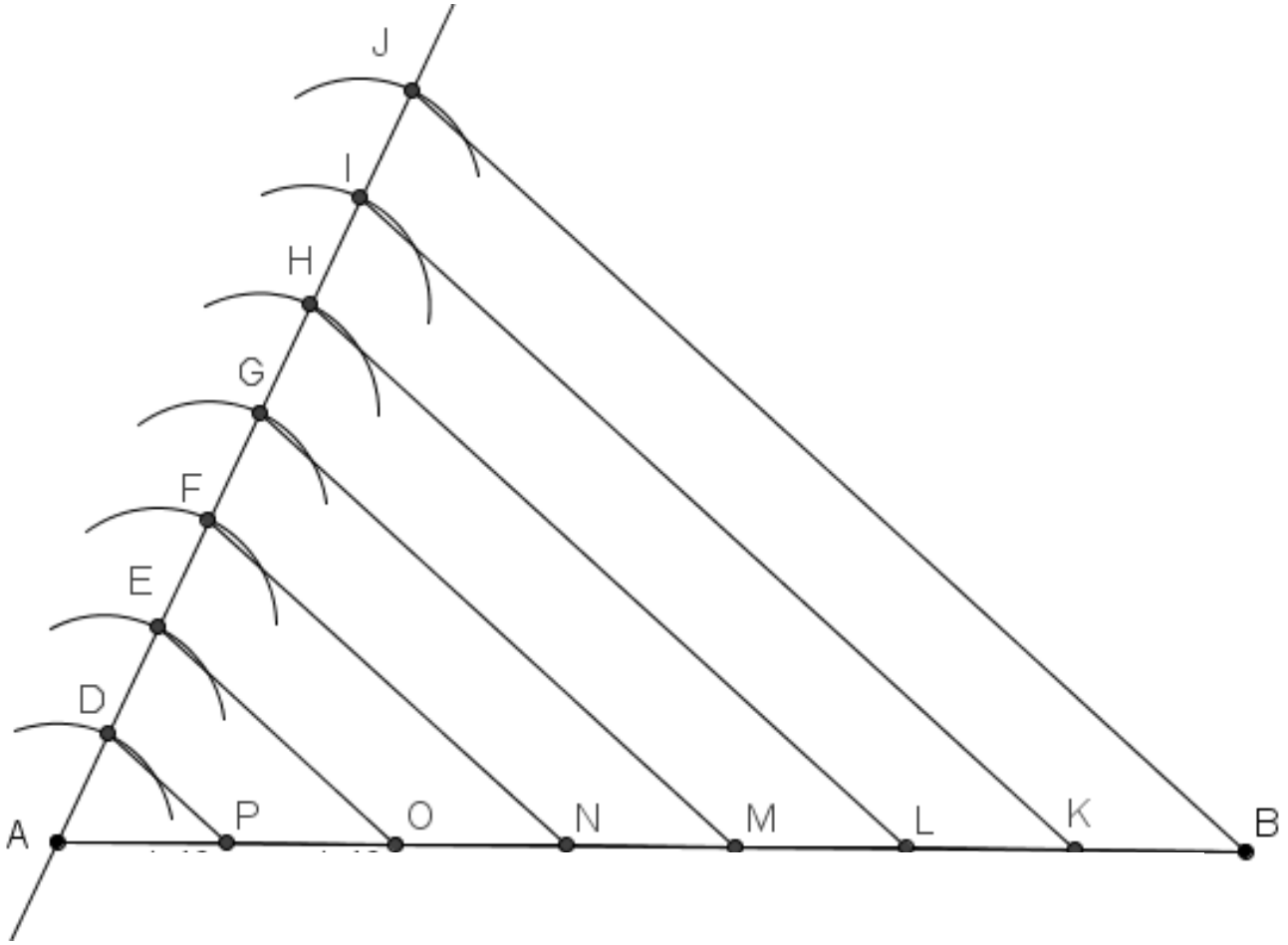


Visualising Rational Numbers

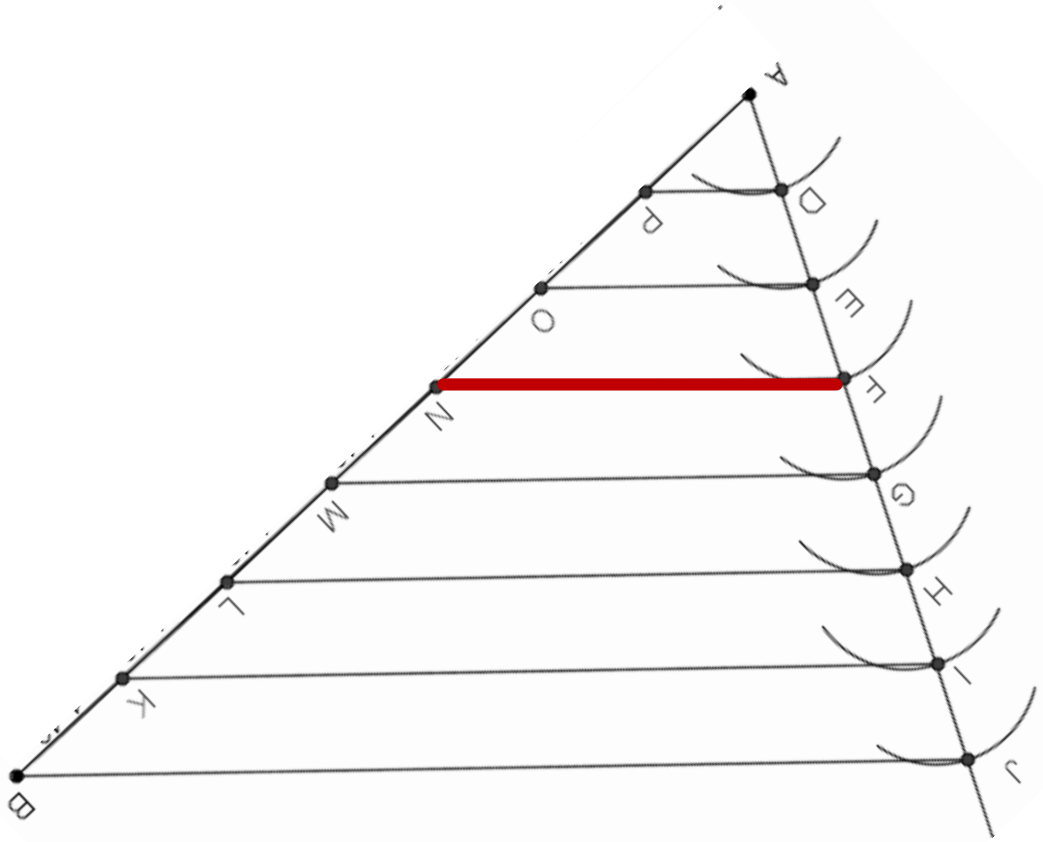
Linking Representations



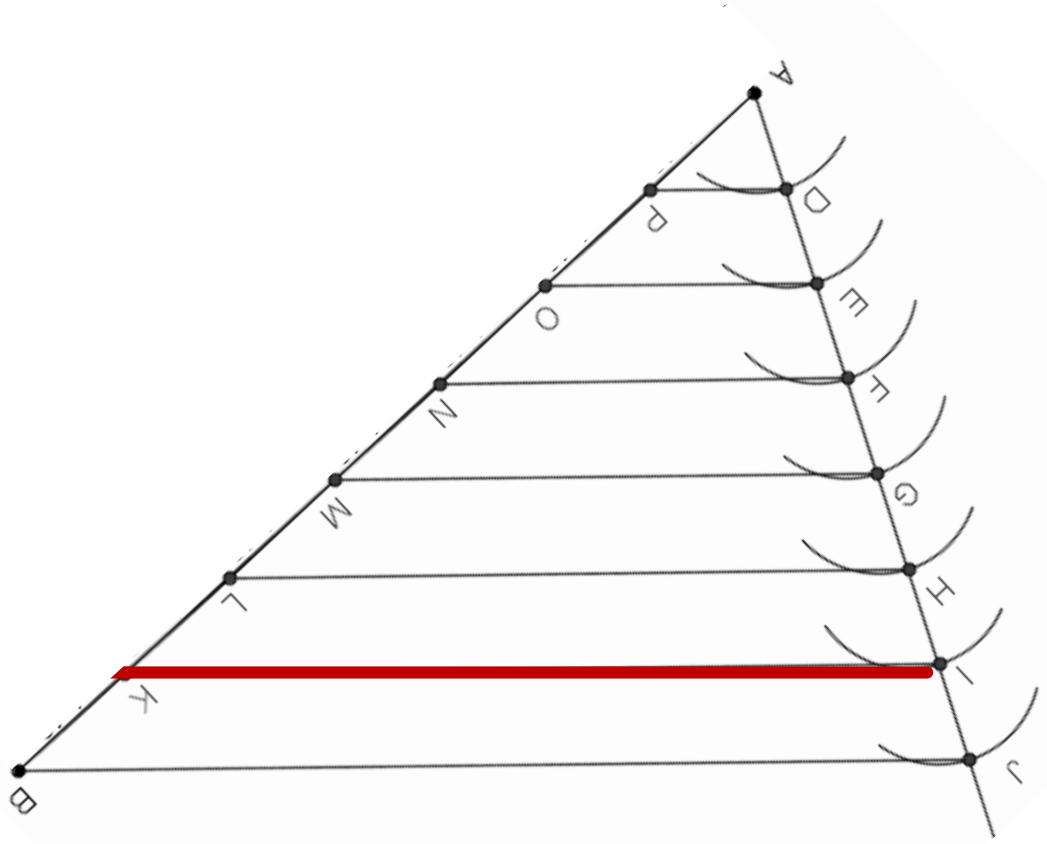
Patterns and Generalisations



Patterns and Generalisations

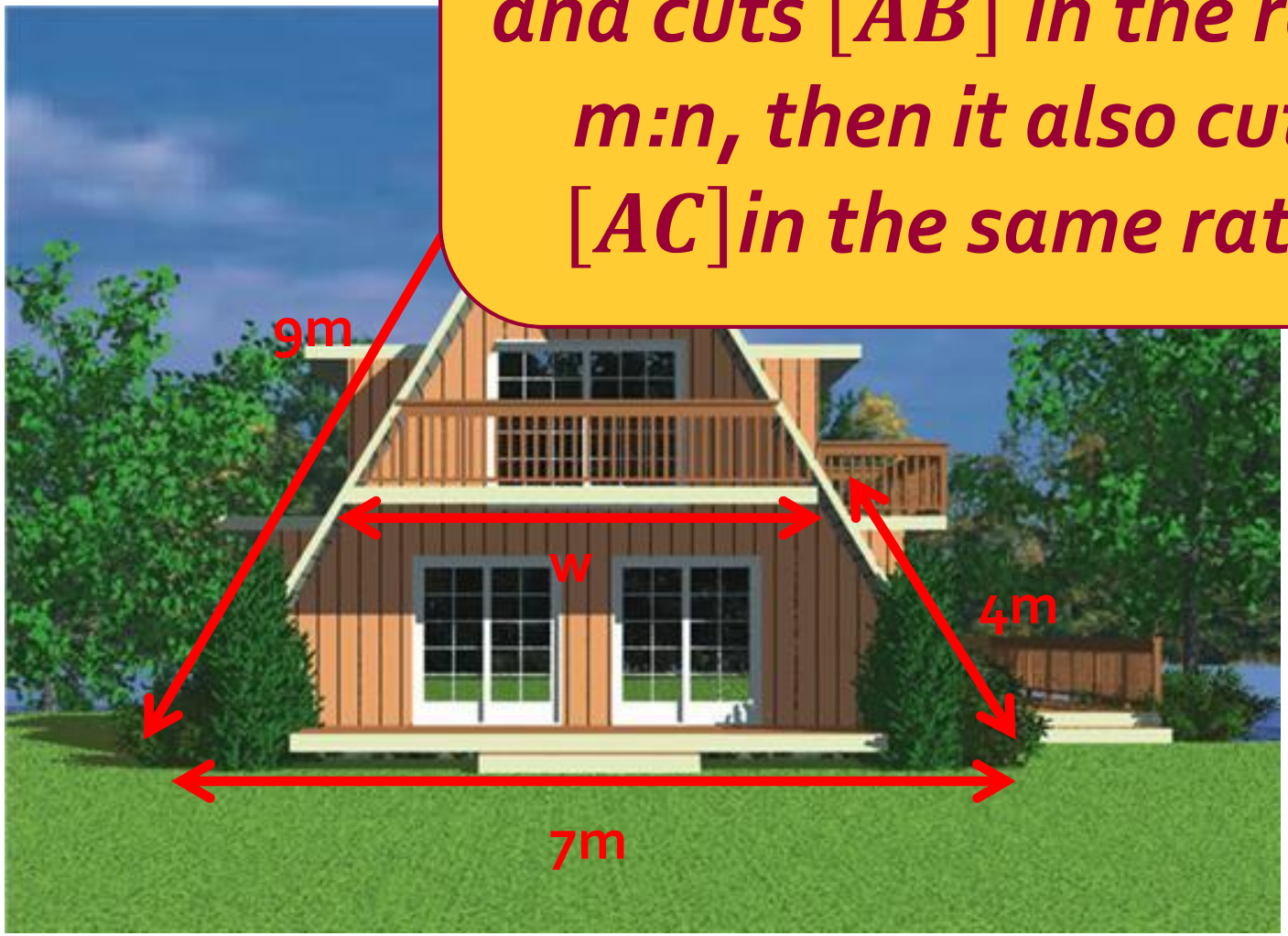


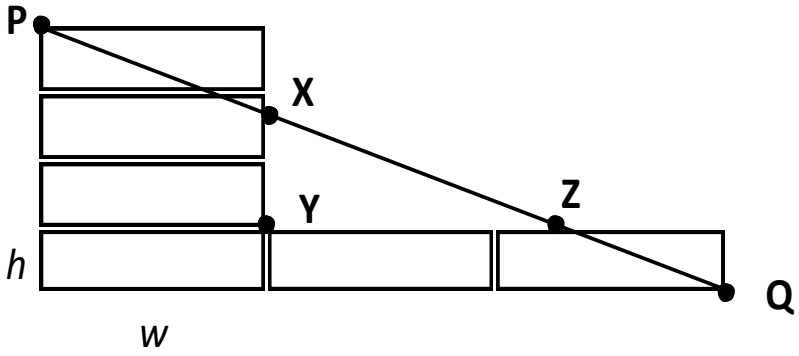
Patterns and Generalisations



"A-frame" house

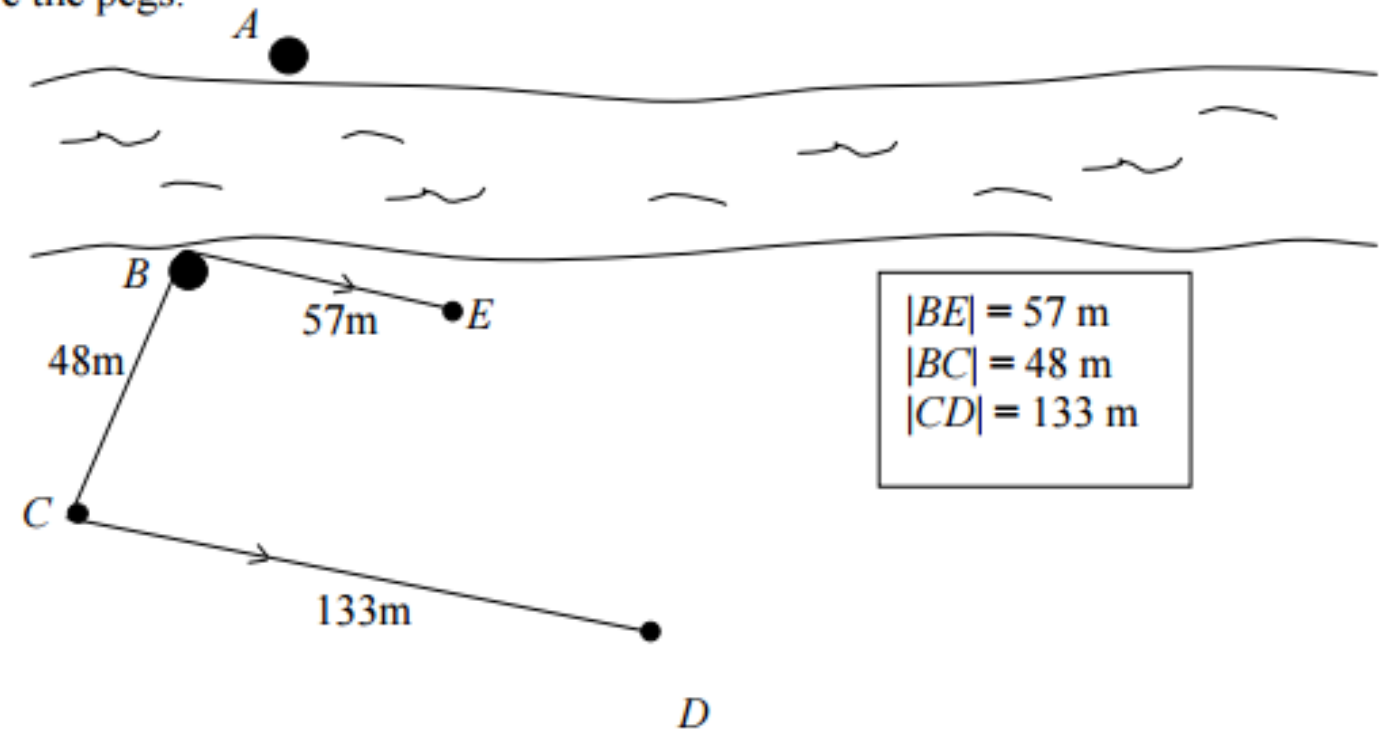
Let ABC be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio m:n, then it also cuts $[AC]$ in the same ratio




P
X
Y
Z
Q
h
w

Strategic Competence

A group of students were trying to find the distance between two trees on opposite sides of a river using pegs, a measuring tape and a large amount of string. They align the pegs in a particular way, take several measurements and sketch this diagram. On the diagram, A and B are the trees and C, D and E are the pegs.



$|BE| = 57$ m
$|BC| = 48$ m
$|CD| = 133$ m

Evaluation