



From policy to practice: an investigation into pre-service teachers' preparedness for implementing literacy and numeracy for mathematics teaching in Ireland

Bernadette O'Donoghue

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UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

DEPARTMENT OF MATHEMATICS AND STATISTICS

From Policy to Practice: An Investigation into Pre-Service Teachers' Preparedness for Implementing Literacy and Numeracy for Mathematics Teaching in Ireland

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For the award of Doctor of Philosophy

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Doras Feasa Fiafraí

(The door to knowledge is to question)

Abstract

This study examines the readiness of pre-service teachers of mathematics to implement reforms in post-primary education and mathematics education, coined by the author as Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI). A definition of LNMTI was developed coupled with an assessment framework to guide the production of a LNMTI survey aimed at examining the proficiency of pre-service teachers of mathematics in this domain of learning. Findings indicate the participants were underprepared for the demands of current reforms in mathematics education in Ireland, despite holding a tertiary qualification in mathematics. Results also suggest difficulties exist in participants' conception and comprehension of numeracy for mathematics teaching, raising issues about the provision of numeracy education for pre-service teachers of mathematics.

Additionally, the findings from the survey and the LNMTI framework guided the design and the development of a LNMTI module in order to investigate whether the knowledge gaps identified could be addressed. A classroom observation sheet for university school placement tutors was also developed as a post-intervention assessment instrument for LNMTI. Findings indicate that both the module and the classroom observation instrument have the potential to provide solution based responses to the problem of pre-service teacher of mathematics readiness for current educational systemic changes in Ireland. The study's contribution to theory is in the development of design principles for translating educational policy into practice.

Author's Declaration

I certify that this thesis is entirely my own work other than the counsel of my supervisors and it has not been submitted for any academic award or part thereof at this or any other educational institution. Where use has been made of the work of other people, it has been acknowledged appropriately and fully referenced.

Name: Bernadette O'Donoghue

Signature:

Date:

Dedication

To my husband, Stephen, and three girls, Ava, Olwyn and Ruth, who always understood; my mom, Beryl, who always believed in the enduring value of education; my sisters, Marguerite and Melody, my brother, Ray and to the memory of my father Ray, who always believed in me;

and

To Prof. John O'Donoghue for making this possible for me and for so many other mathematics education researchers working in Ireland and abroad.

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In my thinking and exploration of literacy and numeracy for mathematics teaching, I had to undergo a process of becoming literate and numerate in the ways of an academic researcher. As mentors of my learning, I want to thank Prof. John O'Donoghue for his guidance, direction and relentless positivity, Dr Paddy Johnson for pushing me to justify and be critical and Dr Máire Ní Ríordáin for enabling me to structure and synthesise my messy thoughts. This was a very difficult journey for me and the reality that I am writing these words as a conclusion to the Ph.D. process is a tribute to their advocacy, patience and kindness.

To EPI*STEM for the financial assistance that not only gave me the opportunity to pursue this study but established a research base where brilliant and supportive people listened to my whining and whinging during the many troubled spots I encountered in my research journey. Thanks for all the academic discussions, the administrative support and the lunch-time laughs! In particular, I would like to acknowledge Prof. Marilyn Goos, Prof. Sibel Erduran and Helen Fitzgerald.

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Chapter 1: Setting the Context of the Study

This chapter begins by establishing the importance of this topic to the author and the education community in Ireland during a period where the educational landscape is fast evolving to accommodate an expanding and diverse cultural, economic and political context. These current developments are situated in the context of historical, national and international factors. It then provides a background to the specifics of education provision in Ireland for students aged 12-18. As a result of the organisational structures inherent in the Irish education system, the relevance of Bernstein's (1990) theory of Social Pedagogic Discourse to the study is outlined. The chapter concludes by outlining the research aims and objectives of the study and provides chapter summaries for the entire dissertation.

1.1 Introduction

The focus on issues of literacy and numeracy in the Irish post-primary mathematics classroom arose when the author, in her fulltime role as instructional leader, from 2009-2013, facilitated workshops for post-primary mathematics teachers on the teaching and learning of 'Project Maths', a revised mathematics syllabus for students aged 12-18 years. It was noted at these workshops that tasks requiring teachers to record a written component to an activity, such as 'explain your reasoning', were frequently left blank indicating a need to understand why Ireland's cohort of practising mathematics teachers lacked experience and confidence with this endeavour.

In addition, the roll-out of the new syllabus and assessment questions were met with much criticism from teachers, students and third level practitioners as demonstrated in the newspaper discourse of the period. Following initial cries of 'dumbing down', (Keane 2012) reports quickly changed to 'teachers say Project Maths is too difficult' (Keane 2013) in the wake of the first Project Maths summative assessment. Concerns regarding 'literacy difficulties' and the 'text heavy nature of the exam questions' (*ibid.*) were raised as impediments to student success in mathematics. Also, it was reported: 'one of my problems with it is that it is less a test of numeracy and more a

test of literacy’ (Bielenburg 2012), as well as: ‘you have kids who are very good at maths who struggle with Project Maths, because the questions are much too wordy’ (*ibid.*).

The author’s interest in the construct of literacy in the mathematics classroom deepened when the government launched ‘Literacy and Numeracy for Learning and Life, the National Strategy to Improve Literacy and Numeracy among Children and Young Children, 2011-2020’ (Ireland, Department of Education and Skills 2011). In this publication, literacy and numeracy are given broad definitions that reference the ubiquitous reading, writing and arithmetic elements in print and digital media as well as encompassing a definition of communication that includes speaking and listening. The strategy also outlines the importance of continuous professional development for teachers in literacy and numeracy skills across the curriculum (*ibid.*). Furthermore, the latest reform in curriculum and assessment in Ireland for 12-15 year olds entitled ‘Junior Cycle’ expresses the necessity of consolidating literacy and numeracy skills in all curriculum subjects (Ireland, Department of Education and Skills 2015a).

Therefore, the author, a practising mathematics teacher, who strongly believed developing numeracy was an integral part of mathematics teaching and an output of a mathematics education, began this research journey with a primary focus on literacy for effective mathematics instruction. However, the numeracy definition from Ireland’s National Strategy (Ireland, Department of Education and Skills 2011) describes young people having number sense and spatial awareness, understanding and interpreting data, patterns and sequences and recognising ‘situations where mathematical reasoning can be applied to solve problems’ (Ireland, Department of Education and Skills 2011, p.8). This is a significantly more complex and multi-layered definition than the conventional conception of numeracy as ‘understanding number’ (Hislop 2011, p.15). Also, a former Minister of Education, Jan O’Sullivan (2014-2016), at the conferring of 288 out-of-field Irish mathematics teachers, referred to literacy and numeracy as a priority area (O’Sullivan 2015) suggesting numeracy for mathematics teachers had a parallel status to literacy.

A short distance into the research journey it became clear that a prior condition for literacy in the mathematics classroom was a deep knowledge and understanding of mathematics in abstraction and in real life contexts: in other words, the elements outlined in Ireland's numeracy definition (Section 2.2). But the converse was also true: requisite knowledge for numeracy development was literacy skills (Section 2.1). Consequently, unlike the Crowther Report, that described numeracy as a mirror image of literacy (Central Advisory Council for Education (England) 1959), one of the key outputs of this study was the construct Literacy and Numeracy for Mathematics Teaching in Ireland or LNMTI. Similar to Irish poet and political Irish figure, W.B. Yeats, in the poem 'Amongst School Children', when he asks: *How can we know the dancer from the dance?*, (Yeats 1969), it was in the synthesis of the domains of literacy and numeracy that a clearer perspective on their purpose and effectiveness for mathematics teaching in the Irish context emerged. Secondly, by focusing on mathematics teaching through the lens of numeracy, more questions arose regarding the numeracy capabilities of pre-service teachers of mathematics.

1.2 Background and Context

For the reader to appreciate the characterisation of Literacy and Numeracy for Mathematics Teaching in Ireland (for brevity the acronym LNMTI will be used from now on) and the knowledge gap to be filled as a result of this study, background information on the Irish educational context is provided.

The Irish educational system is centralised and there is national control of public schooling by the Department of Education and Skills (DES). Post-primary schools serve students aged 12-18 and primary is the name given to the childhood education of students aged 4-12. The DES also funds specific Early Childhood Education initiatives and one year of pre-primary education provided by private or community enterprises (Ireland, Department of Education and Skills 2017a). The National Council for Curriculum and Assessment (NCCA) is a statutory body that decides through a process of collaboration with education stakeholders what is to be taught

and learned at these three levels. It also has a key role to play in decisions related to the assessment of, and for, learning.

Assessment instruments are developed and administered by Coimisiún na Scrúduithe Stáit/The State Examinations Commission (SEC), a non-departmental public body of the DES (The States Examination Commission n.d.). The SEC's role is to set examinations, manage the examination centres, marking of examination scripts and other assessment formats and certifying student achievement. There are two assessment levels at post-primary: Junior Certificate at the end of year 3 and Leaving Certificate at the end of the final year. The Junior Certificate is being phased out and will be replaced by Junior Cycle. In addition, schools can opt for a two or a three year senior cycle, where year 1 of the three year cycle is called 'Transition Year' where students engage in vocational as well as academic training. For mathematics, these summative assessments can be taken currently at three levels: Foundation, Ordinary and Higher. At Leaving Certificate level, the majority of students sit what is described as the 'established' Leaving Certificate, where students sit a minimum of six subjects. Grades derived from these examinations are awarded points from which third level institutions courses are accessed. Currently the maximum number of points a Leaving Certificate student can achieve is 625 from six subjects taken at higher level. The top grade, H1, is equivalent to 100 points but the system awards 25 extra points, introduced in 2012, to students who opt to sit the higher level paper in mathematics. Other Leaving Certificate programmes exist that were designed to help transition learners from the world of school to the world of work. They include Leaving Certificate Vocational Programme which comprises of academic and vocational components and Leaving Certificate Applied, aimed at students who have different learning needs and do not want to attend higher education after the Leaving Certificate. This programme lists literacy and numeracy development as one of its primary objectives (National Council for Curriculum and Assessment 2001).

At post-primary level, students study a national mathematics syllabus which was revised in 2010 with a greater emphasis now being placed on context-based applications of mathematics and conceptual understanding. This syllabus is framed in

a five strand structure to expand on the learning of mathematics at primary level. In comparison to previous mathematics curriculum publications, the current syllabuses give literacy an explicit role by repeating for each of the five strands the learning outcomes:

- *explain findings, justify conclusions, communicate mathematics in verbal and written form,*
- *analyse information presented verbally and translate it into mathematical form,*
- *devise, select and use appropriate mathematical models, formulae and techniques to process information and to draw relevant conclusions.*

(National Council for Curriculum and Assessment 2013a, p.14).

In other parts of the syllabus, specific references to the teaching and learning of mathematical terms and symbolic notation exist (*ibid.*). As previously mentioned, Ireland is currently engaged in a reform of curriculum and assessment referred to locally as Junior Cycle and the new Junior Cycle mathematics will be rolled out in schools from 2018 (Ireland, Department of Education and Skills 2017b). Currently, the intention is to provide summative assessments at two levels: Higher and Ordinary (*ibid.*). To keep in line with other subject assessments at Junior Cycle and the Junior Cycle assessment protocol in general, two classroom-based assessments for all students, will be assessed at a common level. This is the first time classroom-based assessments have been used in Ireland since the introduction of a public examination system at post-primary level in 1879 (State Examinations Commission 2006). Preparations are in place for Junior Cycle assessment reform in mathematics for 2021 (*ibid.*).

In addition, the education culture in Ireland is dominated by the ‘high stakes examination’ which attracts huge media interest (O’Donoghue *et al.* 2017). Conway and Sloane (2005) list a series of negative Irish national newspaper headlines following the publication of poor Leaving Certificate mathematics results in 2005. In 2015, the headlines capturing the reaction to the Leaving Certificate Ordinary Level Project

Mathematics assessment is a case in point such as ‘Leaving Cert Maths: Fury at Ordinary Level Paper’ (McGuire 2015).

The next section will position the study in relation to historical, national and international context for literacy and numeracy.

1.3 Historical, National and International Context for Literacy and Numeracy

The growing impetus of literacy and numeracy as domains of learning in Ireland’s education system necessitated a clear definition of what literacy and numeracy are, and how they are manifested in the mathematics classroom. Various conceptions and definitions exist (Section 2.1.1) that appear to be country-specific, socially, culturally and politically fostered and are on the national agenda in Ireland and other Organisation for Economic Cooperation and Development or OECD countries (Ireland, Department of Education and Skills 2011; Organisation for Economic Cooperation and Development 2013). South Africa, one of five key partners within the OECD (2017), following the end of the political system of apartheid (Jansen and Taylor 2003), produced education documents emphasising people’s rights to access domains of learning in mathematical literacy, mathematics and mathematical sciences (Tirosh and Graeber 2003). In sharp contrast, literacy values are embedded in the cultural identity of Ireland. For instance, after eight hundred years of British rule, the establishment of the Irish Free State in 1922 focused on building a national identity ‘giving language, history, music and tradition of Ireland their natural place’ (Ó Brolacháin 1922 cited in O’Donovan 2013, p.25).

However, Ireland’s economic crisis in the late fifties brought about innovative economic strategies in the sixties (Hogan 2010) and a reform in mathematics education was enacted. One action put an end to gender inequality by discontinuing a basic numeracy assessment for girls entitled: ‘Elementary Arithmetic for Girls Only’ (O’Connor 2007, p.5). Coincidentally, following a severe recession in Ireland in 2008 (O’Connor 2010), Project Maths was piloted in 24 post-primary schools with incentives to students to study the more academically demanding Leaving Certificate

mathematics paper. It was then rolled out nationally in 2010. Next on the continuum of mathematics education reform was the launch of the National Strategy for Literacy and Numeracy in 2011.

The overall aim of Project Maths was to provide a relevant syllabus that promoted teaching and learning interactions for young people involving the development of problem solving skills situated in real-life contexts to prepare them for higher education and the workplace (National Council for Curriculum and Assessment 2015b). In addition, the National Council for Curriculum and Assessment (NCCA) were asked to formally respond to the draft plan for a national literacy and numeracy strategy and in their submission they stated:

the definition of numeracy offered is closely aligned to the rationale for Project Maths and, as such, sits well with current NCCA work. Key to the success of this work is its capacity to bring about and support attitudinal change through the positive impact of the investigative and interactive approaches adopted in Project Maths.

(National Council for Curriculum and Assessment, 2015a, p.13)

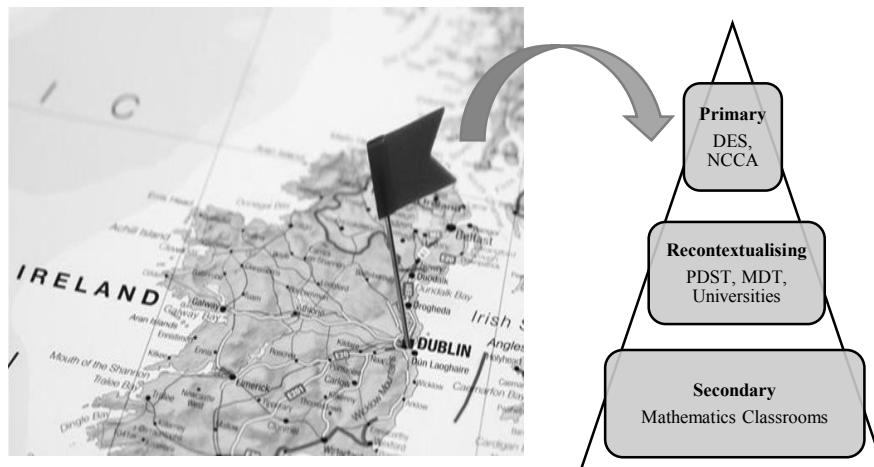
The importance of ensuring pre-service education proposals engage with these reforms to ensure a uniformity of approach was also stressed (*ibid.*).

At the time of this study, the current Minister for Education, Richard Bruton's 'Action Plan for Education 2016-2018' aims, over a ten year period, to reduce the gap in numeracy attainment between Ireland and top European performers (Ireland, Department of Education and Skills 2016a). Continuing to improve the uptake of higher level mathematics at both Junior Certificate/Junior Cycle Level and Leaving Certificate is also presented as a numeracy priority (*ibid.*). How this is achieved practically in the daily workings of classroom activity presents ongoing challenges for the experienced professional teacher, not to mention the novice teacher who is entering the profession for the first time.

1.4 Bernstein Theory of Social Construction of Pedagogic Discourse

Given the Irish context where pedagogic messages from a centralised system of education originate, the dissemination of these messages of reform are conveyed through supporting organisations such as the Professional Development Service for Teachers (PDST) for qualified, practising teachers and Schools of Education in universities for pre-service teachers. Because of this filtering process from macro to meso to micro level, there is a sense that whatever is intended may be misconstrued or lost in translation, or whatever is intended lacks a practical clarity and specificity. Similarly, Sfard (2011) commemorating Paul Cobb's contribution to educational research in mathematics education, writes: 'he gradually extended the object of his study from the individual learner to classroom community to institution' (p. 232). Moreover, the author's study is not concerned with formally critiquing the content of the message such as the definition of literacy and numeracy from Ireland's national strategy or the structure that currently exists to convey that message. What is of interest is the channelling medium; the author wishes to explore variants that may better support pre-service teachers conveying official messages of curriculum and practice. To guide this exploration, Bernstein's (1990) theory of Social Construction of Pedagogic Discourse published in a four part volume on the relationship between language and education, (Clarke 2005), was employed as a structural and analytical device. To explicate the rationale for this theory, Bernstein uses the subject of physics in a post-primary school context as an example. He demonstrates the way physics as a domain of knowledge comprising of logical facts, can be transformed by agents, for example secondary school textbook writers, which in turn is remoulded by teachers in the classroom.

Consequently, Bernstein identified three hierarchical sites for knowledge production: producers (primary), recontextualising, and reproducers (secondary) (Kirk and MacDonald 2001). Figure 1.1 illustrates this model of message transmission, employing examples from the Irish context related to the author's study.



(Maximum Media 2017)

Figure 1.1 Bernstein's (1990) three hierarchical sites for knowledge production in the Irish context

Within each field, agents are active and operate on the same message: literacy and numeracy for mathematics teaching in Ireland. In the primary field, the agents, the National Council for Curriculum and Assessment (NCCA) and Department of Education and Skills (DES) produce the message: the mathematics syllabus, theory of instruction, Junior Cycle Framework and definitions of literacy and numeracy. This message is recontextualised by government funded teacher professional support services. In Ireland, the Maths Development Team (MDT) had the sole remit for teacher support during the roll out of Project Maths but since 2018, it has been subsumed by the previously mentioned Professional Development Service for Teachers, also known as PDST. In addition, university teacher education programmes play a key role in recontextualising pedagogic messages for pre-service teachers by providing subject pedagogy modules where these messages are circulated and analysed to be reproduced in the secondary field of mathematics classrooms in schools around the country.

In addition, the third hierarchical site for this study is the post-primary mathematics classroom in Ireland. Although Pólya (1954) acknowledged the common view of mathematics as a 'demonstrative science' he was quick to dismiss this view as representing only one aspect of mathematics (preface). However, this perspective is

described in detail as a predominant methodology from the data collected in the video study of twenty Irish second level classrooms by Lyons *et al.* (2003). Notwithstanding, the Chief Inspector's report on the quality of teaching and student learning in Irish mathematics classrooms observed in the years 2010-2012, when there was mandatory attendance at Project Maths workshops, that 23% of mathematics teaching and 26% of student learning was less than satisfactory (Ireland, Department of Education and Skills 2013). Teacher-led lessons with a primary focus on skills development were witnessed and student responses to practised procedures were dominant. It was noted that students should be encouraged to reason mathematically and justify their answers (Ireland, Department of Education and Skills 2013).

Furthermore, more recent data from the National Foundation for Educational Research (NFER) report, published in 2013 on the pilot phase implementation of Project Maths, illustrated that pedagogical approaches still remain largely unchanged and students were less positive about mathematics, although students were more confident in their mathematical ability (Jeffes *et al.* 2013). Also, the NCCA commissioned report 'Maths in Practice', which was published the year prior to the 2015 examination that assessed the full syllabus changes, remarked:

Although there is evidence that students are engaging in activities associated with the revised syllabuses, more traditional approaches continue to be widespread.

(National Council for Curriculum and Assessment 2014c, p.12)

Finally, in the Chief Examiners' report, a report written by the managers of the state examinations following the 2015 Junior Certificate and Leaving Certificate mathematics national assessments, it was noted that improvements were observed in candidates' ability to explain and justify their work and work with mathematical terms and notation. However, references to candidates experiencing difficulty in this skill was noticeably prevalent for both cohorts (State Examinations Commission 2016a; State Examinations Commission 2016b).

1.5 The Research Problem

The research problem has three dimensions that can be situated in Bernstein's knowledge production model, previously discussed. With preparation in motion for the reformed Junior Cycle mathematics, the first draft of the message produced by the NCCA and DES was a generic skills framework that explicitly stated:

Literacy and numeracy are not listed as key skills, but are seen as providing the foundation on which the curriculum is built and will be integrated through every level of the curriculum and its assessment.

(National Council for Curriculum and Assessment 2014a, p.3).

However, this position changed and in the current publication (Ireland, Department of Education and Skills 2015), literacy and numeracy as skill domains have been given a specific and separate identity. Therefore there is an issue with content stability and clarity in the longer term.

Continuous change and evolving positions are also issues for teacher education providers in the current climate, a recontextualising field. Present pre-service mathematics teachers, as students in Irish post-primary schools, would have engaged with a different mathematics syllabus, assessment and methodologies. This is acknowledged in the NCCA's response to criticisms of Project Maths (National Council for Curriculum and Assessment 2012). Half of the participants in this study would have completed the Leaving Certificate in 2012 when the first major change to assessment at higher level was implemented whereas the other half were assessed under different circumstances. In light of research conducted on the effectiveness of pre-service education, it was found that 'the influence of past experience on teachers' instructional behaviour in mathematics classrooms, the secondary field, is described by some researchers as influential enough to override teacher education programs' (McMillan 1985, p.85).

In summary, the research problem is centred on the reality that pre-service teachers in Schools of Education in universities are being prepared to teach a mathematics syllabus and assessment they have not experienced themselves. This issue is

magnified by the fact that support for these novice teachers to prepare students for the new Junior Cycle mathematics, with a concentrated focus on literacy and numeracy development, is also required.

1.6 Research Aim, Questions and Phases

In the formulation and scripting of the research aim, the author considered general research aims: to describe, to compare, to evaluate, to explain, to predict or to advise (Plomp 2013). Given the background to the research problem and the nature of research in general, a descriptive and evaluative aim constituted key elements of the research intent. Therefore, the author concluded that the over-arching aim of the research was to evaluate pre-service teachers of mathematics readiness to implement literacy and numeracy for mathematics at post-primary level in Ireland.

Given the present and future position for mathematics education in Ireland, the author composed the following research questions in Table 1.1 using Bernstein's (1990) theory of Social Construction of Pedagogic Discourse as an organisational structure:

Table 1.1 Research questions for the study framed by Bernstein's theory (1990)

<i>Knowledge Production Sites</i>	<i>Research Questions</i>
Primary (Production)	RQ 1: How is Literacy and Numeracy for Mathematics Teaching in Ireland defined and characterised as a knowledge domain for assessing pre-service teachers of mathematics in order to monitor how well prepared they are to meet the challenges of current educational reforms in Ireland?
Recontextualising	RQ 2: Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms? RQ 3: What content and characteristics should a teaching and learning module have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?
Secondary (Reproduction)	RQ 4: What is the potential of a teaching and learning module based on authentic design principles to support pre-service teachers of mathematics develop an understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that implementation of the domain in the dynamic classroom environment is enabled?

In addition, different phases of the research were characterised by Bernstein's model and the research questions. Table 1.2 summarises the key actions taken in the research study that will make a contribution to research into initial teacher education and mathematics education nationally and internationally:

Table 1.2 Key actions taken in the study

<i>Knowledge Production Sites</i>	<i>Key Actions</i>
Primary (Production)	Defining LNMTI Developing the LNMTI framework
Recontextualising	Designing the LNMTI Survey Administering the LNMTI Survey Designing and implementing the LNMTI intervention
Secondary (Reproduction)	Designing and implementing the LNMTI classroom observation instrument

1.7 Significance Nationally and Internationally

As a general point, radical education reforms to meet the needs of a changing society are currently being planned and/or implemented in Ireland and internationally. This research has at its core three major aspects of these current and future changes: pedagogy for literacy and numeracy; initial teacher education; and assessment reforms and thus address key national and international priorities. Specifically, it is anticipated that this study will support future pre-service and current mathematics teachers in Ireland to implement Junior Cycle reforms for mathematics. The study has also the scope to support Schools of Education supervising tutors in providing them with the tools to assess pre-service teachers in the teaching practice environment who are preparing for the world of work during a period of reform.

1.8 Conclusion and Outline of Chapters

Chapter 1: Setting the Context of the Study

This chapter established the need for this research by presenting the reader with the study's background and context. However the key component of the chapter was to outline the compass of the study guided by the research questions which are summarised as follows:

- What is literacy and numeracy for mathematics teaching in Ireland?
- Do pre-service teachers have the requisite knowledge and pedagogic practice to implement LNMTI in their classrooms?
- What can be done to support them in the current circumstances?
- What is the potential of these supporting mechanisms?

Chapter 2: Literature Review

This literature review chapter locates the study in the field of literacy, numeracy and mathematics education. Key themes were identified during the review process which were then used as an organisational strategy to show the need for this research. The reader will also gain an insight into the elements of the theoretical framework that emerged from the literature encased by Bernstein's theory of pedagogic discourse as described in this chapter.

Chapter 3: Methodology

The methodology chapter outlines the arguments to support the philosophical stance of interpretivism for this study. It also presents the theoretical framework to the reader as a model for justifying and evaluating the work of the study as well as explaining why Educational Design Research (EDR) was employed as the dominant paradigm. Guided by EDR, the four phases of the research study are then outlined. The details and outcomes of these phases become the focus of ensuing chapters.

Chapter 4: Literacy and Numeracy for Mathematics in Ireland Definition, Framework and Survey

This chapter concentrates on phase one of the study in which a LNMTI definition, a foundational step for this research, was composed. Once this was in place, the chapter then describes the development of the LNMTI framework and items in preparation for the LNMTI survey.

Chapter 5: Discussion of Findings of Pre-Service Teachers' Prior Knowledge of Literacy and Numeracy for Mathematics Teaching in Ireland

Chapter 5 discusses the findings of the LNMTI survey by focusing on participants' overall performance as well as identifying individual participant anomalies and dominant misconceptions. This second phase of the study generated the need for the design of a LNMTI intervention.

Chapter 6: The Design, Construction and Evaluation of the LNMTI Module

Underscored by the instructional design framework of Herrington and Oliver (2000), this chapter describes the background, aims and development of the LNMTI intervention. This third phase of the study consists of detailed elements from the intervention such as pedagogical considerations, types of tasks and resources developed.

Chapter 7: Post-Intervention - Evaluation and Reflection

This chapter concentrates on phase four of the study where the intervention evaluation is presented. It also describes the rationale for developing a classroom observation instrument based on the TRU framework (Schoenfeld 2013). This chapter concludes with findings from the classroom observation.

Chapter 8: Conclusion, Contribution and Future Work

This final chapter in the study revisits the research questions articulated in this chapter and provides conclusions and outcomes from the study process. It also outlines the study's contribution to theory by generating design principles to translate educational policy statements into practical knowledge for the classroom.

Chapter 2: Literature Review

This chapter consists of a summary of the literature relevant to literacy and numeracy for mathematics teaching in Ireland. The review will focus on three main themes which emerged repeatedly during the literature review process:

- the interpretation of numeracy in content and context and its relationship with mathematics,
- the influence of literacy on numeracy behaviour,
- mathematics teaching and learning: theories and practice.

2.1 Interpreting Numeracy

In this section, various conceptions and relations of numeracy and numeracy in mathematics are described from a historical, international and Irish context. Additionally, features of numeracy assessments are explored in the literature to establish a deeper understanding of numeracy as a domain of learning.

2.1.1 Numeracy: Definitions and Relations

Much of the literature acknowledges that numeracy does not have a precise, concise and globally accepted definition or title (Coben 2000; Maguire 2003; Neill 2001; O'Donoghue 2002 & 2011; Kaye 2009; Hogan 2012a; Clements *et al.* 2013). Academics provide a reason for this ambiguity citing the confluence of social, historical, political and cultural phenomena on numeracy's evolving status as a domain of learning (FitzSimons 2008). Ireland's mathematics syllabus at post-primary equates numeracy with basic arithmetical skills and knowledge of number (National Council for Curriculum and Assessment 2013a). In contrast, Penny's (1984) definition includes an explicit literacy component:

The ability to understand and use mathematics as a means of communication (p.3).

This is also evident in the PISA 2015 definition described as 'mathematical literacy' (OECD 2013b). It extends Penny's (1984) characterisation in greater detail:

an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (p.5).

In contrast, Grawe *et al.* (2010) emphasise the numerical element in numeracy through the use of the phrase 'quantitative reasoning' which they describe as:

the habit of the mind to consider the power and limitations of quantitative evidence in the evaluation, construction and communication of arguments in personal, professional, and public life (p.1).

There are a number of similarities between these definitions: they include the literacy components of communication, description and explanation. In addition, mathematics is referenced directly or indirectly. Steen (2001) however, delivers a clear and unambiguous statement about the relationship between numeracy and mathematics:

Numeracy and mathematics should be complementary aspects of the school curriculum. Both are necessary for life and work, and each strengthens the other. But they are not the same subject (p.108).

Johnston's (1994 cited in Cummings 1996) definition complements Steen's view of numeracy and mathematics having separate entities but her focus is on the cognitive element of numeracy:

A critical awareness which builds bridges between mathematics and the real world (p.13).

Moreover, Goos *et al.* (2012), influenced by Steen's conceptualisation of numeracy as a construct that exists in many contexts, derived a model for numeracy in the 21st century which comprises three elements: dispositions, tools and mathematical knowledge that can be accessed in the personal and social sphere, work and citizenship. To reflect the demands of a changing society, the model also included a critical orientation component to acknowledge complex cognitive processes involved in a person's ability to evaluate results and draw appropriate conclusions from information presented in social or political contexts.

In addition, a growing body of literature asserts knowledge of mathematics is a necessary but not sufficient condition to be numerate (Australian Association of Mathematics Teachers 1997; O'Donoghue 2011). Moreover, results from an interview (Maguire 2003) and survey (National Adult Literacy Agency (NALA) 2013) given to adult literacy and numeracy tutors in Ireland showed that 43% of the respondents did not support the idea that a mathematics education made you numerate. O'Donoghue (2002) writing for the Irish Mathematical Society bulletin also contends: 'numeracy does not seem to be an automatic outcome for many after years of compulsory schooling' (p.49).

A future source of data for this conjecture may be found in Australia's audit of literacy and numeracy standards amongst students in Years 3,5,7,9 with the introduction of a national programme of testing referred to locally as NAPLAN (Hardy 2015). Similarly, the Welsh mathematics curriculum for GCSE as part of the implementation of the statutory national literacy and numeracy strategy examined for the first time a subject titled: GCSE Maths – Numeracy at three levels: foundation, intermediate and higher (Qualifications Wales 2017). This is presented as mathematics for learners for everyday use whereas mathematics is considered an extension of this domain that includes higher order content. In 2017, 72% of the cohort were entered for both Mathematics-Numeracy and Mathematics and the first results demonstrate very little difference in grades awarded in both domains.

2.1.2 Numeracy: meaning on a continuum

As previously demonstrated, there is much debate from international experts on what numeracy actually means. In contrast, Cuomo (2011) gives clarity to what ancient numeracy meant for Greek and Roman civilisations: to count, calculate and measure on a spectrum of abilities. In a lecture for the Gresham College, London, she argues that ancient people with a high level of numeracy might have been able to engage with sophisticated mathematical games where in the following example the objective of the 'game' is to determine a man's age (Demochares) upon death:

*Demochares lived for a quarter of his whole life as a boy
For a fifth part of it as a young man;
For a third as man, and when he reached
Grey old age
He lived thirteen years more on the threshold of eld*.*

**old age (Taub 2017, p.138).*

By contemporary standards, the mathematics used to ‘play the game’, is being studied by 12-15 year olds in Ireland (National Council for Curriculum and Assessment 2013a). Secondly, Devlin (2002) reports on the difficulty that existed for people in antiquity to move cognitively from the concrete to the abstract. He describes clay tokens found in Susa, Iran from 3300BC that were used to represent specific commodities: for example, cones and spheres stood for measures of grain. Then as commerce became more sophisticated these ‘physical counting devices’ (p.17) became redundant and marks on a clay tablet were regarded as the most efficient platform for recording data. However, despite the obvious utility and efficiency of the symbolic representations on clay, most people continued to use both the concrete counting technique as well as the symbolic record to communicate the quantity pointing to a delay and a difficulty in the development of abstract thinking. In Ireland, children engage in this type of task from the age of six or seven.

But Cuomo (2011) is not interested in comparing and creating numeracy hierarchies. She is insistent that Euclid inscribing hexagons in a circle in “The Elements” and the craftsman working out how many panels were needed to tile a floor both exhibited skills in numeracy. Coben (2000) supports this point of view by stating: ‘there is no single point at which one can say that a person is numerate (or mathematic) and that one is not’ (p. 36). Whereas Maguire (2003), as shown in Figure 2.1, highlights that numeracy is a complex concept that has levels of sophistication encompassing three elements on a continuum: formative, mathematical and integrative.

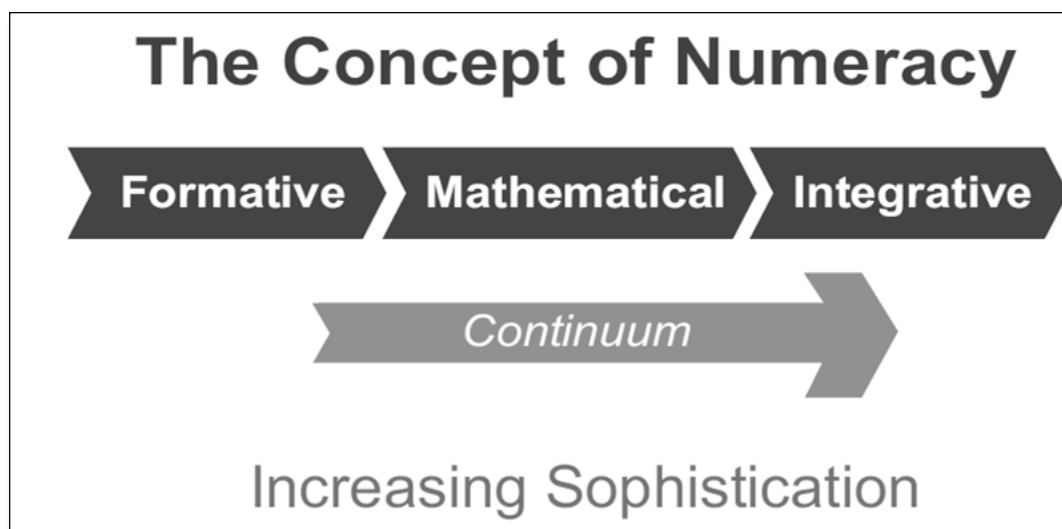


Figure 2.1 Maguire's (2003) conception of numeracy

The first phase describes basic arithmetical skills which is extended to become the mathematical element and finally the integrative phase incorporates 'the mathematics, communication, cultural, social, emotional and personal aspects of each individual in a particular context' (National Adult Literacy Agency 2013, p.14).

This complexity is evident in the statutory definition of numeracy in the National Strategy for Literacy and Numeracy (Ireland, Department of Education and Skills 2011):

Numeracy is not limited to the ability to use numbers, to add, subtract, multiply and divide. Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems (p.8).

It is worth noting that knowledge and cognitive processes conveyed in Ireland's numeracy definition are high order processes. Mapped against Bloom's taxonomy summarised by the Irish support service for teachers, it is clear that understanding and application are explicit in the definition as well as the other cognitive domains of analysing and evaluating (Professional Development Service for Teachers n.d.a).

Another valuable point of interest is that, every strand in the current post-primary mathematics syllabus is explicitly referenced in this definition as well as the overarching competency of problem solving (National Council for Curriculum and Assessment 2013a).

But the question still lingers: where does numeracy end and mathematics start or vice versa; or what are the differences and synergies that exist between these two domains? The next section will review literature that describes numeracy assessments using real world contexts that illustrate the difficulties between a pure mathematical understanding and a culturally focused numeracy understanding.

2.1.3 Assessing Numeracy

In Alberta, Canada, and in neighbouring states such as Saskatchewan, mathematics teachers are obliged to respond to the inclusion of school-going children of the Aboriginal people in Canada known as First Nations, Inuit and Metis. It is a syllabus requirement that mathematical content from these indigenous peoples must be integrated into mathematics classrooms (Alberta Education 2007). Poirier (2007) identifies the social, environmental and cultural aspects that influenced the development of the Inuit numeration system. The Inuit can function quantitatively in their own cultural context. For instance, when taking measurements, they rely on perceptual judgment, using an arm length to measure an object. Also, the mathematical concept of division as equal share in the Inuit world is not culturally relevant as communal ownership of everything is a social norm. Here we have an issue of a real life context where the application of mathematical knowledge, an international numeracy construct, has little meaning, however, the Inuit are legitimately numerate in the context of their worldview.

Benn's (1997) research analysed questions from a survey to ascertain numeracy levels of 21-year-olds in England in 1992. One task required the participants to divide a bill of a specified amount five ways. 40% of the sample of 1,650 who took the assessment, 'could not cope with the division' (p.71). The question should be asked from this result is: is this figure so confounding and should the problem that was asked be

reviewed? Like the Inuit people who don't have a real life and cultural context for the purity of mathematical division into equal parts, social practice has taught us how unsatisfactory this approach is in real life. For example, research on 'The Inefficiency of Splitting the Bill' (Gneezy *et al.* 2004) reported that 80% of subjects preferred to pay individually. However, factors that impeded this practice were 'calculating the portion of the tax and tip that apply to that share' (p.277) and the fear of being viewed as miserly.

Other numeracy survey questions discussed in Benn's (1997) study were the calculation of a dozen bars at the cost of 30p a bar and the area of a 12ft by 8ft wall. 13% of answers were incorrect for the 'bars' question but three times as many, 39%, gave the wrong answer to the area question. Cockcroft (1982) describes mathematics as a clear and unequivocal medium to communicate quantitative ideas where, for example, $20 \times 3 = 60$ can represent a number of contexts. However, what is obvious from these results was the failure of the 21 year-olds who sat this numeracy assessment to see this connection.

The criterion test used for the above assessment was traditional multiple choice format with hit or miss marking. Cai *et al.* (1995) demonstrated how easy it is to transform a multiple choice question into an open ended task by adding the line: justify your answer. The article describes a situation where a middle school student gave an 'incorrect answer' of 31 from the multiple choice selection to the following problem:

An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed?

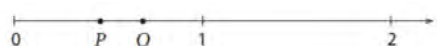
(Cai et al., p. 240).

The student answered 31 and reasoned 12 of those buses could hold one extra soldier. This example demonstrates that in the design of the open ended task, it allowed those being assessed an opportunity to explain a culturally applicable numeracy thought process for a mathematics context.

The Trends in International Mathematics and Science Study (TIMSS) 2015 makes a clear distinction between mathematics and numeracy for fourth grade, 13-14 year-old

students. Numeracy is described as foundational knowledge required to engage with general mathematics assessments. TIMSS numeracy content domains of whole numbers, fractions, geometric shapes and measures, and data displays were prepared for countries with underdeveloped education systems (International Association for the Evaluation of Educational Achievement 2015). In contrast, TIMSS mathematics assessments with content from mathematics curricula from OECD countries, assess cognitive domains of knowing, applying and reasoning. Students understanding of mathematics in each domain is measured by assessment items covering topics from number, algebra, geometry, data and chance. For example in TIMSS 2011, as illustrated in Table 2.1, Number strand items on fractions and decimals, aimed at Grade 8, are given the following cognitive treatment (National Center for Education Statistics 2015):

Table 2.1 Cognitive domains assessed in TIMSS 2011 illustrated with items from fractions

<i>Cognitive Domain</i>	<i>TIMSS 2011</i>
Knowing	Which fraction is equivalent to 0.125?
Applying	Ann and Jenny divide 560 zeds between them. If Jenny gets $\frac{3}{8}$ of the money, how many zeds will Ann get?
Reasoning	<p>P and Q represent two fractions on the number line. $P \times Q = N$.</p>  <p>Show the location of N on the number line.</p>

The item that represented the ‘knowing’ domain: *which fraction is equivalent to 0.125?* from the TIMSS mathematics assessment is equivalent to domains of learning in the TIMSS Numeracy assessment. However, in an Irish context, the domains of learning in the TIMSS mathematics assessment are cognitively comparable to the elements in the numeracy definition, emphasising the synergy between numeracy and mathematics.

The Programme for International Student Assessment (PISA) assessment items in mathematics are devised to examine the ability of 15 year-olds to identify mathematics

in real life contexts, an element in Ireland's numeracy definition (Hogan 2012b, OECD 2003, cited in Shiel *et al.* 2007). This would suggest that the Irish definition of numeracy is aligned with the PISA assessment for mathematics. However, there has been much criticism of the PISA approach from mathematics academics from an Irish university (Grannell 2011). The belief is the influence of PISA on the development and direction of mathematics education in Ireland is damaging. The PISA influence was evident from the style of assessment items produced for Phase 1 of Leaving Certificate Mathematics 2010 which were prepared for students, aged 18 years, in the pilot phase of Project Maths. It was evident from the trialling of sample questions that assessment items set in real life contexts proved extremely difficult for these students.

For example the question designed to test the application of routine procedures in trigonometry in a real life context generated the following response from the State Examinations Commission:

It appeared that candidates had little experience of applying their mathematical knowledge in this way. The trigonometry involved was not difficult and one would expect it to be well within the compass of moderately good candidates. However, at this stage, the candidates clearly have had little experience of planning and undertaking field activities or of discussing the practicalities of using trigonometry to solve real problems.

(State Examinations Commission, 2010a. p.37)

This suggests, from the Irish numeracy position outlined in the National Strategy, these students did not exhibit numerate behaviour.

The concept of numeracy as foundational knowledge of basic mathematics for a teachers' professional role in England is assessed by the government's decision to introduce literacy and numeracy skills screening of pre-service teachers (United Kingdom, Department for Education 2018). England began testing in 2000 following completion of a teacher education programme, but since September 2012 prospective teacher education candidates have to sit this professional skills test. Similarly,

O’Keeffe *et al.*’s (2017) study reports on the introduction in Australia, except New South Wales, of a literacy and numeracy proficiency test that is a requirement for pre-service teachers to graduate from a teacher education programme from 2017 onwards. The study discusses the linguistic challenges that exist in current numeracy assessment items for this proposed test in Australia and they demonstrate that numeracy proficiency in this context is heavily reliant on literacy competence. For example, Figure 2.2 presents question one from the numeracy sample test (Australian Council for Educational Research (ACER) 2015, p.9) that underwent a linguistic analysis and was identified as linguistically difficult and complex.

Numeracy Sample Question 1

EDUCATION EXPENDITURE

Government operating expenditure on education refers mainly to money spent on schools and tertiary education.

Of the total operating expenditure on education in 2011–2012, 51% was spent on primary and secondary education and 36% on tertiary education (universities and TAFEs).

What percentage of the total operating expenditure on education in 2011–2012 was spent on the remaining aspects of the education budget?

_____ %

Figure 2.2 Numeracy sample question for pre-service teachers, Australia

2.1.4 Conclusion to Interpreting Numeracy

In summary, with the various conceptions of numeracy and the many policy demands for the Irish classroom, when the author is referring to numeracy the official definition from Ireland’s national strategy will be employed. However the literature demonstrated on the one hand, numeracy comprises of foundational knowledge required to access mathematics but it also describes real life contexts and applications of mathematics. Therefore, Maguire’s (2003) concept of numeracy as a continuum will be used in conjunction with the numeracy definition from Ireland’s national strategy. Although Steen sees the symbiotic relationship between mathematics and numeracy and Johnston (1994, cited in Cummings 1996) aligns cognitive processes

with problem solving learning outcomes, Maguire's conception validates the learning objectives and cognitive demands in the Irish mathematics syllabus.

2.2 Interpreting Literacy

This section begins by examining the literature that relates literacy to numeracy and the issues of domain identity. This is followed by a synopsis of studies that explore the impact of numeracy on literacy development and finally the section will report on studies that involve literacy in mathematics.

2.2.1 Literacy and Numeracy Identities

The literature on numeracy establishes interesting developments in the relational status between literacy and numeracy. According to the Merriam-Webster dictionary (2018), the word numeracy is derived from literacy. Numeracy, a coined word first used in 1959 in the Crowther Report (England, Central Advisory Committee in Education 1959), is from the Latin 'numerus' meaning number and 'acy' from the word literacy. Moreover, the Crowther Report metaphor of numeracy as 'the mirror image of literacy' (*ibid*, p. 269) indicates a separate but symmetrical connection within the dimensions of literacy and numeracy for different subject domains. This is illustrated in the following quotation:

The complementary elements should be designed to ensure the literacy of science specialists and the numeracy of arts specialists.
(*ibid.*, p.282)

In 1990, The Australian Council for Adult Literacy and the International Literacy Year defined literacy as 'the integration of listening, speaking, reading, writing and critical thinking' (Cummings 1996, p.6). The following year the definition was modified to include a numeracy reference: 'literacy also includes the recognition of numbers and basic mathematical signs and symbols within text' (*ibid.*) pointing to a growing awareness of the connection between literacy and numeracy. However, employing descriptive phrases such as 'recognition of numbers' and 'basic mathematical signs and symbols' characterise literacy encompassing low order skills from the numeracy

domain. But Maguire (2003) identified another issue from interviews with adult literacy coordinators whereby numeracy did not have a separate status and its content was addressed within the domain of literacy:

'the clock, shopping etc. that most of our tutors do is incorporated as part of literacy we call social skills. It is not numeracy rather it is numeracy as part of literacy' (p.203).

In addition, literature from Ireland's Department of Education and Skills (DES) and the National Council for Curriculum and Assessment (NCCA) has shown an increased interest in the domain of literacy and its relationship to numeracy. The national strategy draft plan to improve literacy and numeracy in schools required more attention to be given to the teaching and learning of numeracy than to literacy (Ireland, Department of Education and Skills 2011). This was corroborated by further comments made by the National Council for Curriculum and Assessment (NCCA) (2015a) that criticised the 'coupling' of the terms literacy and numeracy followed by a recommendation to give numeracy greater prominence in the plan 'in order to avoid a propensity to have numeracy thought of as a minor adjunct to literacy' (p.13). In the final draft of the literacy and numeracy national strategy numeracy was given a distinct identity by awarding it equal status and a separate definition in the government publication (p.8).

Despite these developments in distinguishing numeracy from literacy, the first draft of The Junior Cycle Framework 2015 (National Council for Curriculum and Assessment 2014a) identified 'Communication' as one of the six key skills to be taught and assessed as part of this reform and 'Number' was listed as a component of this strand (p.10). However, as outlined in the introduction, the most recent draft (Ireland, Department of Education and Skills 2015a), has increased the skill set to eight with the addition of 'being literate' and 'being numerate' (p.13) therefore legitimising their prominence and independence.

Conversely, the Australian National Numeracy Review Report recognises that effective numeracy programmes include learning content on the literacy demands of numeracy (Council of Australian Governments (COAG) 2008). American Research

conducted by DiGisi & Fleming (2005) argues that students' competency in reading and understanding specific mathematical verbs and nouns such as factorise, evaluate, square, integer as well as language used to describe the mathematics in context is a prerequisite for numeracy success. Moreover, Cockcroft's (1982) definition of numeracy involves literacy elements with the inclusion of phrases such as 'understanding of information which is presented in mathematical terms' and 'the ways that mathematics can be used as a means of communication' (p.11). Here we have a case of numeracy subsuming literacy. In addition, Cummings (1996) addresses this issue directly by stating:

We need to examine first of all the adequacy of the ways in which numeracy is being defined within literacy and secondly the ways in which numeracy is conjoint with literacy and separate in identity (p.9).

This clarity in definition and practice is a priority for this study and the author will follow Cummings' advice by examining the adequacy of ways in which literacy and numeracy is defined for mathematics teaching and learning in the Irish post-primary classroom.

2.2.2 Numeracy in Literacy

Collectively, a recurring theme in definitions of, and discussions on, numeracy is the ability for a numerate individual to identify and interpret mathematics in context (Grawe *et al.* 2010; Maguire 2003; Ireland, Department of Education and Skills 2011; OECD 2013a). But to understand the context, an individual requires literacy skills. The typical example given is a person's ability to interpret information given in newspaper reports or bulletins (Cummings 1996). Bohlmann and Pretorius (2008) researched the relationship between mathematics and literacy with South African senior primary school pupils, arguing that English reading is strongly supportive of mathematics achievement rather than the more general construct of English language proficiency. Similarly, other research conducted with pre-schoolers links numeracy achievement to the literacy domain; Purpura *et al.* (2011) examined whether early literacy skills uniquely predict early numeracy skills development and the study

concluded ‘print knowledge and vocabulary accounted for the unique variance in the prediction of Time 2 numeracy scores’ (p.647).

On the other hand, in a study to design a portion size estimation interface for a low literacy population, Chaudry *et al.* (2011) noted that numerically representing volume amounts helped participants in the study choose between two types of fluid containers. Also, research by Parikh *et al.* (2003 cited in Chaudry *et al.* 2011) describes how a semi-literate population in India, which was experiencing difficulties understanding printed instructions, was able to navigate its way using numeric information, culminating in a recommendation to designers to include numerical data with images of portion sizes. This finding suggests that a low level of achievement in the literacy domain does not preclude learners from engaging with number tasks. Ruddock *et al.* (2006) reported on the reading demands of assessment items from PISA in comparison to TIMSS, GCSE and the English national curriculum tests. The report highlighted the discrepancy between the high reading level in the mathematics context problems and the low level mathematics required to solve them. Mathematics consultants from England anticipated that English students had and would experience difficulties with these questions because of the text heavy nature of the items.

Similarly, literature from the Dyslexia Association of Ireland makes a point of referencing the prevalence of ‘wordy’ problems in the assessment of Project Maths as an impediment to student success in mathematics with this learning difficulty (Ball and McCormack 2014). However, U.S. authors, Schackow and O’Connell (2008) make an unequivocal statement about ‘wordy problems’. They argue that text based problems are necessary to develop problem solving abilities in students because the presentation of an abstract mathematical equation, for example, has done the thinking work for the student making it ‘a rote process’ (p.24). The PISA 2015 results show Irish fifteen year olds ranked third in reading out of 35 OECD countries and maintained a stable position in mathematics in comparison to other years, suggesting that good reading levels support stable results in mathematics (Ireland, Department of Education and Skills 2016d).

The literature in this section strongly advocates the complementary aspects of literacy and numeracy for student learning. This substantiates the DES and NCCA's decision to position these skills in the key skills framework for Junior Cycle (Ireland, Department of Education and Skills 2015a). However, the problem of generalising these domains to all subjects without attaching a specific definition to the position of literacy and numeracy in the subject discipline still exists.

2.2.3 Literacy in Mathematics

Recent literature in mathematics education has pinpointed language and communication as crucial components of mathematical understanding (Fey 1970; Pollak 2007; Schleppegrell 2007; Ní Ríordáin 2011; Usiskin 2012; O'Keeffe *et al.* 2017). This is reflected in the current post-primary mathematics syllabus in Ireland, where the technical language of mathematics is specified: 'students should be able to use the terms theorem, proof, axiom, corollary, converse and implies' (National Council for Curriculum and Assessment (NCCA) 2013a, p 18). Furthermore, 'communicating and expressing' (*ibid.*, p.8) are identified as required skills to develop mathematically.

Another variable in learning mathematics at post-primary in Ireland is the status of the minority language, Gaeilge, in Ireland. Ní Ríordáin's (2011) research focuses on the impact of a Lán Ghaelach education on mathematics learning. Research on immersion education in the language of Irish (Gaeilge), the official language of the state of Ireland, showed that students' understanding of mathematics word problems broke down at the comprehension stage. Mac Mahon (2013) specifies the problems and obstacles that exist to integrate subject discipline literacy into subject content areas internationally and in Ireland. One of the key issues identified was Irish teachers' conceptualisation of literacy: when asked to define literacy, all subject teachers focused on the ability to read, with the majority confining this to the reading of words only. Mac Mahon also remarks: 'initial teacher education programmes...will need to address literacy as a central element of subject pedagogy in all disciplines' (*ibid.*, p.33).

Vygotsky (1986) writes ‘the structure of speech does not simply mirror the structure of thought; that is why words cannot be put on thought like a ready-made garment’ (p. 219). This is reflected in Schlepppegell’s (2007) work on the linguistic challenges of mathematics teaching and learning. She describes language in the mathematics classroom as a multiple semiotic system (p. 141) comprised of oral and written components with symbolic and visual representations. Similarly, Temple and Doerr (2012) focus their study on language in the mathematics classroom, nevertheless, they caution against using explicit language instruction by citing a study by Alder (1999) in a multilingual classroom in South Africa because ‘when the language became the object of attention instead of just a means to the mathematics, it had the potential to distract the students from the mathematics’ (p. 288).

Vygotsky (1986) with specific reference to literacy, the mathematics classroom and PISA mathematics framework, is also referenced in Close’s (2006) article entitled: ‘The Junior Cycle Curriculum and the PISA Mathematics Framework’. He compares the language elements in the mathematics classroom to the knowledge of grammatical rules and structures of a foreign language and notes the mastery of technical aspects of a language may not be sufficient to enable a person to use it effectively.

Furthermore, English and Halford (1995) who set out to define best practice for mathematics education include a chapter titled ‘Numerical Models and Process’ in which they outline the ‘linguistic complexity of place value’ (p.118). They compare Asian and English systems of naming numbers to highlight how language supports understanding and reasoning of place value. They focus on the Asian system of language where Asian children learn ‘ten two’ for ‘twelve’ and ‘six ten’ for sixty. However, the writers are keen to emphasise that Asian students’ superiority in international assessments in mathematics is as a result of various pedagogical practices (p.120). They follow this comment by citing a study by Stigler and Baranes (1988) that showed Japanese teachers and students engaged in more verbal interactions in the classroom (p.120). Also, they write about US teachers’ use of concrete materials in the classroom in comparison with their Japanese counterparts: ‘Japanese teachers’ use

the objects as a topic of discussion, whereas American teachers tend to use the objects as a substitute for discussion' (p.121).

Fey's (1970) research focussed exclusively on patterns of verbal communication for mathematics teaching where he describes the basic unit of classroom discourse as a move, a construct originally developed by Bellack (1968). A move, employed to achieve a pedagogical purpose, can be categorised as: structuring, soliciting, reacting, and responding. See the following Figure 2.3 for Fey's model of patterns of verbal communication with examples:

Move Type	Pedagogical Purpose	Examples
Structuring	Set a context	We will now look at the multiplication of integers
Soliciting	Elicit verbal information or cognitive/physical action	What is ----? If we ----, what is ----?
Responding	Response to a solicitation but it is to fulfil the expectation of the solicitation	Student responses to teachers' questions Teacher responses to students' questions
Reacting	Clarify, synthesise or expand Rate what was said	Yes you could look at it that way That is correct Think about that again

Figure 2.3 Fey's (1970) model of Patterns of Verbal Communication with examples

A central element to Fey's (1970) analysis of mathematical discourse comes from the teachers and students engaging in mathematical activities. Truxaw and de Franco (2007) extended this framework to include exchanges, sequences and episodes in the analysis of Pólya's mathematical discourse in his lesson on 'Let us teach guessing'.

In addition research done on ‘Mathematics Education at Highly Effective Schools That Serve the Poor, Strategies for Change’, Kitchen *et al.* (2007) identified central themes that emerged across the case-studies of schools that earned this accolade. One key feature was the strong focus on communicating in the mathematics classroom (p.107). One teacher explained: ‘I think it is wonderful if the kids can explain concepts, because I think then they personalise it and it helps them understand. Perhaps you internalise it [a concept] because you can repeat it and re-express it’ (*ibid.*). In another school it was noted that many of the teachers ‘valued students communicating their mathematical ideas in class’ (p. 81).

Currently, a review of mathematics learning for 3-8 year olds in Ireland is underway, establishing principles that underpin a view of mathematics that aligns with the objectives of post-primary mathematics syllabuses in Ireland (National Council for Curriculum and Assessment (NCCA) 2014b). The NCCA’s research report quotes Vygotsky (1986) who also emphasises the importance of articulating what works best. He also indicates the importance of the child interacting with adults and peers that have more prior knowledge and experience. This is followed by a detailed example of a child learning how to complete a jigsaw. It describes how a child may observe others (adults or peers) use strategies such as turning pieces, trying pieces, focusing on the shape, size or colour of pieces. Vygotsky’s study associates the verbal expression with action in an effort to promote higher order cognitive skills:

Speech accompanying these actions may then be internalised by the child to provide self-monitoring or self-regulating strategies that can later be called on to solve similar problems.

(ibid, p.30)

This hierarchy of development is reflected in levels of learning in the psychomotor domain from Bloom’s Taxonomy that describes learning of physical skills detailed by Dave (1967, cited in Huitt 2003): imitate, manipulate, precision, articulate, naturalisation. Consequently, as mathematics is a discipline about ‘actions’ characterised by skills to perform procedures, formulate and solve problems, explain, justify and communicate, (National Council for Curriculum and Assessment 2013a,

p.6), the concept of linking language with action in the learning trajectory has relevance.

2.2.4 Conclusion to Interpreting Literacy

In summary, the literature debates the value of employing explicit literacy instruction for mathematics. However, the learning objectives for mathematics in Ireland originated in Kilpatrick *et al.*'s (2001) work embody literacy elements which in turn generate mathematical activity.

The literature also endorses the idea that literacy development is a direct result of learning in action from more experienced others. This theory can inform this study by employing the elements from Fey's (1970) 'Patterns of Verbal Communication' as a lens to explore pedagogic discourse. While Truxaw and de Franco's (2007) analytical tools are more detailed than Fey's, a more simplified framework of classroom discourse is more appropriate for pre-service teachers who are in the process of learning the art and craft of teaching.

The next part of this section will explore literature in the area of mathematics teaching that includes other studies on the impact of expert models.

2.3 Mathematics Teaching

Ed Begle, who is best known for his role as director of the School Mathematics Study Group credited for the New Math movement in the 1960s, wrote: 'we have learned a lot about teaching better mathematics but not much about teaching mathematics better', (Crosswhite 1986 cited in Brumbaugh 1997, p. 26). Also, Fey (1970) argues: 'but even if [such] research produces a comprehensive theory of learning, a teaching theory will not necessarily follow as a routine corollary' (p.2). Therefore, this section begins by citing studies that examined teacher education; it then explores the literature that has conflicting findings with regard to theories of learning and professional practice in real life settings. The section concludes with the literature that describes

teacher knowledge with specific reference to teaching theories for mathematics teaching.

2.3.1 Teacher Education

Anita Straker, who directed England's national numeracy project in 1997, is quoted by the Inspectorates from the North and South of Ireland who worked collaboratively in identifying best practice in numeracy teaching in both jurisdictions as saying: 'numeracy is what you develop when you learn mathematics well' (Ireland, Department of Education and Skills 2015b, p.2). A corollary to this is the following statement: numeracy is what you develop in students when you learn to teach mathematics well. However, Hattie's (2009) seminal work 'Visible Learning' that ranks influences in student learning from 800 meta-analyses in order of effectiveness places teacher education programmes at 124 with students' estimation of their own performance in first place. Other contributions from the teacher that were deemed significant and influential were teacher clarity, ranked eighth, and micro teaching, ranked fourth. Meanwhile, the research does conclude that teacher education programmes play a pivotal role in 'building lenses and conceptions' (p.111) to prepare pre-service teachers for their professional role but greater variation is seen in mathematics teaching than other disciplines. In addition, Coben *et al.*'s (2007) study on effective teaching and learning practices for numeracy for adult learners found traditional teaching methods in classes with both high and low learner performance, but the use of practical activities was more prevalent amongst low attainment groups. Furthermore, participants in the study described good numeracy teachers having the attribute of being able to explain the content well.

Ball (2012) in a lecture entitled: 'Great teachers aren't born..They're taught' identifies features of strong training for skilful responsible practice through a hierarchy of observation, simulation, supervision and independent practice. One theory underpinning the idea of observation and simulation comes from Bandura's (1977) social learning theory and vicarious learning. He argues that most human behaviour is learned by example by a competent or incompetent model. He emphasises the costs

and long term damage if the influencing example is not an expert or positive role model.

These ideas were developed and formalised in a situated learning theory by Lave and Wenger (1991) by delineating positions and roles for individuals in the act of modelling. It is built on the belief that what is to be learned happens in the context in which the learning will be used. There are three constructs that underpin this theory:

- Expert models,
- Community of practice,
- Legitimate Peripheral Participation.

Expert models are key elements in situated learning and collaboration. Activity and interaction are also important processes if learning is to take place (Huggard 2015). These experts are members of a community of practice that interact with each other but also reach out to beginners on a learning trajectory who are on the periphery. The practice aspect is important as it represents that members are administrators or agents of a domain of knowledge and work to sustain and improve it (Wenger-Trayner and Wenger-Trayner 2011). A novice or newcomer in the social learning frame is positioned outside the community of practice and engages in a process of legitimate peripheral participation to advance their position on the novice-expert hierarchy and eventually become an authorised member of the community of practice. This community includes old-timers relative to newcomers and within the old-timer community, there are masters. Wenger (1998) further developed the concept of community of practice by introducing the concept of reification whereby tools and artefacts that articulate and represent the knowledge are produced by community members. However, participation and reification are not mutually exclusive constructs but come as a pair.

Furthermore, Collins, Brown and Holum's (1991) study explores the notion of cognitive apprenticeship: 'to make real differences in students' skill, we need both to understand the nature of expert practice and to devise methods that are appropriate to learning that practice' (p.2). However, O'Connell (2009) discusses the negative

implications of ineffective examples on mathematics teaching: ‘most teachers learned their content through the same ineffective methods that educational reformers are endeavouring to replace’ (p. 967).

Research that emphasises the importance of teacher attitudes, dispositions, efficacy and competence is also relevant here. Golding (2017) compared two mathematics departments working in different schools in England facing reform in mathematics for students in the 14-16 age group. One included experienced and knowledgeable practitioners while the other department was comprised of novice teachers. Interestingly, the less experienced group achieved more positive gains in the implementation of the reform through support mechanisms such as collaborative learning and reflective practice. The other group appeared to be constrained by a drop in state examinations results in the previous year, and perhaps a belief because of their prior knowledge and experience that supporting mechanisms were not required. In addition, Liou *et al.*’s (2017) study on dispositions of pre-service teachers and their performance in mathematics teaching found a positive correlation between peer social and emotional support and self-efficacy with teaching performance. Interestingly in Evans’ (2011) research on teacher self-efficacy in a New York City Teaching Fellows mathematics methods programme for out-of-field teachers, learning in the domains of numeracy and problem-solving were most important.

It is clearly evident from the literature that expert model provision and factors related to supportive collaborative practice are requisite to good teaching outcomes amongst novice teachers.

2.3.2 Learning Theory and Professional Practice

Thompson (1992 cited in Kitchen *et al.* 2007, p.11) described how the purveyors of an academic discipline could view the knowledge of that discipline as an Instrumentalist, a Platonist or as a Problem Solving process as presented in Table 2.2. The problem solving definition could be similarly characterised as constructivism.

Table 2.2 Perspectives on knowledge

<i>Knowledge type</i>	<i>Description</i>
Instrumentalist	views knowledge as an isolated body of skills
Platonist	sees knowledge as a body of connected ideas
Problem Solving	sees knowledge as a process of inquiry that is continuously expanded by human creation

Delaney's (2005) research in an Irish context illuminated how the implementation of the intended curriculum is vulnerable because of how a teacher identifies with each of these three positions. He gives the example of a Primary school teacher who held an instrumentalist view as best practice for mathematics teaching which was in conflict with the Primary Mathematics curriculum that promoted a problem solving/constructivist view of knowledge (National Council for Curriculum and Assessment 1999). The instrumentalist approach shares similar dominance in Irish post-primary classrooms described in the NCCA's response to the debate on Project Maths in October 2012:

while teachers in Ireland reported that they favour a constructivist approach to teaching and learning, there was greater classroom emphasis on the use of structured didactic practices (p.3).

Therefore, this literature offers contradictory findings between Irish teachers' view of knowledge and the unpacking of the knowledge for students in practice. But a Platonist view of knowledge is represented in Ireland's Junior Certificate mathematics syllabus (National Council for Curriculum and Assessment 2013a, p.10) which was also developed in a Maths Development Team workshop for Irish mathematics teachers advocating the value of 'using and valuing connections' (Maths Development Team n.d.c.). However, Guerrero (2014) remarks: 'these types of connections require depth of content knowledge and proficiency in teaching mathematics for understanding that many Irish maths teachers are unfamiliar with at this time' (p.46).

However, this is not a uniquely Irish problem. There is consensus in the literature about the educational value of constructivism (Olson 2001) as a learning theory in

mathematics teaching, but the educational practice of behaviourism from Thorndike to Skinner prevail in many countries (Noddings 1990, English and Halford 1995, Brumbaugh, 1997). Klinger (2009) argues: ‘while constructivism dominates current pedagogy, I suggest that there are profound flaws in the context of mathematics and numeracy education’ (p.157). He describes a constructivist textbook with colour, real world contexts and experiments as a ‘veneer’ covering the same core materials that existed in pre-constructivists textbooks. While Noddings (1990) is an advocate of constructivism for mathematics teaching to develop cognition and method in learners, its shortcomings are also emphasised: ‘classroom conditions force us to think about instructional economies’ (p. 16) and ‘teaching this way requires considerable mathematical knowledge as well as pedagogical skill’ (p. 17).

The issue of content knowledge is also addressed by Ma (2010) whose seminal research, compared the mathematical knowledge of U.S. and Chinese elementary school teachers. Even though U.S. teachers spent more years in education, Chinese teachers’ depth of mathematical understanding and skill in communicating mathematical knowledge was superior. Ma coined the term Profound Understanding of Fundamental Mathematics (PUFM) which she characterises as a domain of knowledge that encompasses mathematics content and pedagogical skill. PUFM is specifically defined as an understanding of ‘the terrain of fundamental mathematics that is deep, broad, and thorough’, (p. 120). This knowledge domain is a pre-requisite to teach using a Platonist or Problem Solving approach previously mentioned.

In addition, Grannell *et al.*’s (2011) report on the promoted methodologies to be practised in Irish classrooms claim the approaches originate in theories of situated learning and constructivism. The authors coined a new phrase ‘context constructivist’ (p.10) to reflect what they consider to be a combined approach of these two learning theories. The Irish academics who co-authored this report welcome this approach but believe it is overemphasised and recommend this method should be practised in Junior Cycle rather than Senior Cycle (p.11). However, Guerrero (2014) identified the school structural deficiencies in the Irish system that mitigate against the implementation of more diverse pedagogies and teacher reflection. She describes her own teacher

education scenario in the U.S. where she would prepare one lesson but teach it three times in a day to different class groups for a longer period. This afforded an opportunity to reflect on the success of the learning and the methodologies. Recently efforts to support Irish teachers to reflect on their teaching of mathematics is provided through the provision of lesson study and reflections on practice groups which is supported by the Maths Development Team but participation is not mandatory (n.d.e).

In Australia, the New South Wales Department of Education (n.d.) curriculum support for numeracy and mathematics showcase Newman's work (*ibid.*), whose Newman Error Analysis and Newman's Prompts have given Australian primary teachers practical tools to support and guide pupils when they cannot understand a word problem. Her research illustrated that addressing mathematical difficulties with repetitive algorithm practice without any attention given to comprehension and transformation was futile (White 2010). However, from her experience and research, she identified structured guidance was necessary to facilitate the development of reasoning and strategic competence in children (Newman 2015).

In summary, the literature illustrates that human inquiry needs strong elements of clarity, direction and guidance and although constructivism is a very valuable learning theory to be enacted in mathematics classrooms, there are numerous variables such as teacher expertise, curriculum timetabling and lack of teacher reflection that mitigate against its success.

2.3.3 Teaching Mathematics and Teaching Theory

Recent studies in mathematics education have provided significant evidence that teaching mathematics better must be central to the debate to address improvement in mathematics education outcomes for students (Tatto and Senk 2011). One such study, the Learning Mathematics for Teaching (LMT) (2011) project, described a mathematics lesson to teach the concept of circumference and pi where the teacher began the lesson by reading from a story book entitled: 'Sir Cumference and the Dragon of Pi'. The students were immediately engaged and this was followed by an activity where pies were cut from paper and the circumference was measured. Yet,

despite the novelty elements of the lesson, the team felt the lesson was weak: the teacher's definition of the circumference lacked precision, the circumference/diameter relationship was not examined and 'it was also unclear whether the use of 'pies' to talk about 'pi' [had] created confusion' (p.27).

Hodgen (2011) further illustrates the issue of teacher knowledge of mathematics by giving a detailed account of the explanation given by an experienced primary school teacher in representing the following numerical expression with a story, diagram or model:

$$1\frac{3}{4} \div \frac{1}{2}$$

This teacher was also a numeracy consultant responsible for developing lessons on multiplication of fractions for professional development programmes. However, in this situation, the teacher performed the calculations correctly but was not successful in generating models and had difficulty devising stories to represent the problem (p.31). Using the same fraction division problem, Ma's (2010) research, first published in 1999, demonstrated Chinese teachers' ability to extend student learning with the construction of a mathematical proof by adopting different approaches. This was an example of their 'Profound Understanding of Fundamental Mathematics' (PUFM), as previously mentioned, in contrast to their U.S counterparts who spent more years in formal education but finished with less mathematical knowledge for teaching. In addition, Hodgen (2011) describes a group of prospective mathematics teachers, several of whom had doctorates in mathematics encountering difficulties in explaining why 'one multiplies the numerators and the denominators to multiply fractions but one does not add them to add' (p. 27). In Ireland, the Teaching Council of Ireland's (2013) approach to addressing the issues of mathematical content knowledge has been to stipulate the study of specific advanced courses in mathematics as a prerequisite to register to teach secondary school mathematics in Ireland from 1st January 2017.

According to Shulman (1986) the above examples of teacher content knowledge affordances and deficits were the ‘missing paradigm’ in teacher education research in the eighties. He sought to fill this gap and launched Pedagogical Content Knowledge (PCK) as a construct that has since infiltrated many teacher education programmes in Australia and elsewhere (Finger *et al.* 2014). Shulman (1986) defines PCK as:

the most regularly taught topics in one’s subject area, the useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing the subject that make it comprehensible to others (p.9).

However, PCK is not only a contemporary phenomenon in education and historical evidence demonstrated the existence of competency tests in the area of subject content knowledge and pedagogy from the late nineteenth century (*ibid.*).

O’Meara (2011) describes five models of teacher knowledge that include: Shulman; Ernest; Fennema and Franke; Rowland; Ball, Thames and Phelps and includes her own model titled ‘the ladder of knowledge’ (p. 169). This study emphasises that despite their differences, each model positioned content knowledge as a foundational and anchor component to other elements. Similarly, the classroom observation instrument that was used to assess ‘Mathematics Education at Highly Effective Schools that Serve the Poor’ (Kitchen *et al.* 2007) included dimensions such as teacher depth of knowledge (p.176) and mathematical discourse (p.177).

Exploring the theme of mathematical discourse further, Meyer (2007) lists attributes of great teachers (clear writer, confident speaker, eloquent communicator, lucid thinker) that can be correlated with elements of Ireland’s definition of literacy, which references reading, writing, listening, understanding and critical appreciation as key proficiencies (Ireland, Department of Education and Skills 2011). One of the key aims and objectives of the current mathematics syllabus was for teachers to highlight the central role mathematics has in facilitating the development of thinking habits, communication skills and solving problems for life and work and to engage in pedagogies that promoted these skills and competencies. Usiskin (2012) believes being familiar with the language of mathematics is a precursor to all understanding, a

finding substantiated by Schleppegrell (2007). What also emerges once again in the literature is the idea of working with ‘experienced interlocutors’ as essential for students to engage meaningfully with the language of mathematics, as well as the learning of mathematics (*ibid.*). Furthermore, Pollak (2007), the pioneer of mathematical modelling in mathematics education, asserts knowing how to explain mathematical work is a necessary part of the learning process. That is, it is not enough to know how to do mathematics without knowing how to explain it.

A study by Kinach (2002), that explored ways to promote better instructional explanations of integer addition and subtraction among pre-service teachers of mathematics, produced interesting findings. Version A had pre-service teachers researching and debating specific methodologies for integer addition using visuals and concrete materials such as the number line and algebra tile unit squares to guide the learning. Version B allowed the pre-service teachers to use any method of their choice. Version A, the more guided approach, was more successful in deepening teacher knowledge.

The Learning Mathematics for Teaching (LMT) (2008) project, mentioned previously, examined mathematics teachers’ knowledge in the dynamic context of the classroom. The work emerged from the Study of Instructional Improvement (SII) (University of Michigan 2010) that examined whole school reform initiatives on teaching and learning practices in the U.S. The Learning Mathematics for Teaching project developed video coding measures for mathematics teaching titled ‘Mathematical Quality of Instruction’ or MQI (Hill *et al.* 2008). This observation instrument was developed to examine, analyse and assess mathematical instruction for elementary and middle school teachers. The novel factor of this instrument was its concentration on the quality of the mathematics being taught, in contrast to the identification and measurement of pedagogical strategies such as group work or activity based learning (Learning Mathematics for Teaching 2011). The Mathematical Quality of Instruction (MQI) instrument has five domains (Harvard University Center for Education Policy Research 2018a):

Classroom Work is Connected to Mathematics;

- (1) Richness of the Mathematics;
- (2) Working with Students and Mathematics;
- (3) Errors and Imprecision;
- (4) Common Core Aligned Student Practices.

Under each domain there is a taxonomy. For example, the domain ‘richness of mathematics’ has two elements *meaning-making* and *mathematical practices* (*ibid.*). Meaning-making comprises of observable factors such as illustrating which answer, or why a mathematics procedure or solution works, linking between representations of a concept and making connections between mathematical ideas. These components align well with Ireland’s Project Maths Continuous Professional Development pedagogic messages from 2010-2015 delivered in workshop format. For instance, *Workshop 4 – Patterns* introduced the practice of making connections in the teaching of algebra (Maths Development Team n.d.c). This idea was developed in *Workshop 6 – Exploring connections and reasoning leading to proof* and *Workshop 9 – Connections and integral calculus* (*ibid.*). In addition, the features of the element *mathematical practices* had a similar focus in the previously mentioned workshops as well as referencing literacy and numeracy components. This element describes the facilitation of multiple ways of arriving at a solution (workshop 6), generating patterns and generalisations (workshop 4) and precision in the use of mathematical language (workshop 9). Guerrero’s (2014) study also identifies similarities between the reform in mathematics education in the U.S. referred to locally as the Common Core Standards Initiative (2018) and Ireland’s Project Maths. She argues for examining affordances and hindrances in the Irish rollout of Project Maths to support teacher change and curriculum implementation both in Ireland and the U.S.

2.3.4 Conclusion to Mathematics Teaching

In summary, the dimensions of the U.S. conceived Mathematical Quality of Instruction (MQI) instrument are applicable to the Irish post-primary level context because the objectives in the Irish syllabuses (National Council for Curriculum and Assessment

2013a, 2013b, 2017) align with the U.S. (Common Core Standards Initiative 2018). Components from the literacy and numeracy definitions are also referenced. Since the overarching aim of this study is to explore literacy and numeracy strategies to improve mathematics instruction in post-primary mathematics classrooms, the Mathematical Quality of Instruction (MQI) protocols have the capacity to guide and support this work.

2.4 Conclusion

As illustrated, an abundance of literature exists on numeracy, literacy in mathematics, teachers' knowledge of mathematics and pedagogical teaching knowledge for the mathematics classroom. However, the author's interest is in knowledge and pedagogical knowledge of literacy and numeracy in the mathematics classroom with specific attention to the Irish context and no research has been found that has explored this construct. Also, while the study is situated in an Irish context, the literature does illustrate that problems and issues exist regarding the coherent transmission of pedagogical messages from policy documents to classrooms.

However, the predominant issue that did emerge from the literature is that a definition of Literacy and Numeracy for Mathematics Teaching does not exist and without this, the study has no foundation. For the remaining chapters, the study will demonstrate how the separate and unconnected elements for LNMTI were woven together and unified as a singular domain of research and learning. The next chapter will outline the methodological approach and the instruments that were employed to achieve this key objective in the first phase of the research as well as details of the other phases that were generated as a result.

Chapter 3: Methodology

3.1 Introduction

The author's study is in the field of mathematics education. If the two key purposes for mathematics education research, as Schoenfeld (2000) names them, are to understand the nature of mathematical thinking, teaching and learning and to apply such understandings with the aim of improving instruction then these components are captured in the following aims for this study:

- (1) To understand the nature of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI);
- (2) To investigate if pre-service teachers of mathematics are prepared to implement LNMTI;
- (3) To explore strategies for supporting pre-service teachers to implement LNMTI;
- (4) To assess the potential of an LNMTI intervention.

The means or processes by which these aims are achieved are the subject of this chapter. However, Cohen *et al.* (2011) describe research as processes not in a mechanistic and technical sense but as a craft that has inherent guidelines and procedures (Romberg 1992). As part of the crafting process, Creswell (2009) stresses the need to identify the philosophical assumptions that underpin the practice of research, the paradigm that guides the research and the theoretical framework that informs the procedures of the research (Creswell 2007). Therefore, this chapter will describe for the reader these three processes as well as the concepts and author's world view that are expressed in this study. As illustrated in Figure 3.1, the philosophical assumptions are defined by the author's ontological and epistemological positions which informed the interpretivist paradigm. Secondly the theoretical framework was framed by three structures presented by Kieran (2016) as grand, intermediate and domain specific.

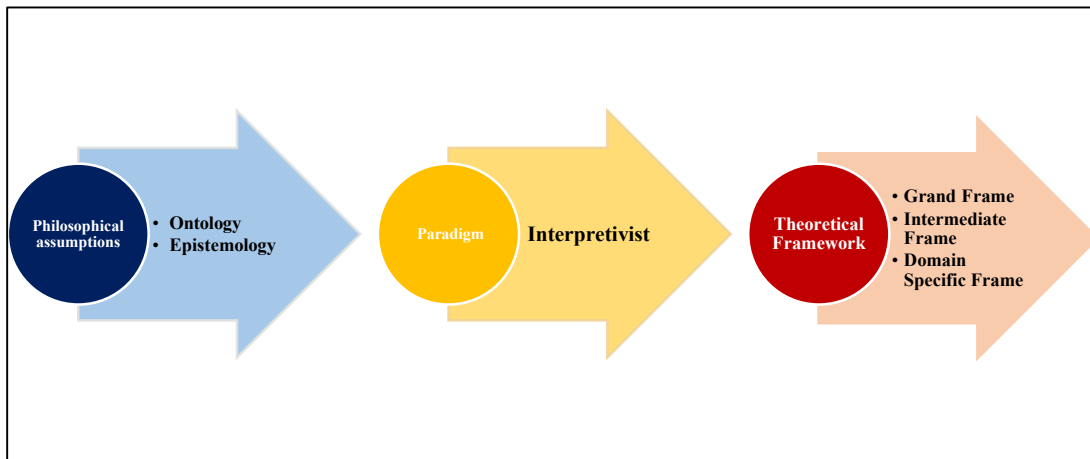


Figure 3.1 Philosophical processes that underpin the author's study

The reader will also gain insight into the rationale for implementing Educational Design Research (EDR) as the epistemic methodology that underpins the study and the data collection instruments used. Issues of credibility, trustworthiness as well as ethical obligations are also described.

3.2 Purpose and Main Research Question of this Study

The purpose of this research is to investigate how to support pre-service teachers of mathematics' learning and understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that they can enact this knowledge in post-primary mathematics classrooms in Ireland. To embark on achieving this purpose, firstly, the domain of Literacy and Numeracy for Mathematics Teaching in Ireland would require a valid definition and secondly practical teaching and learning tools would need to be constructed. What is meant by 'valid' in this case is that the definition must be aligned with current Irish educational policy. In addition, the definition must be conveyed into practical applications if pre-service teachers are to teach mathematics using literacy and numeracy as foundational knowledge. In line with this purpose the overarching research question is:

What is the construct of Literacy and Numeracy for Mathematics Teaching in Ireland and what are the characteristics and the potential of teaching and learning strategies to teach pre-service teachers of mathematics about literacy and numeracy in such a way that they experience coherence between mathematics, literacy, numeracy and teaching in an Irish context?

3.3 Philosophical Assumptions

Ontology is defined as the nature of the reality or phenomenon (Cohen *et al.* 2011) and epistemology is referred to as the theory of knowledge or more simply put: ‘what can be known’ (Bates and Jenkins 2007, p. 62). In this study, ontology is the nature of Literacy and Numeracy for Mathematics Teaching in Ireland and epistemology is the justification and evaluation of this knowledge (Carter and Little 2007). The next section of this chapter will describe these positions in relation to the author’s study in more detail.

3.3.1 Ontology

The author is conducting this study in the domain of mathematics education in the Irish educational context. The perspective on mathematics education expressed by the government funded education bodies in Ireland, National Council for Curriculum and Assessment (NCCA 2017), Professional Development Service for Teachers (PDST n.d.b), Maths Development Team (MDT n.d.c), in a reform oriented climate embodies features such as inclusive learning environments and constructivist pedagogies but also emphasises the utility of mathematics ubiquitously referred to as numeracy. This is not a new idea and Kilpatrick’s (1992) ‘History of Research in Mathematics Education’ devotes a section to ‘The Social Utility Movement’ (p.17) where he describes various reports written from 1845-1930 on the importance of context based, useful mathematics to be taught in American schools. This philosophy stems from the Piagetian notion of knowledge as an evolving system that adapts to environmental circumstances in order to survive (Ernest 1997).

Von Glasersfeld (1995) developed Piaget’s theory by describing the interaction between acquiring knowledge and knowing. Given the same knowledge stimuli, individual learners will assimilate and construct knowledge elements based on prior knowledge that has been clearly understood and deeply processed. Implicit in this description is the notion of subjective knowledge unique to each individual, context and circumstance. Therefore, there is no ultimate reality but many realities. The title of this study implicitly acknowledges the existence of many realities whereby the Irish

situation is specified because the nature of Literacy and Numeracy for Mathematics Teaching conveys different meanings to different countries. For example, TIMSS numeracy was developed as a more straightforward mathematics assessment for learners in education systems less developed than Ireland's (International Association for the Evaluation of Educational Achievement n.d.). Moreover, philosopher and mathematician, Blaise Pascal expressed a similar ontological position 'what can be true on this side of the Pyrenees [can also be] false on the other' (Tubbs 2016, p.3).

Furthermore, this ontology is particularly relevant to the teaching and learning context in Ireland where new programmes, new approaches and new assessments are evolving continually; and during a typical teaching career, syllabus changes will occur and the ability to adapt and integrate new standards is a professional requirement (Meyer & Paxson 2016). However, a recurring theme exists in the literature on the varied enactment of an education policy in the dynamic environment of a classroom. Boesen *et al.* (2014) write:

if a curriculum includes content goals such as arithmetic, then arithmetic is indeed taught but if the curriculum includes competency goals such as problem solving ability, then the effect on teaching may vary considerably (p.73).

In the same way, Gal (2000) highlights the issue of competency versus content goals in numeracy education. Bernstein's (1990) Social Construction of Pedagogic Discourse offers an explanatory theory for this phenomenon whereby educational policy is disseminated through continuous professional development programmes, university teacher education programmes and ultimately to schools and classrooms. Because of this filtering process, a unit of knowledge conceived at a macro level will ultimately undergo a metamorphosis producing many variants.

3.3.2 Epistemology

Braa & Vidgen (1999) refer to three epistemological approaches that aim for (1) prediction, (2) understanding and (3) change brought about by an intervention. They align the first and second orientations with positivism and interpretivism respectively whereas the third approach is not categorised. As previously described, the

construction of knowledge for this study, Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI), is localised and lacks uniformity. Baxter *et al.*'s (2006) study demonstrate this phenomenon where a conflict arose between the researchers' objective view of real life activities which they categorised as mathematical, whereas those enacting these activities did not believe this to be the case. Consequently, an interpretivist epistemology had to be considered (Cohen *et al.* 2011). This worldview was also demonstrated in the writings of Ancient Greece in Plato's Socratic dialogues. In one example, Meno, the slave-boy is seeking the answer to the question 'can virtue be taught?' and Socrates replies:

For I literally do not know what virtue is, and much less whether it is acquired by teaching or not? (Plato 2008, Meno)

In the same way for this study there is an absence of an objective, singular reality (Creswell 1994) and prior to conducting this research the author did not know what Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI) was and much less whether it required teaching or not!

The next section will elaborate on the author's interpretivist viewpoint for this study.

3.4 Research Paradigm: Interpretivist

From a static perspective, the study sought to define Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI). This was a subjective interpretation, in keeping with the interpretivist tradition. This view is endorsed by Ernest (1997) who notes that qualitative research in mathematics education in recent decades is driven by a view of mathematics as 'inseparable from human contexts and practices' (p.26). However, the dynamic element to this study heralds a change oriented approach in practice and pedagogy in mathematics education. Steen (1988) uses a tree and rain forest metaphor to convey the different conceptions of mathematics as a subject. He describes scientists and engineers viewing mathematics as a domain that produces knowledge (formulas and theorems etc.) like ripe fruits ready to be harvested, which underpins positivism. On the other hand, he believes mathematicians see mathematics

as a rain forest ‘contributing to human civilisation a rich and ever-changing variety of intellectual flora and fauna’ (p.611), an interpretivist viewpoint. Dossey (1992) describes how these two opposing world views impact on the teaching and learning of mathematics. Goldkuhl (2012) asserts there are many forms of interpretivism, therefore the author has resolved this conflict by relying on ‘a paradigm of choices’ called post-positivism (Wildemuth 1993, p.451). Willis (2007) describes the goal of this philosophy ‘is to find the truth about something’. However, one study is not enough to uncover the truth but the research becomes part of a larger body of work that gets closer and closer to the nature of that reality. In addition, post positivism promotes scientific inquiry that reveals which beliefs are justified and which are not well-founded. For instance, the results of the data analysis in Chapter 5 of this study challenge the belief that mathematics teachers are automatically numerate.

Creswell (2007) outlines post-positivism in a qualitative research context with quantitative research attributes. The logical aspect of this belief system is emphasised in the way the researcher conducts the inquiry by designing a process that has cogent steps. It was this belief system the author adhered to when carrying out the research for this study.

Philips and Burbules (2000) delineate five key aspects to this paradigm. Table 3.1 lists these features with an example from the author’s study that situates it in this tradition.

Table 3.1 The post-positivism tradition for the author's study

<i>Post-positivism</i>	<i>Author's Study</i>
Knowledge is conjectural	The study aims to explore and understand the construct of literacy and numeracy for mathematics teaching in Ireland (LNMTI)
Research is the process of making claims and then refining or abandoning them for others	In an earlier draft of the research study, the focus was on Literacy for mathematics teaching in Ireland. This was refined to Literacy and Numeracy for Mathematics Teaching in Ireland
Data, evidence and rational considerations shape knowledge	The choice of Educational Design Research (EDR) as a methodology guided the production of knowledge artefacts
Research seeks to develop relevant, true statements	Literacy and numeracy for mathematics teaching are two sides of the same coin
Being objective is an essential aspect of competent inquiry	The author has chosen to write in a formal style by employing a third person narrative. Issues of reliability and validity are directly addressed

The next section will present the theoretical framework that was used to ensure that all beliefs for this scientific study were well-founded

3.5 Theoretical Framework

In this section, as illustrated in Figure 3.2, the three part structure of the theoretical framework: grand frame, intermediate frame and domain specific frame (Kieran 2016, Creswell 1994) is described. The grand frame is an overarching theory such as socio-constructivist, psychological etc. employed to explain in an abstract and general way the major elements of the variables in the study. The middle frame contains theoretical elements and definitions specific to this study and provides heuristic devices and tools to guide and evaluate. The final frame is domain-specific, which for this study included domains of literacy, numeracy and the enactment of mathematics teaching in an Irish context.

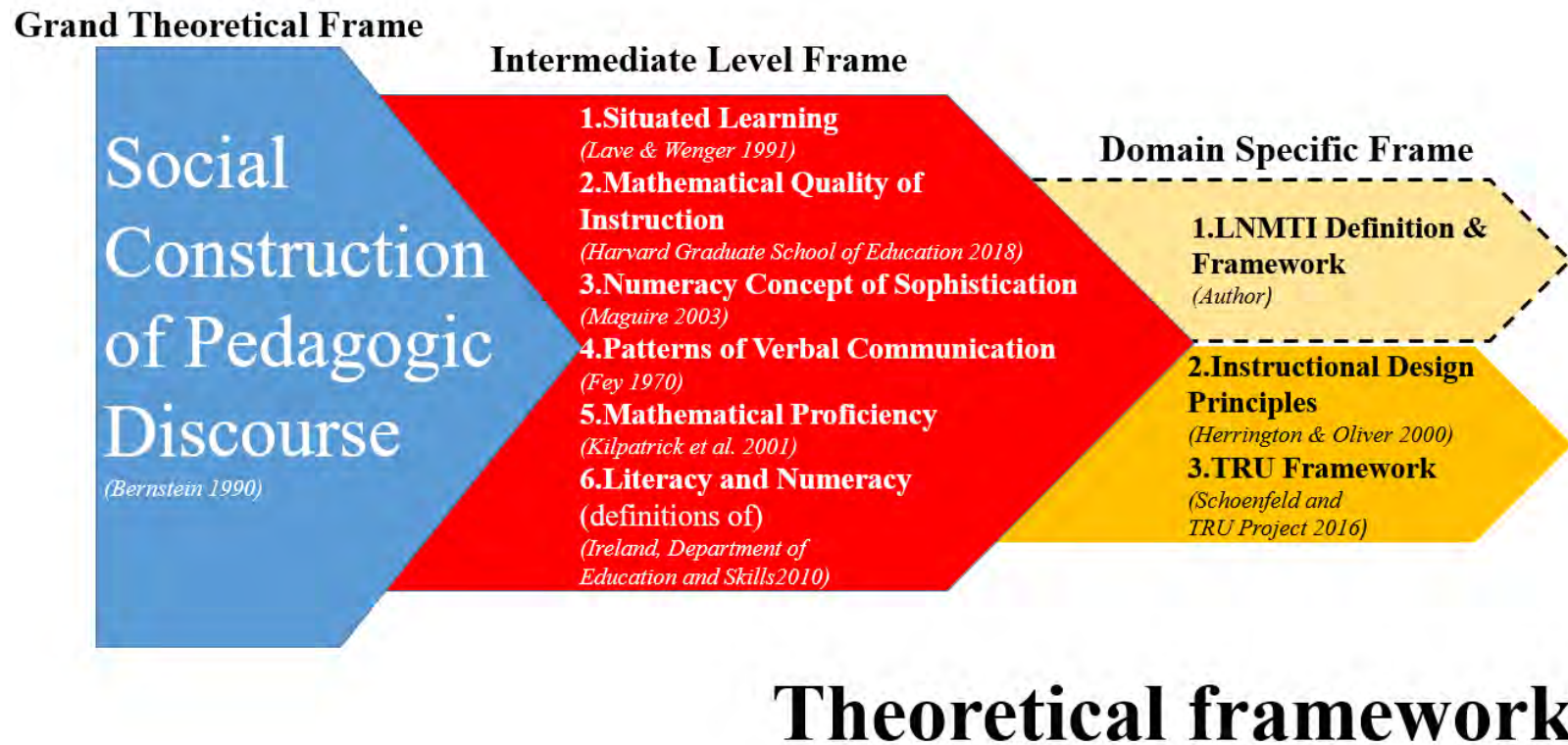


Figure 3.2 The theoretical framework for the author's study

3.5.1 Grand Theoretical Frame

As previously mentioned, the constructivist and social constructivist positions of Irish education policy makers focusses on student construction of knowledge in a collaborative way (University College Dublin, n.d.). These are presented in written documentation such as syllabuses, frameworks and specifications and expressed in Continuous Professional Development supports through workshop presentations and resource materials (National Council for Curriculum and Assessment 2017, 2013a, 2013b, 1999 and Maths Development Team n.d.c.). As shown by Lerman *et al.* (2002) from an audit of the *Education in Mathematical Studies* journal and the author's literature review, very few studies in mathematics education research explore discourse from policy and/or curriculum and how messages are transmitted to the education community. Secondly, Michel Foucault developed theories on discourse which he defined as statements belonging to a system (Heller 2016). This has a direct relevance to this study because the group of statements involved in expressing literacy and numeracy for mathematics teaching has been generated by the Irish education system. However, Bernstein's (1990) Social Construction of Pedagogic Discourse concentrated on both the message and how the message was being relayed (Clark 2005). This was important for this study because of the way educational reform is being relayed through written documentation coupled with the recontextualisation of the message by organisations such as university education programmes. Consequently, Bernstein's theory was chosen as the grand frame or macro level theory for this study as a lens to guide the exploration of the hierarchal transmission of messages from government agencies to university education programme providers, and then on to pre-service teachers in supporting them to facilitate the decoding of pedagogic messages for improved student learning.

3.5.2 Intermediate Level Frame

The intermediate frame can have theoretical or craft based elements (Kieran 2016) but it has a specific focus that includes heuristics, elements and tools to support and evaluate the research methods, processes and artefacts of the study. The author

included six sub-frames to make up the intermediate frame for this study because Literacy and Numeracy for Mathematics Teaching in Ireland is composed of separate components united under the one domain.

- (1) Situated Learning (Lave and Wenger 1991);
- (2) Mathematical Quality of Instruction for Literacy and Numeracy in Mathematics Teaching in Ireland (*MQI for LNMTI*);
- (3) Numeracy Concept of Sophistication – An Organising Framework (Maguire 2003);
- (4) Patterns of Verbal Communication (Fey 1970);
- (5) Mathematical Proficiency (Kilpatrick *et al.* 2001);
- (6) Definitions of Literacy and Numeracy from Ireland’s National Literacy and Numeracy Strategy (2011).

The following sections describe the relevance of the individual intermediate sub-frames for this study.

3.5.2.1 Situated Learning

Based on the principle that learning is a process of assimilation from doing and apprenticeship from experienced others in an authentic context, situated learning theory assigns roles to individuals and or organisations to examine this learning interaction. As outlined in Section 2.3.2, the learner is the new-comer who takes the position of legitimate peripheral participation, and through a process of collaboration and engagement enters the community of practice following a learning trajectory that is compliant with the culture of the community to become old-timers and/or masters (Lave and Wenger 1991). This study employed situated learning devices to assign personnel in this study a specific role to enable the evaluation and justification process, presented in Figure 3.3. For example, the pre-service teachers were in the legitimate peripheral participation role as newcomers to the mathematics teaching community at post-primary level; the author, teacher education programme tutors, for example, are practising members of the community. The author also engaged advice and expertise from other ‘old timers’ or masters in this community who had specialist knowledge

and experience in the field of this study in an Irish context. For instance, in deciding on the appropriateness of items for evaluating participants' knowledge of Literacy and Numeracy for Mathematics Teaching in Ireland as well as the rating of participant answers from the chosen questions, the author used an Expert Panel of individuals who were more skilled than the author in the relevant domains of literacy, numeracy and mathematics teaching.

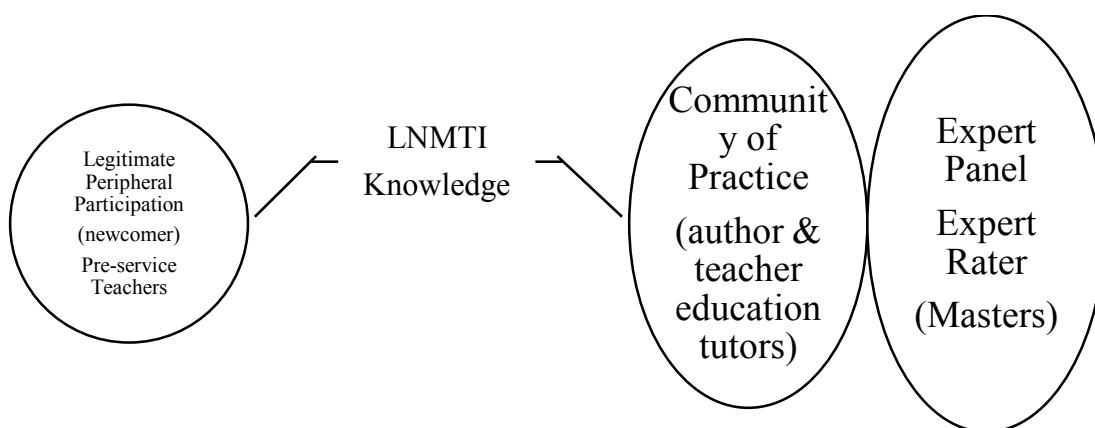


Figure 3.3 Situated learning theory for the author's study

3.5.2.2 Mathematical Quality of Instruction

Reviewing the literature on mathematics teaching revealed the existence of the Mathematical Quality of Instruction (MQI) observation instrument that specified attributes of mathematical instruction that focused on the teaching and learning of precise and accurate mathematics. While other reliable teacher observation protocols existed that were designed for post-primary classrooms such as the Reformed Teaching Observation Protocol and UTeach, (Gibbons 2016), these instruments were designed to accommodate both mathematics and science lessons whereas the Mathematical Quality of Instruction framework had a purely mathematics focus. This was created and developed by the Learning Mathematics for Teaching (2011) project from the University of Michigan who the author added to the 'master' category in the situated learning model because members of this project have an international reputation in mathematics education research. Also, one member of the team, an Irish academic, adapted aspects of this work for the teaching of mathematics in an Irish

context (Delaney 2010). Consequently, the pedagogic messages relayed from the U.S. and Ireland were comparable (Guerrero 2014), making the instrument viable for Irish mathematics classrooms. The heuristic underwent revisions (Learning Mathematics for Teaching (LMT) 2011; Learning Mathematics for Teaching (LMT) / Hill 2014) and the current Mathematical Quality of Instruction instrument has the following five dimensions (Harvard University Center for Education Policy Research 2018a):

- (1) *Common Core Aligned Student Practices* refers to how students in the class are engaged by the mathematics;
- (2) *Working with Students and Mathematics* is the extent to which the teacher addresses student misconceptions and/or mathematical contributions appropriately;
- (3) *Richness of the Mathematics* as described in Section 2.3.3 refers to how the teacher engages in mathematical practices such as using precise mathematical language or exploring patterns and generalisations and meaning-making by linking different representations of a mathematical idea or making connections between different strands of mathematical knowledge such as ratio and trigonometry;
- (4) *Errors and Imprecision* describes a teacher's lack of clarity and understanding while delivering the mathematical content;
- (5) *Classroom Work is Connected to Mathematics* refers to the extent to which the teacher facilitates a lesson that is focused on developing mathematics.

Hill (2011) emphasises the utility of the Mathematical Quality of Instruction instrument and suggests it can be used as outlined above or certain dimensions can be eliminated to accommodate a specific focus. Consequently, out of the five categories, the author has chosen two: *Richness of Mathematics* and *Working with Students and Mathematics*. They converge around aspects that directly reference the domains of literacy and numeracy and comply with pedagogic messages presented at Continuous Professional Development workshops delivered by the Maths Development Team during the national roll-out phase of Project Maths from 2009-2015 (Maths Development Team n.d.c; Guerrero 2014).

3.5.2.3 Concept of Numeracy

Numeracy as a domain of knowledge is central to this study but as previously outlined in Section 2.1, there are a multitude of published concepts of numeracy which represent the construct in a narrow sense; for instance, in the Irish Junior Certificate syllabus, numeracy is equated with basic skills in number (National Council for Curriculum and Assessment 2013a) while Australians Goos *et al.* (2012) offer a more broad, expansive interpretation. However, Maguire's (2003) *Numeracy Concept Sophistication – An Organising Framework* for adult numeracy accommodates both the narrow and the broad components of the numeracy concept. It includes three hierarchical phases known as: formative, mathematical and integrative:

- (1) *Formative* - describes basic arithmetical skills;
- (2) *Mathematical* - refers to real life applications of mathematics;
- (3) *Integrative* - 'incorporating the mathematics, communication, cultural, social, emotional and personal aspects of each individual in a particular context' (p.329).

These phases will support the positioning of numeracy moments for mathematics teaching as well as reflect the reform oriented approach to numeracy as a complex and sophisticated construct.

3.5.2.4 Patterns of Verbal Communication

The literature reveals research into the literacy aspect of mathematics teaching and learning is important (Pollak 2007; Schleppegrell 2007; Ní Ríordáin 2011; Usiskin 2012; O'Keeffe *et al.* 2017). The study centres on pre-service teachers of mathematics preparedness to embed literacy and numeracy skills in the mathematics classroom. Consequently, unlike Truxaw and DeFranco (2007), Fey's (1970) *Patterns of Verbal Communication in Mathematics Classes* offers a more simplified model for categorising the nature of mathematical discourse in the classroom setting. Originally developed by Bellack (1968), Fey investigates the patterns of verbal interactions to achieve a pedagogical purpose using moves that are sub-categorised as structuring, soliciting, responding, and reacting. As illustrated in Section 2.2.3, structuring refers

to how the lesson is launched; soliciting describes the extent to which verbal information or cognitive/physical action is obtained; responding indicates a response to a solicitation and finally reacting is demonstrated by providing clarification of a concept or rate what was said. The soliciting and responding moves also provide useful structures to verbalise pedagogic messages embedded in Ireland's current mathematical syllabus and Junior Cycle Framework.

3.5.2.5 Mathematical Proficiency

The elements of mathematical competence delineated by Kilpatrick *et al.* (2001) in their construct 'mathematical proficiency' are replicated in the written objectives in Ireland's mathematics syllabuses, see Figure 3.4. The metaphor of the strands intertwining in the visual representation of these elements is reflected in the message from the Department of Education and Skills' National Council for Curriculum and Assessment and the Mathematics Development Team about the discipline of mathematics itself and the learning of the subject. The issue of making connections has received much attention in the workshop materials provided to support teachers in alternative pedagogies in the classroom as well as the state examinations. The official message was that while the syllabus is presented in five distinct strands titled: Statistics and Probability, Geometry and Trigonometry, Number, Algebra and Functions, students should not experience the learning of mathematics in discrete blocks and opportunities to make connections between strands and within strands should be exploited (National Council for Curriculum and Assessment 2013a, p.10) Therefore mathematical proficiency heuristics for this study are necessary as they underpin the content, learning outcomes and direct mathematics instruction.

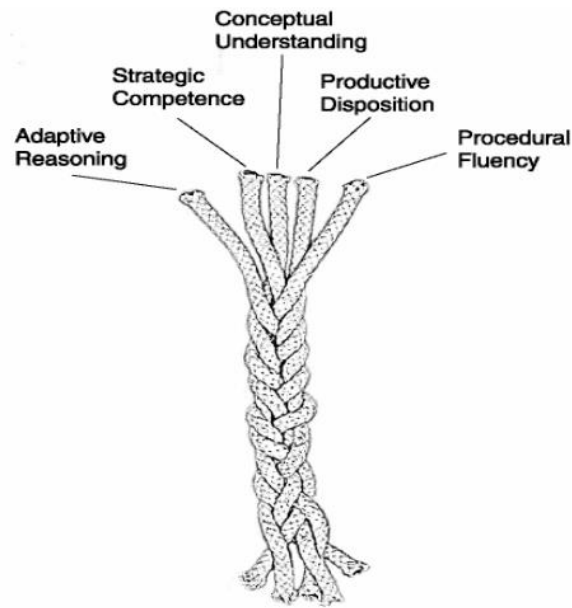


Figure 3.4 Kilpatrick et al. 's (2001) conception of mathematical proficiency

3.5.2.6 Definitions of Literacy and Numeracy

As Maguire's (2003) research showed by carrying out an international comparative study of how various countries conceptualised numeracy, Ireland was positioned in the formative stage with the available definition at that time taking on basic and foundational characteristics. Currently, in Ireland's National Strategy for Literacy and Numeracy, a much broader and more holistic view of both of these constructs are presented (Ireland, Department of Education and Skills 2011):

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media (p.8).

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems (p.8).

However, within this holistic approach fundamental elements can be identified such as cognitive domains of understand, use and critically appreciate; content domains such as patterns and sequences; and literacy form domains, for example digital media. These definitions are essential components of the theoretical framework for this study as they represent the message from policy makers in its purest form. They will be utilised to construct the definition and explain the meaning and nature of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI).

3.5.3 Domain Specific Frame

The domain of Literacy and Numeracy for Mathematics Teaching in Ireland was a field of study that did not exist before the author embarked on this research journey, therefore it had to be constructed from elements of the intermediate frame. Chapter 4 gives a detailed account of how the Literacy and Numeracy for Mathematics Teaching in Ireland framework emerged to help clarify and position the LNMTI concept and educational artefacts such as the LNMTI survey and intervention to express the concept. Moreover, the LNMTI framework was not sufficient to guide all of the research decisions for this study. For example, the design of the LNMTI module relied on the instructional design principles developed by Herrington and Oliver (2000). The four element intervention framework, effectiveness, integrity, acceptability and validity, to evaluate interventions developed by Shapiro (1987) was considered for guiding the development and the evaluation of the LNMTI module. However, the following nine design principles, presented in Table 3.2, were chosen because this domain specific frame emerged directly from situated learning theory, a component of the study's intermediate frame (Herrington and Oliver 2000).

Table 3.2 Instructional design principles (Herrington and Oliver 2000)

1	authentic contexts that reflect the way the knowledge will be used in real life
2	authentic activities
3	access to expert performances and the modelling of processes
4	multiple roles and perspectives
5	collaborative construction of knowledge
6	reflection to enable abstractions to be formed
7	articulation to enable tacit knowledge to be made explicit
8	coaching and scaffolding by the teacher at critical times
9	authentic assessment of learning within the tasks.

The 9th principle, ‘authentic assessment of learning within the tasks’, for this study prescribed an evaluation to take place in participants’ classrooms to observe the potential enactment of LNMTI. Consequently, another framework was sourced to guide this work. Schoenfeld’s Teaching for Robust Understanding (TRU) Framework (2016) for observational analysis in mathematics classroom was chosen because it had a number of relevant features to the Mathematical Quality of Instruction instrument. However, the Mathematical Quality of Instruction instrument was developed for observations from videoed classes whereas the Teaching for Robust Understanding framework focussed on live classroom interactions which was the case for this phase of the study. A more detailed treatment of this component of the theoretical framework can be found in Section 7.3.2.1.

3.6 Research Methodology: Educational Design Research

The theoretical framework supporting the study is built on published educational theories, key concepts, frameworks and models from the areas of literacy, numeracy and mathematics teaching. The purpose of this section is to demonstrate why the choice of Educational Design Research methodology that aims to provide practical solutions to educational problems (Sloane 2017) was the most suitable design methodology for the author’s study. However, during the early phase, the author

considered following a case-study design (Blatter and Haverland 2012) for three reasons:

- (1) The real life context aspect of the study with the opportunity to look at the contextual factors was valuable,
- (2) The research had exploratory, descriptive and explanatory elements,
- (3) Case study allowed the researcher to focus on the outcomes and assess the overall effectiveness of the strategies.

Moreover, Hattie (2009) and other researchers report on the multitude of research and studies rarely used by teachers (Design-Based Research Collective 2003; Nutall 2004; Open Universiteit 2013). Therefore, as the study evolved it became apparent that the design of artefacts (LNMTI definition, survey, module and classroom observation sheet) as stimuli for dialogue was necessary for the phenomenon under study to develop and grow as a body of knowledge. Secondly, as the research paradigm influences the research methodology (Van Merriënboer and de Bruin 2014), ‘reification’ is a characteristic of situated learning theory from the study’s theoretical framework referred to as the physical tools, devices or artefacts where meaning is expressed (Wenger 1998). Subsequently, the author decided to use Educational Design Research methodology as it was imperative to incorporate the design of educational artefacts as a product of the research process (Bakker and Van Eerde 2015).

Educational Design Research has many identities (Open Universiteit 2013):

- Design Based Research,
- Designed Experiments,
- Formative Research,
- Development Research.

Bakker and Van Eerde (2015) opt to use the title ‘Design Based Research’ (p. 437) because the emphasis is on the *Research* element. In the same way, this author has chosen Educational Design Research as the label for this study because it suggests that

it is predominantly *Education Research* based on design mechanisms. Also, Richter and Allert (2017), proponents of Educational Design Research, reference the former president of the Educational Research Association who views education not as a natural phenomenon but a man-made process designed to meet human needs requiring innovative solutions to make it better (p.1). Therefore, McKenney and Reeves (2012) describe Educational Design Research's dual purpose:

- (1) The design and development of an educational artefact: a practical contribution
- (2) A contribution to theory

As illustrated in Figure 3.5, they also present a generic model that describes the Educational Design Research process (p.77).

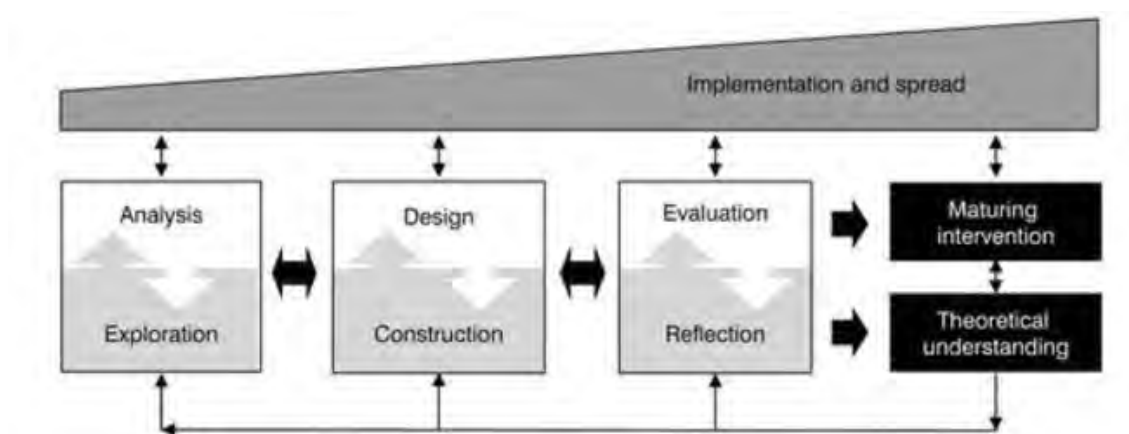


Figure 3.5 Generic model to conduct Educational Design Research (McKenney and Reeves 2012)

Distinct but iterative elements of the generic model for Educational Design Research have been positioned in three distinct geometric shapes: squares, rectangles and trapezoids. The squares comprise the three core phases of Educational Design Research: analysis/exploration, design/construction, evaluation/reflection; the rectangles incorporate the outputs of a design process which are labelled 'maturing intervention' and 'theoretical understanding' or as previously described: an educational artefact and a contribution to educational theory. Finally, the trapezoid points to the Educational Design Research's robust connection with practice and is

aply named ‘the implementation and spread’ phase. In addition, McKenney and Reeves (2012) describe the completion of one of the core phases, i.e. analysis/exploration, design/construction, evaluation/reflection as a micro-cycle. A meso-cycle is one completion of the three core phases or alternatively it can be a completion of two micro cycles such as design/construction followed by evaluation/reflection. A macro-cycle is generated when at least two meso-cycles are completed.

As summarised in Table 3.3, the author’s study completed one full macro cycle during the four phases of the research. A more detailed treatment on these Educational Design Research cycles will be discussed in the next section.

Table 3.3 Four phase Educational Design Research: macro-cycle and meso-cycles

Macro Cycle	<i>Phase</i>	<i>Analysis/ Exploration</i>	<i>Design/ Construction</i>	<i>Evaluation/ Reflection</i>
	<i>1</i>	Literature Review	LNMTI Definition	Expert Panel
	<i>Meso</i>	Content Analysis	LNMTI framework LNMTI Survey	Pilot
	<i>2</i>	LNMTI Survey		Post Survey Focus Group
	<i>3</i>	Priority Issues from	Prototype	LNMTI framework
	<i>Meso</i>	LNMTI Survey and Focus Group Context Analysis	LNMTI Module Paper Evaluation	Instructional design principles Hypothetical learning trajectories Paper Evaluation
	<i>4</i>	Survey	LNMTI Classroom	Contribution to Theory
	<i>Meso</i>	Expert Appraisal Post Intervention Focus Group	Observation Sheet	

3.7 Research Design

The aims of this research study reflect Sierpinska *et al.*'s (1993) description of key purposes for mathematics education research as 'fundamental' and 'pragmatic' (p. 275). Implicit in these two purposes are approaches that are exploratory and practical in nature. Consequently, an exploratory study was designed for three specific purposes which are captured in phase 1 and phase 2 of the study:

- to define and characterise the new knowledge domain: LNMTI,
- to determine the scope of pre-service mathematics teachers' strengths and difficulties in LNMTI,
- to record data that might indicate a gap that exists between current LNMTI knowledge and the desired level set out in syllabus/specification documents, Junior Cycle Framework and Ireland's National Strategy for Literacy and Numeracy.

The results of the exploratory study informed phase 3, which was the design of an intervention that aspired to:

- identify, name and translate literacy and numeracy skills into learning outcomes for the mathematics classroom,
- support pre-service teachers of mathematics to enact LNMTI in the classroom.

This was followed by phase 4 where the observation of pre-service teachers in real classroom settings to ascertain the level of enactment of LNMTI was designed and analysed.

Figure 3.6 presents the chronology of the research, the logical development in the research phases guided by the key research questions and also highlights the various milestones that were achieved during the research process. The narrative that follows includes a detailed description of the research phases and the various data collection instruments that were utilised but it will begin by describing the sampling method that was employed.

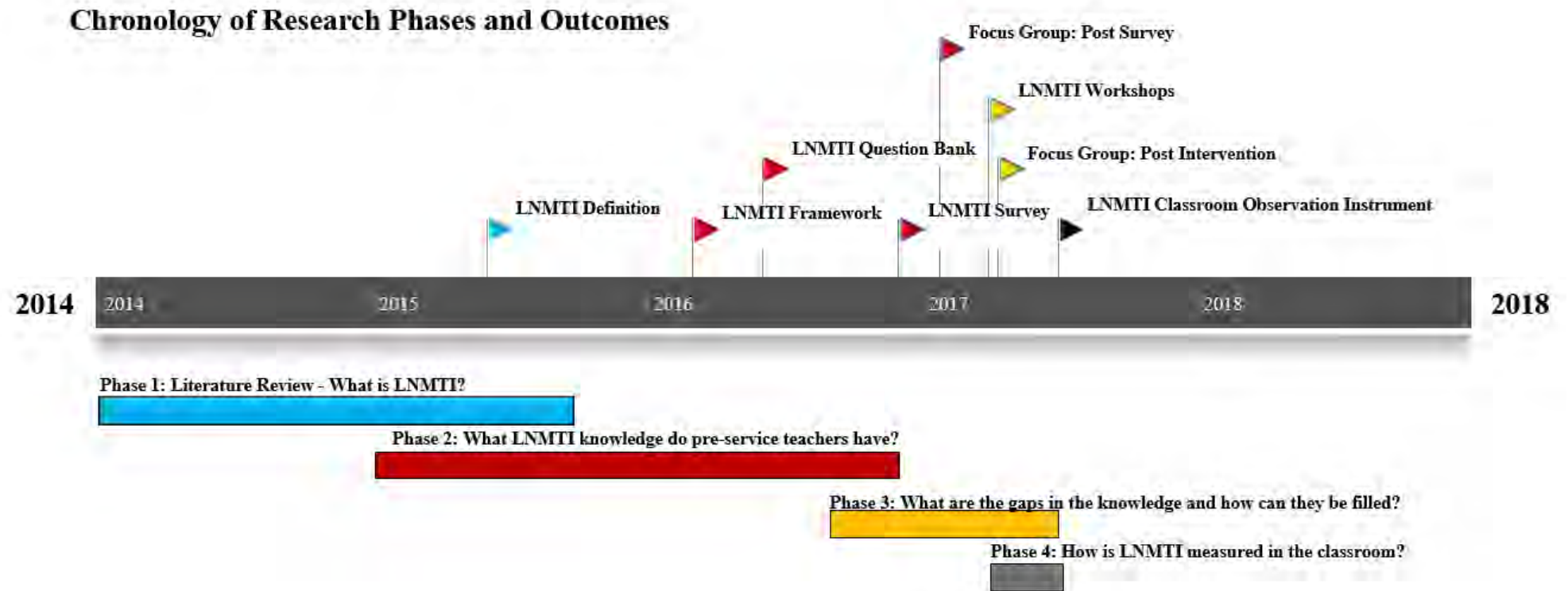


Figure 3.6 Chronological overview of the author's study

3.7.1 Sampling Method

As is common in qualitative inquiries, a non-probability sample was chosen (Cohen *et al.* 2011) and as is common in choosing a non-probability sample, it is undervalued, a target for criticism and even disqualification (Gobo 2013). Cohen *et al.* (2011) suggest in qualitative studies where the focus is on the idiosyncrasies and uniqueness of a phenomenon or a group then the term ‘sample’ is not even appropriate because representativeness to a wider population is unimportant (p. 161). Therefore, the following paragraphs will describe decisions taken regarding the context and the participants of the study.

The study was conducted at a university in Ireland from 2016-2017 that offers a two year Professional Master of Education (PME). The participants were full-time Professional Master of Education students studying to qualify them to teach mathematics at post-primary (12-18 year olds). These participants were in their second year of studies. The study participants comprised of a purposeful/purposive sample (Cohen *et al.* 2011) defined as ‘the identification and selection of information-rich cases related to the phenomenon of interest’ (Palkinas *et al.* 2015, p.533). However, Patton (2002) describes all sampling as purposeful in a qualitative inquiry and outlines fifteen strategies to acquire a purposeful/purposive sample, one of which is directly applicable to this study: criterion sampling. This strategy of sampling involves the study of cases that adhere to predetermined benchmarks. Two criteria informed the selection of this sample:

- (1) In contrast to the population of teachers teaching mathematics in Ireland where it was found in 2011 (Ní Ríordáin and Hannigan 2011) that 48% of teachers were unqualified to do so, this cohort were selected for suitability based on the most up-to-date teaching registration criteria, i.e. for persons applying on and after January 1 2017, established by the Teaching Council of Ireland, a professional standards body for teaching (Teaching Council 2013).
- (2) The university Initial Teacher Education programme introduced the two year Professional Master of Education in 2014 with the first cohort graduating in

2016. As part of pre-service teachers' education, literacy and numeracy was introduced in a generic way but the extended and modified core module on *Literacy and Numeracy Development in the Post-Primary Classroom* was developed for the current cohort requiring all final year Professional Master of Education students to attend 12 x 2 hour lectures and submit two reflective research papers on each of the domains of literacy and numeracy. Provision for literacy and numeracy was also compulsory for subject specific lesson planning and the teaching and learning interaction in a classroom-based context. At the time of this research, other university Initial Teacher Education programmes in Ireland had comparable requirements in subject planning and school placement performance but unlike this institution, they did not provide an independent module on literacy and numeracy with mandatory attendance and credit rating (Appendix A).

These criteria suggest another sampling category that acknowledges the uniqueness of this cohort of participants: revelatory case sampling, 'in which individuals are approached because they are the first members of a particular group' (Cohen *et al.* 2011, p. 157). This is the first group of pre-service mathematics teachers who have two information-rich attributes: they have engaged with the extended and deeper focussed literacy and numeracy module, associated assessments, classroom planning and practice as well as having the pre-requisite content knowledge in mathematics. Consequently, the results of this study would tend to be an over-estimation of the current levels of literacy and numeracy for mathematics teaching amongst pre-service teachers. Finally, in keeping with the qualitative tradition, this revelatory case sample size for this study was numerically small, $n = 12$ (see Walsh 2015, p.109). To put the small number of participants for this study in context, the author contacted the seven universities in Ireland who offered a Professional Master of Education programme during 2015-2017, the same time frame as this study, regarding number of applicants who completed a similar course. There was an 86% response rate. Table 3.4 presents the data from the universities apart from University 7 who was contacted on three separate occasions in an effort to acquire the figures (Appendix B). From the

remaining six universities, there was an average of 9 applicants making it acceptable to argue that the sample of 12 for this study is representative of the larger population of pre-service teachers of mathematics studying on a Professional Master of Education programme in Ireland.

Table 3.4 Pre-service teachers of mathematics who graduated from the Professional Master of Education programme 2017 from an Irish university

<i>University in Ireland</i>	<i>Pre-service teachers of mathematics who graduated from the Professional Master of Education programme 2017</i>
University 1	12*
University 2	7
University 3	8
University 4	4
University 5	7
University 6	16**
University 7	No data

*Number of Professional Master in Education students who participated in the study.
 **Two cohorts: 9 from this total completed the mathematics methods module in 2015-2016, year 1 of the programme.

The next section will describe phases 1-4 of the research process.

3.7.2 Phase 1: Analysis/Exploration, Design/Construction, Evaluation/Reflection

Phase 1 consisted of one Educational Design Research meso-cycle, which comprised of an analysis/exploration stage followed by a design/construction stage and completed by an evaluative/reflection cycle. The analysis/exploration step encompassed a literature review examining three broad areas of literacy, numeracy and mathematics teaching. The outcome of this process confirmed that no specific definition for Literacy and Numeracy for Mathematics Teaching in Ireland existed. Consequently, the first Research Question was composed that directed the development of a practical solution to this problem:

RQ1: *How is Literacy and Numeracy for Mathematics Teaching in Ireland defined and characterised as a knowledge domain for assessing pre-service teachers of mathematics in order to monitor how well prepared they are to meet the challenges of current educational reforms in Ireland?*

To guide the research three new questions emerged:

RQ1(a) *Is the Project Maths syllabus the appropriate vehicle to disseminate literacy and numeracy outcomes or is a separate literacy and numeracy syllabus necessary?*

RQ1(b) *What are the elements in instructional practice that endorse the assimilation of skills in literacy and numeracy in the mathematics classroom?*

RQ1(c) *Are the survey items and survey instrument effective in measuring LNMTI?*

In the following section the author describes the processes and methods used in this first phase of this research which is summarised in Figure 3.7.

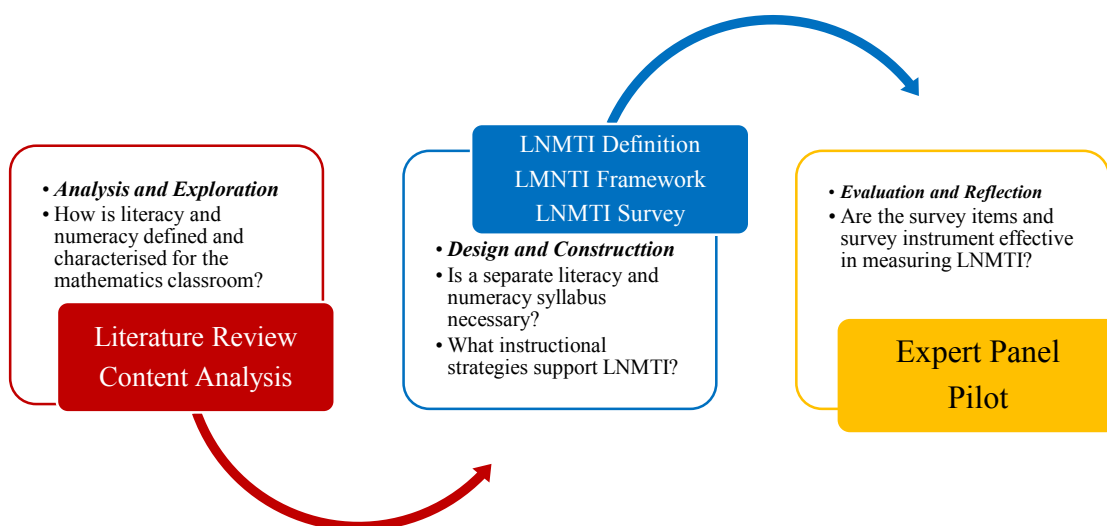


Figure 3.7 Phase 1 of the author's study

The author used content analysis, whereby a text is analysed regarding its context, content and message (Krippendorff 2004). It seeks to understand latent content by decomposing the text into component parts for analysis and coding. This is relevant where the focus of the research is on concept building (Drisko and Maschi 2016). The author will give an example of how this method was employed in constructing a working definition of Literacy and Numeracy for Mathematics Teaching in Ireland

(LNMTI). Firstly, the author examined the text of the definitions of literacy and numeracy published in Ireland's national literacy and numeracy strategy (Ireland, Department of Education and Skills 2011) and the Junior Certificate Mathematics syllabus (National Council for Curriculum and Assessment 2013a) to answer Research Question 1(a):

RQ1(a): Is the Project Maths syllabus the appropriate vehicle to disseminate literacy and numeracy outcomes or is a separate literacy and numeracy syllabus necessary?

The procedure adopted was domain and taxonomic coding (Saldana 2009). A domain is a category and a list of elements associated with that category is a taxonomy. For example, from the literacy definition, a domain was created labelled 'literacy' and two sub-domains labelled 'cognition' and 'text formats' were established. A hierarchal taxonomy associated with the sub-domain label cognition was identified: 'read/write/listen/speak', 'use' and 'critically appreciate'. The text format sub-domain contained four elements:

- Spoken Language,
- Printed Text,
- Digital Media,
- Broadcast Media.

Secondly, a mapping relation to identify shared content between the texts was established. A mapping method was used extensively by the Teacher Education Development Study-Mathematics (TEDS-M), a cross national study of teacher preparation to explore common features from education systems in different countries (Tatto 2013). For instance, the content reference in the numeracy definition, 'patterns', was mapped to the Algebra strand and the Functions strand in the mathematics syllabuses where linear, quadratic and exponential patterns/functions are listed content. In addition, the synthesis and problem solving learning outcomes from the syllabus, for example, 'communicating mathematically' were mapped to the literacy domain.

However, during the course of the author's study, a draft specification for mathematics to address the syllabus and assessment needs of the new post-primary programme for 12-15 year olds in Ireland, Junior Cycle, was published (National Council for Curriculum and Assessment (NCCA) 2017). Therefore, the Junior Cycle Mathematics specification was examined, post data collection and analysis, to explore whether the positive result for Research Question 1(a), regarding the Junior Certificate Mathematics syllabus as the appropriate vehicle to disseminate literacy and numeracy outcomes, had changed. The mathematics content of the Junior Cycle Mathematics specification aligned with the Junior Certificate Mathematics syllabus but the strand structure was updated. The new specification merged the Algebra and Functions strands which mirrored the mapping relation the author had originally identified, allowing for future proofing. For subsequent research questions, the Junior Cycle Specification was used together with the Junior Certificate Mathematics syllabus. For example, a new strand in the Junior Certificate Mathematics specification, titled, the 'Unifying Strand' (Section 4.4.1) is an organisational structure for the teaching and learning of the mathematics content. This strand had significance for Research Question 1(b) which is outlined in the next paragraph.

To answer Research Question 1(b),

RQ1(b): *What are the elements in instructional practice that endorse the assimilation of skills in literacy and numeracy in the mathematics classroom?*

the author undertook professional training in the use of the Mathematical Quality of Instruction (MQI) instrument provided by the Graduate School of Education in Harvard (Appendix C). Studying the maths-specific rubric and coding videos of mathematics lessons identified the pedagogy domain that will be referred to in the study as Mathematical Quality of Instruction for LNMTI or *MQI for LNMTI*. From the pedagogy domain, the author identified shared elements from the Junior Cycle Mathematics specification, Ireland's Continuous Professional Development programme for Project Maths (Maths Development Team n.d.c) with the relevant content of Mathematical Quality of Instruction (Harvard University 2018b). For example, 'representations' is an element in the Unifying Strand from the Junior Cycle

Mathematics specification and it also exists in the *Richness of Mathematics* domain of the Mathematical Quality of Instruction protocol. This work, combined with the mapping of the syllabus to the literacy and numeracy definition, enabled the formulation of the LNMTI definition.

Furthermore, the author developed a framework for assessing knowledge needed for LNMTI (McKenney and Reeves 2012). Items were chosen to represent the curriculum content of post-primary level mathematics up to Leaving Certificate Ordinary Level which is a level below what pre-service teachers of mathematics in Ireland would be qualified to teach (Teaching Council 2013). This decision was taken as all of the participants were in school placements teaching classes at Leaving Certificate Ordinary Level or below, adhering to Herrington and Oliver's (2000) design principle for authentic contexts.

Moreover, in the design and construction cycle for the LNMTI survey, items were chosen to represent the domains generated by the LNMTI definition. These comprised of (a) content referred to as *Numeracy in Mathematics*, (b) cognitive processes, described as *literacy processes*, (c) the pedagogy domain was labelled *MQI for LNMTI* and (d) text formats i.e. printed text, digital media etc. were subsumed under the banner *Literacy Forms*. The items were adapted from existing instruments such as TEDS-M (Tatto *et al.* 2012), Irish Continuous Professional Development materials (Maths Development Team n.d.c) and State Examinations assessment questions. This step generated the last research question for the evaluative cycle of this phase:

RQ1(c) *Are the survey items and survey instrument effective in measuring LNMTI?*

To ensure the validity of the LNMTI survey instrument, a data bank of 20 items to reflect the four numeracy in mathematics domains (Number, Algebra and Functions, Statistics and Probability and Geometry and Trigonometry) was compiled (Appendix D). By validity, the author means that the items are aligned with the prevailing reform agenda for literacy, numeracy and mathematics for an Irish context. Consequently, the items with the LNMTI definition and framework were sent for review by an expert

panel of three mathematics educators in Ireland (Appendix E). The panel consisted of a senior inspector of mathematics from Ireland's Department of Education and two Regional Development Officers from the Maths Development Team, one of whom had the remit to develop the literacy and numeracy aspect of a teaching and learning workshop for post-primary mathematics teachers during the reform period. This Regional Development Officer is also a part-time lecturer and school placement tutor for Professional Master of Education mathematics students. Given that the final survey would have to be completed in 50 minutes, the panel of experts were asked to:

- Choose two items from each classification,
- Rate the two items as easy/moderate/difficult,
- Estimate how much time it should take to complete the two items,
- Give a reason(s) for choosing these two items.

Following the panel review feedback, a draft of the LNMTI survey was constructed and piloted with two newly qualified teachers of mathematics (Appendix F). The author initially sought to pilot the survey with a similar cohort of students studying in other universities in Ireland but then took the decision to complete this work with newly qualified teachers who had completed one full year in professional practice. This decision was made because as previously described in Section 3.6.1, this study's participants experienced more focussed literacy and numeracy training as part of their Initial Teacher Education programme, therefore the pilot results may not have been paradigmatic.

The evaluation of the responses from the pilot survey was guided by a LNMTI survey rubric adapted from the Mathematical Quality of Instruction rubric (Learning Mathematics for Teaching (LMT)/Hill 2014). It comprised of a four point ordinal scale where the responses were graded 0 for 'Not Present'; 1 for 'Low'; 2 was the grade for 'Mid' and a fully correct response was awarded a grade 3 or 'High'. An educational consultant working in the Irish context and a key figure in designing and constructing the reform programme for post-primary mathematics education in Ireland acted as an expert-rater, referred to as an Education Expert in this study, for the pilot

surveys. Following the pilot, there were no adjustments to be made to the survey questions and layout, however the analytic scoring rubric was slightly amended. The Education Expert suggested extending the scale to accommodate other possible responses to the tasks. For example the rubric for the LNMTI survey question on Geometry Definitions where the participants were to examine a photograph of a tiled floor and identify and define geometric shapes was revised (Appendix G). There were two different aspects to this task: recognition of shapes and giving definitions of these. The recognition of the following five 2D shapes encompassed ‘all’ shapes, identifying ‘most’ referred to at least 3 ‘some’ meant 1 or 2 shapes:

- Triangle: isosceles/right-angled,
- Quadrilateral: square/rhombus, rectangle, parallelogram, trapezium/trapezoid.

In addition, exact mathematical definitions in each case, particularly in relation to the important distinctions between a parallelogram, a rectangle and a square was required.

The first draft of the rubric for this item was as follows:

0 = Not Present (Identifies some shapes/Incorrect definitions)

1 = Low (Identifies some shapes/Some correct definitions)

2 = Mid (Identifies all shapes/Mostly correct definitions)

3 = High (Identifies all shapes/Correct definitions)

The Education Expert correctly identified another possibility which allowed for ‘identifies most shapes and all definitions are correct’. However, instead of adding another point to the scale, the author extended the description in the coding manual under the ‘Mid’ category. An extended discussion on the scoring rubric can be found in Section 4.7.

3.7.3 Phase 2: Analysis/Exploration and Evaluation/Reflection

In this section the author describes the second Educational Design Research meso-cycle that involved an analysis/exploration cycle followed by an evaluation/reflection stage. This phase moved from desk to field research, a necessary move to make in this theory to practice study because ‘the field under study becomes clear only when one has entered it’ (Cohen *et al.* 2011, p. 179). Research Question 2 guided this phase:

RQ2: *Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms?*

Figure 3.8 summarises phase 2 of the study, framed by new research questions that were generated from initial data analysis. The two research methods, context analysis and focus group interviews, were used to collect data. The following section describes in detail the methods used.

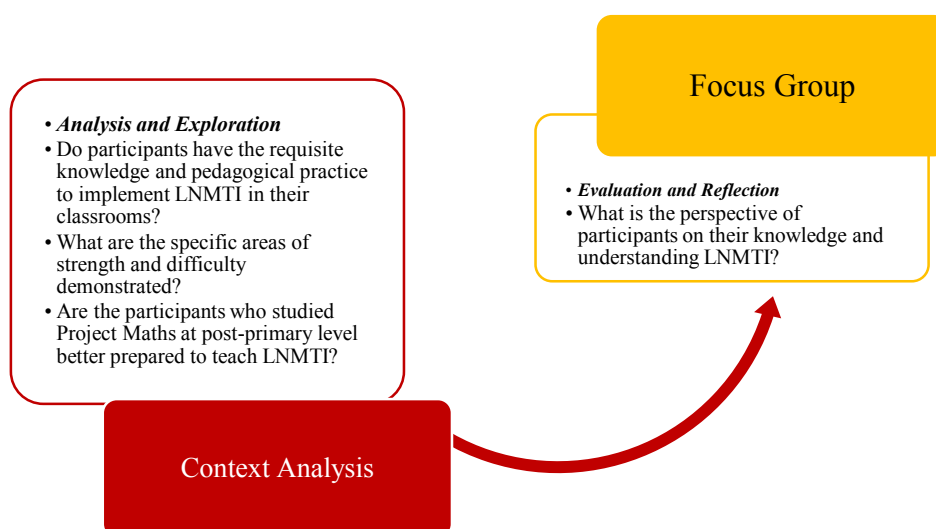


Figure 3.8 Phase 2 of the author's study

3.7.3.1 Context Analysis

Context analysis was used to identify the ‘base conditions in which the inquiry is situated’ (McKenney and Mor 2015, p.266). Part of the context was to evaluate the participants’ LNMTI knowledge using the LNMTI survey that was distributed to eleven participants on November 28th 2016. The responses were evaluated using the four point scale previously described. The author and the Education Expert independently rated the responses. Cohen’s kappa statistic was calculated to assess inter-rater reliability (Appendix H). A moderate agreement between the two raters was recorded ($Kappa > 0.5$). As no discussion took place about the ratings between the author and the Educational Expert, the author took the decision to use the grades awarded by the Educational Expert in the next step of the analysis. This was in line with the post-positivist nature of enquiry for this study that embraces ‘multiple, coexisting realities’ (Cohen *et al.* 2011, p.27).

In the next step to answer the Research Question:

RQ2(a) *Do participants have the requisite knowledge and pedagogic practice to implement LNMTI in their classrooms,*

descriptive statistics, including frequency counts and percentages were used to compare the allocation of awards in each of the categories. Summaries and comparisons of participant knowledge of LNMTI were generated by grouping ‘Not Present-Low’ and ‘Mid-High’ categories to form two dichotomous variables: *LNMTI Adequate*, to indicate a particular response demonstrated adequate LNMTI knowledge, and *LNMTI Inadequate*. Following this process, patterns in participant LNMTI knowledge base was explored framed by Research Question 2(b):

RQ2(b): *What are the specific areas of strength and difficulty for LNMTI demonstrated?*

Moreover, as variations existed in participants’ overall performance, they were divided into two further dichotomous variables: participants who had experienced the first Project Maths assessment at post-primary level in 2012 and those who did not. A Mann-Whitney U-test, a rank based test, was performed (Appendix I) to investigate if

a statistical difference between the two groups existed (O'Loughlin 2016). This data answered Research Question 2(c):

***RQ2(c):** Are the participants who studied Project Maths at post-primary level better prepared to implement LNMTI?*

Moreover, as part of the context analysis process and the evaluation and reflection cycle of this phase, the author spoke in detail to the staff member at the University who designed, delivered and assessed the generic literacy module. Also, the numeracy module materials on the student learning management system were made available to the author for examination. This information was used as baseline data to devise questions for the first focus group interview guided by the final Research Question for this phase:

***RQ2(d):** What is the perspective of participants on their knowledge and understanding of LNMTI?*

The following sub-section will outline the rationale for choosing the focus group interview as a research method for data collection.

3.7.3.2 Focus Group Interview

The decision to conduct a focus group interview instead of individual interviews was to get a sense of the collective views of the participants who had engaged in a uniform learning experience in literacy and numeracy and mathematics methods classes. Focus groups as a data collection method allows the interaction between the participants to generate relevant data (Cohen *et al.* 2011) and dilute the researcher's dominance (Kamberelis and Dimitriadis 2013). For example, for this study, the most surprising aspect of the data was when the participants collectively acknowledged the extreme shortcomings in their knowledge and application of numeracy to the subject of mathematics.

The focus group interview was conducted in early January 2017. The interview took place in a classroom in the university used for the mathematics methods tutorial. The session began by clarifying that the purpose of the interview was for the author to learn

from them about their experiences, thoughts, observations and ideas of literacy and numeracy for mathematics teaching (Appendix J). Respect was also shown for the participants' privacy and anonymity by making it perfectly clear that the session would be audio taped and transcribed but the transcript would not include any written or any other information that could identify them and that these would be destroyed following published results. The focus group interview had four clear and specific objectives, which helped in the facilitation and management of the interview (Cohen *et al.* 2011):

- (1) As a method of data triangulation following the survey,
- (2) To elicit participants' current understanding of LNMTI,
- (3) To obtain evidence of LNMTI practice in participants' teaching,
- (4) To learn about participants' experience of the institutions provision of literacy and numeracy training (investigator triangulation).

The focus group audio was transcribed by the author (Appendix K). This is a practice that is recommended and endorsed by Braun and Clarke (2006) as it allows the researcher to gain an initial familiarity with the content. Further themes emerged in the analysis of the focus group interview such as the necessity for clarity on planning for numeracy in the subject of mathematics.

3.7.4 Phase 3: Analysis/Exploration, Design/Construction, Evaluation/Reflection

From a situated learning perspective, the participants by means of the intervention would be instructed in the practices of the LNMTI community. Applying Bernstein's Social Construction of Pedagogic Discourse theory (1990), the intervention was the recontextualised frame to move 'the unthinkable' into the 'thinkable' (p.162). This phase was directed by Research Question 3:

RQ3: What content and characteristics should a teaching and learning intervention have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?

Figure 3.9 summarises the main steps taken and outputs generated in completing this research meso-cycle underpinned by three research questions that were generated at each stage:

RQ3(a): *What are the design requirements and design constraints for the LNMTI intervention?*

RQ3(b): *What type of intervention should be employed to support participants to gain the relevant expertise for LNMTI?*

RQ3(c): *How should the LNMTI module be structured to enhance participants' understanding of LNMTI?*

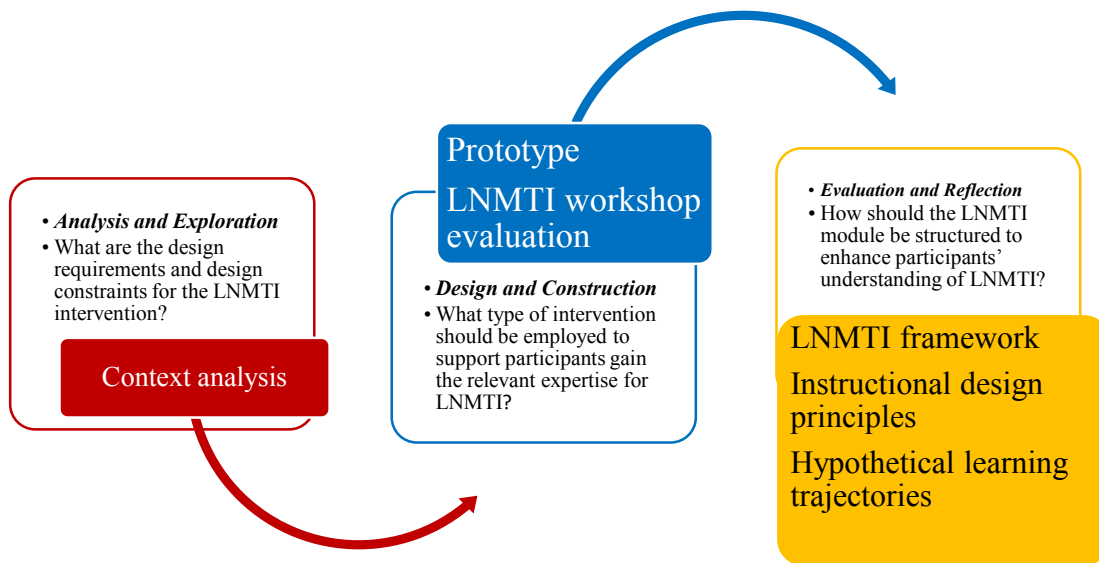


Figure 3.9 Phase 3 of the author's study

The analysis and exploration stage was guided by Research Question 3(a):

RQ3(a): *What are the design requirements and design constraints for the LNMTI intervention?*

The context analysis was derived from empirical data from the LNMTI survey that assessed participant prior knowledge of LNMTI and from the physical, cultural and academic environment of the participants (Patton 2002). Priority issues were identified, such as facilitating participant understanding of numeracy in mathematics,

and design constraints were noted, for example six one-hour classes were provided by the University for the Intervention.

The second question initiated the design and construction stage:

***RQ3(b):** What type of intervention should be employed to support participants to gain the relevant expertise for LNMTI?*

This was addressed by exploring Continuous Professional Development models and working within the contextual constraints such as classes were to be held on the university campus every Monday from 4-5pm during the pre-service teachers' maths methods tutorial. The author developed a prototype of the LNMTI intervention which comprised of a four hour workshop for practising teachers that focused on the derivation of the LNMTI definition and teaching and learning tasks for the purpose of facilitating literacy and numeracy skills in the mathematics classroom in Ireland. This developed into the following six workshop programme for LNMTI (Appendix L):

- (1) Understanding LNMTI,
- (2) Literacy and Numeracy for Geometry and Trigonometry Teaching in Ireland,
- (3) Literacy and Numeracy for Number Teaching in Ireland,
- (4) Literacy and Numeracy for Algebra and Functions Teaching in Ireland,
- (5) Literacy and Numeracy for Statistics and Probability Teaching in Ireland,
- (6) Literacy and Numeracy for Problem Solving Teaching in Ireland.

Following each workshop, the teaching and learning materials were made available to the participants on the university's learning management system. In addition, participants were asked to rate each workshop using a Likert-style question with the descriptors: excellent, very good, good, poor, very poor (Appendix M).

Moreover, the development of each LNMTI workshop was guided by three frameworks/principles:

- LNMTI framework,
- Instructional design principles,
- Hypothetical learning trajectories,

in response to Research Question 3(c):

RQ3(c): How should the LNMTI module be structured to enhance participants' understanding of LNMTI?

The content of each LNMTI workshop was mapped to elements in the LNMTI framework. This was achieved by developing a matrix to systematically align each workshop with the four domains in the LNMTI framework (Section 6.4.1). This was accomplished by hand.

- (1) *Literacy Processes* refer to three cognitive processes of 'understand', 'use' and 'critically appreciate',
- (2) *Numeracy in Mathematics* describes the numeracy content that is aligned with Irish mathematics syllabuses such as Statistics and Probability; Geometry and Trigonometry; Number; Algebra and Functions,
- (3) *Mathematical Quality of Instruction for Literacy and Numeracy for Mathematics Teaching in Ireland (MQI for LNMTI)* encompasses the strategic work of teaching to enable literacy and numeracy development in the mathematics classroom,
- (4) *Literacy Forms* describe the modes of communicating messages such as printed text, spoken language, digital media and broadcast media.

Secondly, eight out of the nine elements in Herrington and Oliver's (2000) Instructional Design framework, previously discussed in Section 3.5.3, were utilised to frame the content and the facilitation of the content for each workshop. The elements were grouped thematically under four headings: authentic, facilitation, perspectives and reflection:

- *Authentic (1,2)*

Authentic contexts and activities were designed to be authentic, addressing element 1, authentic contexts that reflect the way knowledge will be used in real life and 2, authentic activities. This was realised by preparing materials that were directly relevant to their current teaching situation. Also, despite the

fact the learning environment was a university classroom a direct focus was continually maintained on implementation strategies for the classroom.

- *Experts, Coaching and Collaboration (3,5,8)*
Elements related to Expert Modelling (3), Collaborative Construction of Knowledge (5), Coaching and Scaffolding (8) were addressed by researching materials by individuals who are internationally regarded as masterful teachers who practise the above named strategies to facilitate mathematical thinking in students. The author studied verbal discourses in George Pólya's (1957) dialogue between teacher and student on the 'rates' problem in printed text and Dan Meyer's (2011) Three Acts of a Mathematical Story using 'Pyramid of Pennies' in digital media using heuristics from Fey's (1970) pattern of verbal communication.
- *Perspectives and Articulation (4,7)*
Multiple roles and perspectives (4) and Articulation to make tacit knowledge explicit (7) was facilitated by making connections between content areas such as ratio and proportion in the number strand and similarity in Geometry and Trigonometry. In addition, the university Initial Teacher Education programme approved and promoted Goos *et al.*'s (2012) model for numeracy as a framework for lesson planning and numeracy implementation. An activity was designed to connect this model with elements from the problem solving outcomes in the mathematics syllabus (Section 6.4.2).
- *Reflection (6)*
Reflection to enable abstractions to be formed (6) was a promoted practice by enabling the participants to record their experience of the intervention following each workshop through an evaluation survey.

In addition, Hypothetical Learning Trajectories (HLT), a structured plan that contains the learning goal, learning tasks and learning processes, were used to chart the course of the workshops. The word 'hypothetical' is important as what might be planned may not be achieved in the live lesson. The author charted the hypothetical learning trajectories using two heuristics, 'structuring' and 'soliciting' (Fey 1970) from the

study's theoretical framework. The 'structuring' element set the context of the learning which was conveyed by the choice of learning task and the sequencing of the tasks. 'soliciting' elicited verbal/cognitive or physical actions from the learners while they engaged with the learning tasks.

Finally, the assessment of elements in these three frames/principles was achieved by a pen and paper evaluation following each of the six workshops. Participants were asked to respond to the following two questions:

- What aspects of the workshop did you find most useful?
- What aspects of the workshop did you find least useful?

For example, one participant's response to the most useful aspect of workshop 1 on the concept of LNMTI was:

Being able to relate numeracy to the syllabus. Clearly explaining numeracy in maths. Been given examples of the different aspects of numeracy within the Junior Cert. Group work activity - could discuss and validate answers. Very enthusiastic. Clear explanations.

This response suggested the elements of coaching and scaffolding as well as collaborative construction of knowledge from the Instructional design principles was achieved. In contrast, the following participant response on the least useful aspect of workshop 3 on Number demonstrates the Hypothetical Learning Trajectory for ratio was not achieved:

Although I got what you meant with describing the difference between $\frac{2}{3}$ of the sweets and $2:3$ of her sweets, I'd be afraid highlighting [it as] that could confuse them.

3.7.5 Phase 4: Analysis/Exploration, Design/Construction, Evaluation/Reflection

The final phase of the study was framed by the fourth Research Question:

RQ4: *What is the potential of a teaching and learning module based on authentic design principles to support pre-service teachers of mathematics develop an understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that implementation of the domain in the dynamic classroom environment is enabled?*

To answer this question the following four questions were developed:

RQ4(a): *To what extent did the LNMTI intervention contribute to improving pre-service teachers' knowledge of literacy and numeracy for mathematics teaching in Ireland?*

RQ4(b): *To what extent did the enactment of the LNMTI intervention align with the design principles for authentic learning?*

RQ4(c): *How do we measure pre-service teachers' LNMTI skills in the mathematics classroom?*

RQ4(d): *To what extent were LNMTI skills enacted by pre-service teachers in the classroom setting?*

Figure 3.10 presents an overview of this phase that includes the above mentioned research questions and the methods and frameworks that were utilised to guide the processes:

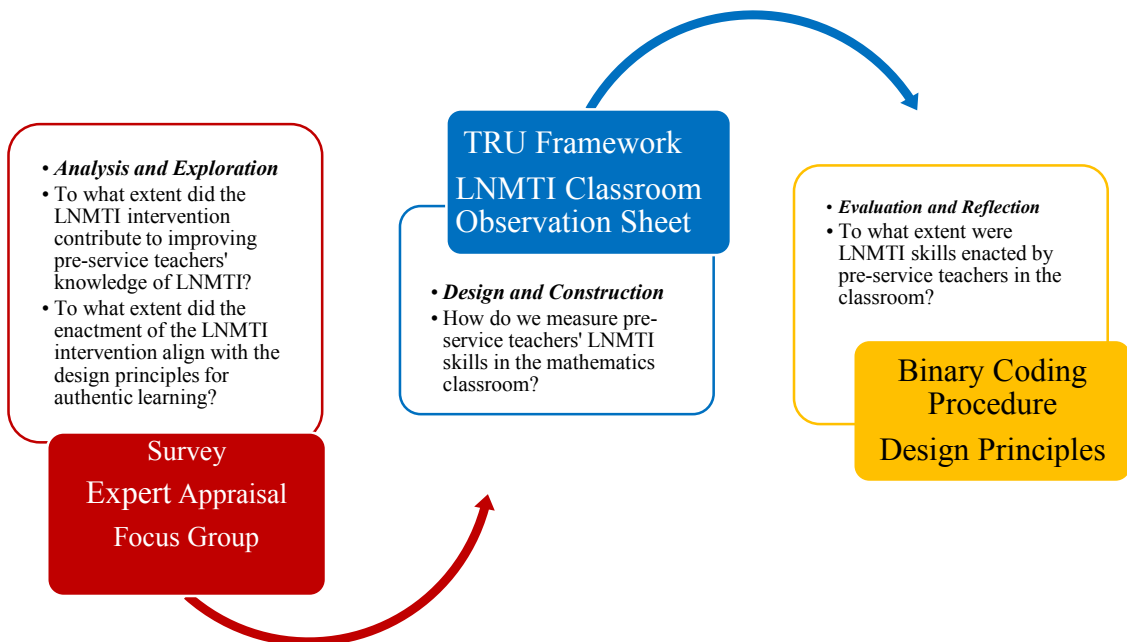


Figure 3.10 Phase 4 of the author's study

Following the final workshop in the LNMTI module, a pen and paper survey was given to the participants to evaluate its overall effectiveness (Appendix N). By effectiveness the author means the extent to which the module addressed the LNMTI priority issues of the participants, the extent to which LNMTI was facilitated in the context of participants' teaching and the sustainability of the intervention. Secondly, an Expert

Appraisal of the LNMTI module was sought. This Expert is a fully qualified post-primary mathematics teacher in Ireland who is also doing research in the area of numeracy. The Expert was asked to outline her understanding of LNMTI before observing the workshops (Appendix O) and then to write an appraisal following completion of the workshops (Appendix P). Both perspectives, participant and expert, were used to answer the first Research Question, RQ4(a) from this phase:

RQ4(a): To what extent did the LNMTI intervention contribute to improving pre-service teachers' knowledge of literacy and numeracy for mathematics teaching in Ireland?

To answer the second Research Question, RQ4(b),

RQ4(b): To what extent did the enactment of the LNMTI intervention align with the design principles for authentic learning,

a focus group interview was also conducted with the same participants who volunteered for the first focus group interview in Phase 1 (Appendix Q and R). However, two participants were unable to attend due to school placement commitments therefore data was collected via individual telephone interviews using the same question content and sequence from the focus group (Appendix S). The responses were coded using the thematically grouped Instructional design principles, outlined in Section 3.7.4.

However, one of the principles was not addressed in this analysis: 'authentic assessment of learning within the tasks'. It was important for the aims and objectives of this study to assess explicit LNMTI knowledge in situ and for this study, this entailed observing participants' in live lessons. This generated the development of the final artefact for this study, the classroom observation instrument (Appendix T), guided by the third Research Question in this phase, RQ4(c):

RQ4(c): How do we measure pre-service teachers' LNMTI skills in the mathematics classroom?

Schoenfeld (2013) describes in detail the difficult process involved in producing a classroom observation instrument for the mathematics classroom that is workable,

focussed on important dimensions, comprehensive and comprehensible, valid and reliable (p.610). That said, Schoenfeld and the Teaching for Robust Understanding (TRU) (2016a; 2016b) project are in the process of clarifying and characterising successful teaching in the dynamic environment of the classroom. They have identified five dimensions which are described in detail in Section 7.3.2.1:

- (1) The Content,
- (2) Cognitive demand,
- (3) Equitable Access to Content,
- (4) Agency, Ownership and Identity,
- (5) Formative Assessment.

These dimensions were aligned with the four categories of the LNMTI framework, Literacy processes, Numeracy in Mathematics, *MQI for LNMTI* and Literacy forms. For example, the Content component from the TRU guidelines was mapped to Numeracy in Mathematics content in the LNMTI framework. Unlike the TRU classroom observation instrument that offers generic themes to observe in the work of teaching, the author took the decision to itemise the elements from the LNMTI framework and apply a binary coding system: ‘Present’ or ‘Not Present’. This meant if the participant demonstrated an element from the LNMTI domain it was coded ‘Present’ and if not, then that element was coded ‘Not Present’. This practice aligned with the Learning for Mathematics Teaching (LMT) project (2011) coding procedures. However, the latter allowed for rating whether an element in the context was ‘Present and Appropriate’ or ‘Present and Inappropriate’ (p.33). The author took the decision to allow a space for comment whereby the classroom observer could offer a reason for the coding decision. This was a feature in the TRU classroom observation sheet.

Further fine graining of this preliminary coding procedure was applied by taking the comments from the ‘Present’ codes provided by the classroom observer and assigning four codes: Present, P+, P++, P+++. The ‘Present’ code used here now denoted the starting point of the scale and indicated that the element was observed to a limited extent; P+ that it was observed to some extent; P++ indicated the element was observed

extensively and P+++ that it was observed to a high degree (Coben *et al.* 2007). Overall, a five point scale was generated from the classroom observations with frequency counts and percentages. For example, the scores for one participant are presented in Table 3.5.

Table 3.5 Ratings for one participant from the LNMTI classroom observation five point scale

Five Point Scale	Not Present	Present	P+	P++	P+++
% Relative Frequency	12.5%	50%	0%	37.5%	0%

To compute an overall score for the enactment of LNMTI skills in a live lesson, frequencies from P+, P++, P+++ categories were totalled and percentages were calculated. Comparisons were made between overall scores from the LNMTI survey and LNMTI classroom observation sheet to identify patterns or trends in individual participant performance and in elements from the LNMTI framework.

At the conclusion of the Educational Design Research process or one macro-cycle, a practical contribution was made in the design and construction of the LNMTI definition, framework, module and classroom observation sheet. A contribution to theory was also achieved by devising design principles for translating policy to practice in educational settings.

3.8 Research Issues

In the following section, the reader will gain insights into the fundamentals of research practice as relevant to this study:

- ethical considerations,
- issues of validity and reliability.

The section will then conclude with a discussion on the limitations of the research.

3.8.1 Ethics

Cohen *et al.* (2011) discuss ethical issues that are an intrinsic part of educational research by firstly focusing on Frankfort-Nachmia's 'cost/benefits ratio' (1992 cited in Cohen *et al.* 2011, p.75). This is described as a dilemma where benefits arising out of research may cost the participants or the institution loss of dignity and embarrassment but by not conducting the research the society or in the case of this study, the mathematics educational community in Ireland, would not benefit from its findings. The author was cognisant of this situation where participants would be put in a position to reveal information about the possible deficits in their knowledge of LNMTI at a stage in their studies where they were five months from their final assessments for qualification to be a practising mathematics teacher in Ireland. Consequently, various steps were taken to obviate individual harm in the study.

Firstly, participation was voluntary and secondly while anonymity was not possible because of the small sample size, participants' rights were protected by promising to maintain confidentiality at all times. An information sheet was given to the participants that described the purpose of the study and details of the participation. It was also explained they had the right to withdraw from the study at any stage (Appendix U).

Furthermore, the University of Limerick also requires researchers to submit a proposal to the Research Ethics Committee that details the extent of human participation in the study and the prescription of participation involvement inherent in the Research Design. Copies of the participant information sheet, consent forms and draft survey were also submitted. Approval was granted on 19th May 2016.

3.8.2 Issues of Validity and Reliability

The criteria of validity and reliability are critical elements in the conceptualisation and operationalisation of the research design and process (Cohen *et al.* 2011). However, throughout the research design narrative, the author positioned herself as a member of the community of practice because of her professional experiences as a mathematics educator in Ireland. This position and the fact the author used the qualitative paradigm

throughout the research process where ‘the researcher is the instrument’ (Patton 2001 cited in Golafshani 2003, p.600) could have impacted on the truthfulness and the trustworthiness of the findings. Consequently, throughout every phase of the research, the author employed strategies to ensure the threat of invalid and unreliable results, interpretations, inferences and conclusions was minimised (Cohen *et al.* 2011, p.133). The paragraphs that follow will elaborate on the strategies summarised in Table 3.6 to provide assurances to the reader that the research findings are ‘worth paying attention to’ (Lincoln and Guba 1985 cited in Golafshani 2003, p. 601).

Table 3.6 Validity and reliability strategies from the author’s study

<i>Elements from Author’s Study</i>	<i>Strategy</i>
LNMTI Definition and Framework	Theoretical triangulation from literature and Mathematical Quality of Instruction (MQI) training
LNMTI Survey	Three expert panel members Pilot of survey
LNMTI Survey Results	Expert-Rater
Focus Groups/Interview	Bracketing Member checking
LNMTI Module	Prototype of module Participants’ evaluation of module Expert appraisal
LNMTI Classroom Observation Instrument	Theoretical triangulation Expert evaluation

The process of qualitative research involves the researcher engaging in thinking patterns to understand social phenomena but in doing so they must bracket their worldview to reliably accomplish a respectful and rigorous understanding of others and the object of study (Saldana 2015). This was accomplished at the desk research stage where triangulation or the application of multiple perspectives, was employed. Theoretical triangulation was used in the conception and development of the LNMTI definition, framework and classroom observation instrument. Various theoretical viewpoints from the literature were evaluated through the mapping strategy outlined in Section 3.7.2. In addition, despite the author’s experience as a mathematics educator in Ireland, further training was sought from international researchers in the

field (Harvard Graduate School of Education 2018b) in an effort to deepen the author's theoretical viewpoint on mathematics education reform.

Secondly, in preparation for and during the field research phase, expert panels, expert evaluators and pilot testing were used to maximise the validity and reliability of the various LNMTI instruments and artefacts developed for this study. Pilot testing (see Carroll 2011, p.113 and Walsh 2015, p.125) the LNMTI survey validated its usability and relevance for the study. The LNMTI module was also endorsed by participant evaluations following each of the six workshops. In addition, in keeping with situated learning theory, an expert appraisal from an experienced teacher and Ph.D. researcher in numeracy education attended, participated in and evaluated the workshops.

Furthermore, three experts were drawn from the mathematics education specialists' community to perform various tasks, such as the selection of appropriate questions for the LNMTI survey that would capture the assessment of LNMTI knowledge amongst pre-service teachers. One Education Expert rated the pre-service teachers' answers from the survey, however this individual carried the higher status in the community of practice of what Lave and Wenger (1991) called an 'old-timer'. He also held a primary domain position in Bernstein's hierarchal sites for knowledge production (1990).

An expert evaluator also performed the LNMTI classroom observation instead of the author in an effort to preclude bias. However, it must be reported, to allay conflict of interest issues, that the expert for this aspect of the data collection phase was also one of the three member PhD supervisor team of the author's. Nonetheless, the variability of data generated from this instrument indicates the results are trustworthy.

Finally, the focus group interview as a method of data collection was chosen to dilute the researcher's dominance, (Section 3.7.3.1). Bracketing, the process whereby the researcher's viewpoint, is suspended to exclusively focus on the participants outlook and frames of reference (Cohen *et al.* 2011, p. 370) was also engaged. This was achieved by the author's rigorous adherence to the interview questions that were composed prior to conducting the interview. This is evidenced in the transcription of the interview data.

In addition, since the author also transcribed the interview data, member checking, a method to ensure the validation by the participant of recorded and transcribed data by the researcher, was employed to ensure the views of the participants were represented accurately (Birt *et al.* 2016). Following the transcription of interview data, the participants were invited to review the transcripts and check what was transcribed by the author was accurate. If subsequently, their viewpoint had changed on a particular issue, they were invited to update that change (Appendix V).

3.8.3 Limitations of the Research

As numbers of student teachers studying to be mathematics teachers is low nationally and a small sample size is a characteristic of a qualitative inquiry, low participant numbers in the study was not considered a limitation (Morrow 2005). Nevertheless, gender representation was a limitation. Out of the twelve participants, three were male, reflecting the trend of gender ratio in education graduates in Ireland (CSO 2014) but these individuals were not participants in the focus group discussion or classroom observation.

Secondly, it was evident from the participants' experience of the university's Initial Teacher Education programme literacy module that significant progress was made in their understanding and application of that domain in their lesson planning and classroom enactment. This module was allocated sixteen hours with a mandatory written assignment and presentation on the application of literacy in their planning and teaching. However, six timetabled hours were allocated to the LNMTI module, but in reality, the classes were fifty minutes long as participants had mandatory attendance at a lecture before the maths methods class that was a ten minute walk across the university campus. Furthermore, participants' workload was high at the time of the intervention, therefore it was not possible to give an assignment after each workshop that would have allowed time for reflection and consolidation of the ideas presented (Herrington and Oliver 2000). Finally, as only one macro-cycle of Educational Design Research was completed, an opportunity to improve the LNMTI module was not possible.

3.9 Conclusion

Given the research problem, this chapter set about clarifying for the reader the philosophical assumptions, paradigm and framework that informed the research design to explore the phenomenon of interest, LNMTI. The author established the subjectivist ontology of Literacy and Numeracy for Mathematics Teaching in Ireland that prompted engaging an interpretivist paradigm. Within this tradition, the applicability of post-positivism principles by Philips and Burbles (2000) was outlined. This was followed by a description of the three-tiered hierarchal theoretical framework that comprised of the grand, intermediate and domain specific frames that guided, justified and evaluated the knowledge generated by the research process. The reader was then informed about Educational Design Research as the methodology of choice to situate the study firmly in a practical domain. Next, the four phases of the research design were described which included details on the sampling procedure, participant involvement and the instruments used for data collection. The chapter concluded with a section on research issues regarding ethics, validity and reliability and limitations of the research.

In the next chapter, the reader will gain more detailed information on the specific conceptualisation and operationalising of phase one of the research design where the LNMTI definition, framework and survey were produced.

Chapter 4: Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI): Definition, Framework and Survey

4.1 Introduction

Chapters 1 and 2 described the first micro-cycle, Analysis and Exploration, of Educational Design Research for this study. The major outcomes of this process were three fold:

- a multitude of definitions of literacy and numeracy as well as various conceptions of the relationship between these domains exist and were considered,
- a discordant relationship exists between mathematics and numeracy,
- explicit pedagogical descriptions of how literacy and numeracy in a mathematics classroom setting could be enacted are lacking.

In response to these issues, this chapter concentrates primarily on presenting the reader with details of the Design and Construction phase of the research methodology (McKenney and Reeves 2012) to answer Research Question 1:

***RQ1:** How is Literacy and Numeracy for Mathematics Teaching in Ireland defined and characterised as a knowledge domain for assessing pre-service teachers of mathematics in order to monitor how well prepared they are to meet the challenges of current educational reforms in Ireland?*

Two sub-questions were composed to focus the research methods:

***RQ1(a)** Is the Project Maths syllabus the appropriate vehicle to disseminate literacy and numeracy outcomes or is a separate literacy and numeracy syllabus necessary?*

***RQ1(b)** What are the elements in instructional practice that endorse the assimilation of skills in literacy and numeracy in the mathematics classroom?*

The chapter begins with a summary of the Analysis and Exploration phase then a detailed explanation of how the author constructed the following definition of Literacy and Numeracy for Mathematics Teaching (LNMTI) will be presented:

Literacy and numeracy for mathematics teaching in Ireland (LNMTI) encompasses the ability to understand, use and critically appreciate number, algebra and functions, geometry and trigonometry, statistics and probability in various forms including spoken language, print, broadcast media and digital media as well as the enactment of rich instruction while working with students and mathematics.

This author generated literacy, numeracy and mathematics teaching focused definition was derived from the official literacy and numeracy definitions in use by the Department of Education and Skills in Ireland and published in the National Strategy to Improve Literacy and Numeracy among Young People (Ireland, Department of Education and Skills 2011). From this definition an instrument to measure pre-service teachers of mathematics' skill level for LNMTI through the use of a survey was designed and constructed. To enable this process, the author documents for the reader the methods employed to support potential users of this instrument to evaluate its quality and validity. Therefore, this chapter also describes how the vision and direction for the LNMTI framework was underpinned as described in the definition. The chapter concludes with a detailed overview of the LNMTI items.

4.2 Summary of the Analysis and Exploration Phase

The Analysis and Exploration phase of the Educational Design Research process aimed to answer the first draft of the research question:

What is literacy and numeracy for mathematics teaching in Ireland?

Firstly, the Analysis phase derived from examining this research question established the following based on documentary evidence: from the point of view of Ireland's Department of Education and Skills, the definitions of literacy and numeracy are intentionally complex; for the Irish context, stakeholders emphasised a broader view - mathematics as an example of numeracy should not be seen as a set of mechanical

procedures or presented through a ‘reductionist approach’ (Hislop 2011); also, developing numeracy as an integral part of mathematics teaching and learning is assumed (Ireland, Department of Education and Skills 2015b) and developing literacy and numeracy skills in students is every teacher’s responsibility (Ireland, Department of Education and Skills 2011).

Furthermore, Ireland generally scores well in PISA reading assessments (Ireland, Department of Education and Skills 2016d). The latest results from PISA 2015 position Irish 15 year olds 5th of all 65 participating countries in reading but mathematics scores vary. Irish students are underachieving in higher level mathematics tasks; there is a higher percentage of students attaining a basic level of mathematics proficiency but, in general, scores are above the OECD average (Ireland, Department of Education and Skills 2016d). This evidence is a contributory factor in the decision to set ambitious targets for literacy and numeracy for future educational planning (Ireland, Department of Education and Skills 2017c, 2017d; Shiel and Kelleher 2017).

In England, literacy and numeracy standards amongst prospective teacher education candidates are evaluated by the Department for Education (2016), who set a professional skills test in literacy and numeracy. Similarly, the Australian education ministers mandated the personal literacy and numeracy standards of teacher graduates to be in line with the top 30% of the population (Australia, Department of Education and Training 2016). Ireland’s literacy and numeracy strategy recommends measuring teacher knowledge of literacy and numeracy, but it acknowledges the work that has been done by Irish universities to support future teachers in implementing literacy and numeracy across subject areas. However, a lack of consistency regarding approach and provision is also mentioned (Ireland, Department of Education and Skills 2011).

Other relevant knowledge about literacy and numeracy in an Irish post-primary context existed in the history of literacy and numeracy provision in curriculum documents, professional development programmes for teachers and assessment of literacy and numeracy for teachers. In 1995, the National Council for Curriculum and Assessment

(NCCA) designed a context-based curriculum titled ‘Leaving Certificate Applied’ with the aim to facilitate students who did not want to follow a strictly academic programme and to help the student transition more seamlessly from the school environment to the world of work and adulthood. One of the underlying principles of this programme was developing literacy and numeracy skills with the mandatory study of subjects Mathematical Applications, and English and Communications (National Council for Curriculum and Assessment (NCCA) 2001). However, candidates in the 2014 state assessment demonstrated numeracy deficits and unfamiliarity with real life context documents (State Examinations Commission 2014). These comments convey issues surrounding classroom interactions and resource materials in the implementation of Ireland’s explicit numeracy syllabus at second level.

In addition, 2011 and 2014-15 Continuous Professional Development (CPD) support for Irish post-primary mathematics teachers promoted a relations-based approach to algebra as well as a student competency test on algebraic reasoning to help address student underachievement and difficulties in this domain (Maths Development Team n.d.c). The message the Department of Education and Skills, Maths Development Team, Ireland’s national teacher support service for mathematics 2008-2018, attempted to convey was that formal symbolic manipulation of algebraic expressions and equations should happen after much exposure to a relations-based approach to algebra. Establishing and generalising patterns should enable numerate behaviour while the formal manipulation of algebraic expressions is mathematical (Ireland, Department of Education and Skills 2011; Maguire 2003; Ontario Ministry for Education 2013; National Council for Curriculum and Assessment (NCCA) 2013a). However, Prendergast and Treacy (2017) report on the major deficits that exist in current university students’ performances in basic algebraic manipulation. The study also reports on the confusion amongst practising post-primary teachers about establishing an equitable pedagogical balance between teaching algebraic skills and the relations-based approach to algebra. Furthermore, through the experiences of current teachers in the system, this study also highlights the polarisation of this domain in Irish mathematics classrooms and mathematics textbooks. Consequently, this

research shows considerable divergence between the intended and the implemented curriculum in the Irish context (Boesen *et al.* 2014; Mac Mahon 2013). In addition, research conducted on the notable influence of past pedagogical experiences of pre-service teachers on teacher performance (McMillan 1985; O’Connell 2009) is exacerbated by the fact that all of the participants in this study engaged with a different mathematics syllabus, assessment and methodologies because of the phased implementation of the new mathematics curriculum known locally as Project Maths or they completed post-primary education prior to the introduction of syllabus reforms (National Council for Curriculum and Assessment (NCCA) 2012).

Turning now to the Educational Design Research exploration phase for this study, McKenney and Reeves (2012) direct that a problem and its solution from other research that has similar attributes to the problem under study should be examined. For this study, the author’s attention was drawn to research that focussed on the teaching of mathematics in classroom settings. As previously discussed in Chapter 2 the research that the author deemed significant to this study was Mathematical Quality of Instruction (Hill 2011). This was a video coding instrument that was designed to:

- (1) evaluate the quality of mathematics being taught to students,
- (2) understand the nature and quality of mathematical content communicated in various classroom interactions,
- (3) identify the elements of mathematical work teachers engage in.

Consequently, using the learning theory of situated learning where the focus is on the individual’s induction and enculturation into a community of practice (Lave and Wenger 1991), the author, as a novice researcher, completed a certified online course provided by the Harvard School of Education on the Mathematical Quality of Instruction. This certification enabled the author from 2016-2018 to use the Mathematical Quality of Instruction (MQI) instrument (Appendix B). The practical aspect of this phase provided the author with a clearer understanding of how literacy and numeracy skills could be manifested in the work of mathematics teaching. A

further discussion on the relevance of the Mathematical Quality of Instruction instrument to this study can be found in Section 4.5.

Following the Analysis and Exploration phase, a revised problem definition was drafted: *How is Literacy and Numeracy for Post-Primary Mathematics Teaching in Ireland characterised?* This was followed by an outline of partial design requirements and initial design propositions (McKenney and Reeves 2012). The next section describes this process.

4.3 Introduction to the Design and Construction Phase

Gravemeijer and Cobb (2006) argue from a design research and learning perspective, ‘one simply cannot adopt the educational goals that are current in a given domain’ (p.48). However, to thoroughly understand the problem at hand, a detailed understanding of the official definition of literacy and numeracy given in Ireland’s national strategy on literacy and numeracy (Ireland, Department of Education and Skills 2011), as it pertains to the post-primary mathematics syllabus in Ireland, was a design requirement for this study. The design proposition born from this constraint was clarity for teachers on teaching literacy and numeracy for mathematics teaching in Ireland. This facilitates what Hattie (2009) describes as a ‘visible learning classroom’ where what the teacher is teaching and what the students are learning are explicit.

4.3.1 Theoretical Framework in the Design and Construction Phase

DiSessa and Cobb (2004) indicate that grand theories such as Piaget’s theory of intellectual development are incomplete and ‘inadequate’ (p.80) to fully support the researcher using an Educational Design Research methodology in the design of educational artefacts. However, for this study Bernstein’s (1990) grand theory of Social Construction of Pedagogic Discourse, as previously discussed in Chapter 1 and the three sets of rules: hierarchal, sequencing and criterial (p.56), which will be explained in the following paragraphs, guided and justified the design phase of the

LNMTI definition. In addition, there were three major by-products of this endeavour: the LNMTI survey framework, the survey and the survey rubric.

In Bernstein's (1990) description of the hierarchal rule, he gives the example of a patient communicating with a doctor. If the patient's verbal discourse is too general such as 'I feel really, really bad today', then the doctor indirectly coaches the patient to give more information such as 'where is the pain? How long have you had it? In other words, the patient acquires the knowledge of the diagnosis from sharing specific information about his/her condition (p.57). The doctor transmits the knowledge in response to the acquirer's inputs. This is a useful example to demonstrate the research process for this study where the author interrogated policy documentation to elicit literacy and numeracy content.

Bernstein follows the hierarchal rule with the implementation of sequencing rules if the transmission of knowledge is in a progression. Bernstein uses the example of a child developing from the ages of five to six demonstrating competences in an explicit way to signal progress has occurred. This is applicable to this study in so far as a conception of literacy and numeracy for mathematics teaching in Ireland is a prerequisite to characterising this domain in, for example, LNMTI survey items.

Finally, in this context, criterial rules exist to facilitate the understanding of what is 'legitimate' and acceptable communication in a domain of learning (p.58). Bernstein uses the example of a teaching and learning interaction between teacher and child to demonstrate these rules. A child is drawing a picture of a person and the teacher praises the child for his/her efforts but also adds: he only has three fingers? It is in the articulation of the omission, not in a repressive way, that the child becomes aware of it. However, as Shiel and Kelleher's (2017) report demonstrates, huge variations exist in mathematical instruction and implementation of the mathematics syllabus amongst Irish teachers. Guided by Bernstein's three rules, the development of the LNMTI definition was the starting point to clarify, synthesise and support the goals in the National strategy for Literacy and Numeracy (Ireland, Department of Education and Skills 2011), Junior Cycle Framework (Ireland, Department of Education and Skills

2015a) and Junior Cycle Mathematics (National Council for Curriculum and Assessment (NCCA) 2017).

4.4 LNMTI Definition and Framework: Method

Since the composition of an LNMTI definition was the starting point, a logic model as proposed by EDR proponents (McKenney and Reeves 2012) was used to map out the design process in a clear and unambiguous way. The core elements of the model are as follows: problem statement, inputs, processes, outputs, and measurable change. The key issue centred on the lack of clarity from the literature surrounding what literacy and numeracy for mathematics teaching in Ireland actually meant in practical terms. The new Junior Cycle Mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017, p.8) attempts to address this problem by citing one example of literacy and numeracy in mathematics, as illustrated in Table 4.1.

Table 4.1 Literacy and numeracy in the Junior Cycle Mathematics specification

<i>Being literate</i>	Expressing ideas clearly and accurately	Students explain their thinking and justify their reasoning, using mathematical terminology appropriately and accurately
<i>Being numerate</i>	Using digital technology to develop numeracy skills and understanding	Students use digital technology to analyse and display data numerically and graphically to display and explore algebraic functions and their graphs; to explore shapes and solids; to investigate geometric results in a dynamic way; and to communicate and collaborate with others.

However, teachers' performance, management and communication of the learning in the classroom are central to the development of these skills. Therefore, the integration of literacy, numeracy, mathematics, and teaching into a unified whole will enable the process of understanding what skills and knowledge pre-service teachers of mathematics need to know before it is enabled in teaching and learning interactions in the classroom. The inputs are the knowledge base that exists in the literacy and

numeracy definitions, the mathematics syllabus documents and Mathematical Quality of Instruction (MQI) protocol. Using content analysis, the main output was the LNMTI definition. The rationale for and application of this method will be discussed in the next section.

4.4.1 Content Analysis: Literacy and Numeracy in Mathematics

Content analysis is a scientific method ‘for making replicable and valid inferences from texts’ (Krippendorff 2004, p.18) that exist in a certain context. As a technique it performs, using specific coding rules, a systematic analysis of texts. For this study it was employed to answer the more refined research question:

Is the current post-primary mathematics syllabus the appropriate vehicle to disseminate literacy and numeracy outcomes or is a separate syllabus necessary during this phase of educational reform in Ireland?

The documentation studied were as follows:

- Definitions of Literacy and Numeracy from the National Strategy 2011-2020 (Ireland, Department of Education and Skills 2011)
- Junior Certificate Mathematics syllabus with implementation nationally in schools from 2012 (National Council for Curriculum and Assessment (NCCA) 2013a)
- Junior Cycle Mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017)

Content analysis on the third text was performed post data collection because this syllabus, which will be rolled out nationally in post-primary schools in September 2018, was not available for review until November 2017. However, the author views this action as a form of future proofing and data triangulation as it validates the findings of the initial examination of the texts.

Secondly, the author chose to employ directed content analysis whereby the first cycle of coding was driven by theory or research findings (Hsieh and Shannon 2005). Deductive categories were identified using domain and taxonomic coding methods

(Saldana 2009) derived from the definitions of literacy and numeracy from Ireland's National Strategy (Ireland, Department of Education and Science 2011). Influenced by the work of ethnographers McCurdy *et al.* (2005), this coding method as a process was employed to uncover subjects' acquired knowledge and how they use this to interpret their lived experiences. Although this approach originated in cultural sociology, the lens of social construction of pedagogic discourse and situated learning from this study's theoretical framework positions mathematics teaching and learning and its participants as a micro culture/community of practice with a common vision making this method relevant. Moreover McKenney *et al.* (n.d.) highlight the importance of achieving clarity and coherence of the educational message amongst all stakeholders (p. 112-3).

The domain taxonomic coding method focuses on informants, 'individuals who have required a repertoire of cultural behaviour' (Spradley and McCurdy 2012, p. 4) to communicate a common vision. For this study the informants include the writers of the post-primary mathematics syllabus and the definitions of literacy and numeracy. The coding categories originate in their definitions. By applying this approach, it also brackets the author's interpretation of the key concepts in this study (see Section 3.8.2 on issues of validity and reliability). Furthermore, the hierarchal organisation of textual features of the definitions also affirmed the domain and taxonomic coding procedure as the most appropriate method (Saldana 2009).

The process began by identifying literacy and numeracy as distinct domains of learning and examining the definitions separately. The definition of literacy comprises of two statements:

- (1) *Literacy includes the ability to use and understand spoken language, print, writing and digital media.*
- (2) *Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.....when we refer to "literacy" we mean this broader understanding of the skill, including speaking and listening, as*

well as communication using not only traditional writing and print but also digital media.

(Ireland, Department of Education and Skills 2011, p.8).

By making literacy the domain, cognition and text formats (OECD 2016) became sub-domains. Statement 1 adopts ‘use’ and ‘understand’ as different phases in cognition (Maguire 2003), in other words, taxonomies of the sub-domain cognition. These verbs demonstrated thinking at different levels which is reflected by Bloom’s Taxonomy in Practice, a practical document compiled by Ireland’s Department of Education and Skills Professional Development Service for Teachers (n.d.) itemising Bloom’s hierarchal classification of learning outcomes with sample teacher question stems and student tasks. The definition also includes a classification of texts such as printed texts, digital texts, etc. where skills can be expressed. These became the taxonomies of the sub-domain, text formats. The second statement is similar but it must be noted the cognitive skills have been revised: *read, understand* and *critically appreciate*. Skills of speaking and listening are also mentioned and the text format ‘broadcast media’ has been added. Table 4.2 illustrates the final domain and taxonomic codes for this dimension.

Table 4.2 Domain and taxonomic codes for the literacy definition

<i>Domain</i>	<i>Sub-Domain</i>	<i>Taxonomy</i>
Literacy	Cognitive	Read/Write/Listen/Speak Understand Critically Appreciate
	Text Formats	Spoken Language Printed Text Digital Media Broadcast Media

Unlike the literacy definition, the numeracy domain comprises of mathematical content, which became a sub-domain. Cognition was also generated as a sub-domain from this definition:

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

(Ireland, Department of Education and Skills 2011, p.8).

For the content analysis process, taxonomy codes were derived from the noun form of words from the definition, for example ‘quantity’ was substituted for ‘quantitatively’ and ‘space’ replaced ‘spatial’. Cognitive elements were categorised as skills such as ‘to have a spatial awareness’, ‘mathematical understanding’, ‘recognise situations where mathematical understanding can be applied’ as well as ‘solve problems’. Table 4.3 shows the final domain and taxonomy elements for numeracy.

Table 4.3 Domain and taxonomic codes for the numeracy definition

<i>Domain</i>	<i>Sub-Domain</i>	<i>Taxonomy</i>
Numeracy	Cognitive	Mathematical Skills Mathematical Understanding Recognise Situations where mathematical thinking can be applied Solve Problems
	Content	Quantity Data Space Patterns and Sequences Problem Solving

Using the semantic relationship ‘strict inclusion’, where X is a kind of Y in this coding procedure (Spradley 1979 cited in Saldana 2009, p.134), content categories cited in the numeracy definition corresponded directly to the strand classifications in the mathematics syllabus (see Table 4.4). In the Leaving Certificate syllabus, the functions strand (Strand 5) is not specifically mentioned in the definition however the relation-based approach to teaching algebra (Maths Development Team n.d.c) is conveyed in the patterns and sequences dimension. Moreover, a new feature of the

new Junior Cycle Mathematics is the coupling of Algebra and Functions under one strand category.

Table 4.4 Numeracy definition mapped to mathematics syllabuses and specification

<i>Numeracy Definition</i>	<i>Junior Cycle Mathematics specification</i>	<i>Junior/Leaving Certificate Mathematics syllabuses</i>
Quantity	Number	Strand 3 (Number)
Data	Statistics&Probability	Strand 1 (Statistics&Probability)
Spatial	Geometry&Trigonometry	Strand 2 (Geometry&Trigonometry)
Patterns and Sequences	Algebra&Functions	Strand 4 (Algebra) Strand 5 (Functions)
Problem Solving	Problem Solving (Unifying Strand)	Synthesis and Problem Solving

Secondly, the literacy and numeracy elements that make up the definitions are manifestly present in the synthesis and problem solving standards that are repeated in each of the five strands in the Junior and Leaving Certificate syllabuses. A distinguishing feature of the Junior Cycle Mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017) is the inclusion of a ‘Unifying Strand’. This strand has six elements: building blocks, representation, connections, problem solving, generalisation, and proof and communication. The syllabus states there is no content attached to this strand but ‘its learning outcomes underpin the rest of the specification’ (p.10). As can be seen in Table 4.5, the specification has a distinct title, ‘communication’ (a literacy outcome) and ‘problem solving’ (a numeracy outcome) to aspects of the synthesis and problem solving outcomes in the current syllabuses. The other learning outcomes were subsumed under literacy and/or numeracy.

Table 4.5 Mapping literacy and numeracy to problem solving components

<i>Synthesis and Problem Solving from syllabuses</i>	<i>Elements in the Unifying Strand</i>	<i>Literacy & Numeracy</i>
Communicate mathematics verbally and in written form	Communication	Literacy
Explain findings and Justify Conclusions		
Explore Patterns and Formulate Conjectures	Problem Solving	Numeracy
Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts		
Analyse information presented verbally and translate it into mathematical form		Literacy and Numeracy
Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions		

The next stage in the content analysis process was to explore the content of the syllabuses. As pre-service teachers are traditionally assigned a first year group for teaching practice and this was the case for 91% of this study's participants, a purposive sample of the texts was taken, which took the form of the 'Common Introductory Course' (CIC) (National Council for Curriculum and Assessment (NCCA) 2013a). This was a common first year course designed as a minimum mathematics content requirement for 1st year learners (12-13 year olds) in post-primary. The Common Introductory Course was a subset of the Junior Certificate Syllabus and each learning outcome could be found in the main text of the Junior Certificate Syllabus.

As illustrated, the knowledge and cognitive processes conveyed in Ireland's literacy and numeracy definitions are sophisticated and broad (Hislop 2011). Consequently, in an effort to explore the possible convergence of the mathematics syllabus content and literacy and numeracy as domains of learning, code titles were generated from sub domains of the domain and taxonomic coding method on the literacy and numeracy definitions. The cognition dimension had four elements for numeracy: Mathematical Skills, Mathematical Understanding, Recognise Situations where Mathematical

Thinking can be applied, and Solve Problems. These elements were assigned node titles N1, N2, N3, and N4 respectively in QSR NVivo 11. Three nodes were generated for literacy: Read/Write/Listen/Speak; Understand; and Critically Appreciate, and were labelled L1, L2, and L3 respectively.

The unit of analysis was a learning outcome, for example: *list all possible outcomes of an experiment* (National Council for Curriculum and Assessment (NCCA) 2013a, p.33). There were 46 learning outcomes in total in the Common Introductory Course and each learning outcome was examined for suitability for inclusion under a literacy and numeracy node. For example, the ‘coordinate the plane’ (p.33) learning outcome was categorised under N1 (mathematical skills) and L1 for reading, writing, listening and speaking as it is a practical and visual activity. The reading element is expressed in the student’s ability to interpret the order of the ordinates to successfully plot a point also known as graph literacy (Zucker *et al.* 2015). Similarly, the learning outcome ‘present and interpret solutions, explaining and justifying methods, inferences, and reasoning’ (p.35) was coded as N4, *Solve Problems* and L3, *Critically Appreciate*.

As the main goal of coding is to explore patterns and identify consistencies in the data (Saldana 2009), the author noted there was consistency in the cognition domain in assigning a learning outcome to literacy and then to numeracy. As can be seen from Table 4.6, learning outcomes that were coded *Critically Appreciate* L3 (9 units), also appeared in both higher order learning outcomes such as *Mathematical Application* (5 units) and *Problem Solving* (4 units).

Table 4.6 Coding learning outcomes from the Common Introductory Course to literacy and numeracy components

<i>Code Name</i>	<i>Frequency</i>
Literacy	
L1-Read/Write/Listen/Speak	17
L2-Understand	20
L3-Critically Appreciate	9
Total	46
Numeracy	
N1-Mathematical Skills	23
N2-Mathematical Understanding	14
N3-Mathematical Application	5
N4-Solve Problems	4
Total	46

Moreover, 37 units were quite evenly distributed between L1 (17 units) and L2 (20 units). The same learning outcomes were weighted in favour of N1, Mathematical Skills (23 units). For example, the learning outcome ‘use drawings to show central symmetry, axial symmetry and rotations’ (p.33) was coded as a *Mathematical Skill* using Van Hiele’s visualisation level of geometric thought (Walsh 2015) but was also coded as L2 *Understand*, as a classification task was implied (Leinhardt *et al.* 1990).

Irrespective of the unit distribution, there is evidence that the current post-primary mathematics syllabus functions as the literacy and numeracy syllabus for mathematics. The Junior Cycle Mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017) has included graphics of the Key Skills (p.7) as well as exemplars of ‘being literate’ and ‘being numerate’ as previously illustrated. Furthermore, because the cognitive processes for literacy and numeracy had similar attributes which was confirmed in the coding process, the literacy processes of ‘understand, use and critically appreciate’ were chosen to represent the cognitive demands required for LNMTI and became the first building block of the LNMTI definition. Table 4.7 demonstrates where literacy and numeracy definitions are compared to Bloom’s Taxonomy in Practice document (Professional Development Service for Teachers n.d.).

Table 4.7 Bloom's Taxonomy in Practice mapped to literacy and numeracy processes

<i>Bloom's Taxonomy in Practice</i>	<i>Numeracy Processes</i>	<i>Literacy Processes</i>
Knowledge	Have/make sense of/understand	Understand
Comprehension		
Application	Use/Communicate	Use
Analysis	Recognise situations where mathematical reasoning can be applied	Critically Appreciate
Synthesis		
Evaluation	to solve problems	

In addition, if mathematics as a mechanism for communication is an explicit theme in the official syllabus documents then numeracy can also be regarded as an instrument of communication. This idea is not new and is a prevalent theme in the numeracy literature and the Junior Cycle Mathematics specification (Penny 1984; Ciancone 1988; O'Rourke and O'Donoghue 1997; Maguire 2003; National Council for Curriculum and Assessment (NCCA) 2017). Also, Usiskin (2012) believes being familiar with the language of mathematics is a precursor to all understanding. In addition, working with 'experienced interlocutors' (Schleppegrell, 2007, p.147) is essential for students to engage meaningfully with the language of mathematics as well as the learning of mathematics. Harel (2014) is keen to emphasise the intellectual need for communication as a key aspect of mathematical activity.

Consequently, if numeracy can be regarded as a form of communication and the literacy definition provides the cognitive processes 'understand, use and critically appreciate', then the communication frames of 'spoken language, print, broadcast media, digital media' outlined in the definition can be employed to express the learning domains of numeracy in mathematics (Number, Algebra and Functions, Statistics and Probability, Geometry and Trigonometry and Problem Solving). As previously mentioned, one example is evident in the Junior Cycle Mathematics specification where the key skill 'being numerate' was conveyed through the medium of digital media (National Council for Curriculum and Assessment (NCCA) 2017). Figure 4.1

shows the literacy and numeracy in the mathematics framework developed by the author based on the research just outlined:

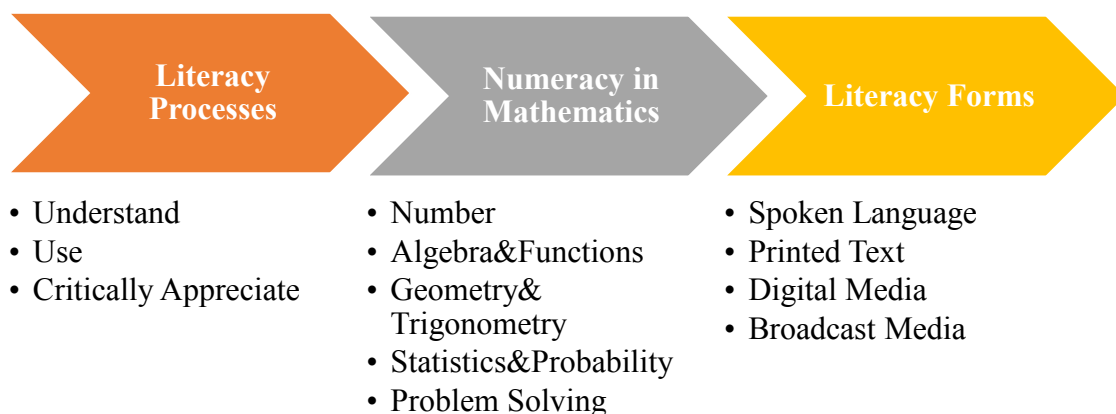


Figure 4.1 Author's conception of literacy and numeracy in mathematics

4.4.2 Enacting Literacy and Numeracy in the Mathematics Classroom

The previous section explored literacy and numeracy in mathematics and the next element to integrate into the definition and framework was the enactment of this knowledge in a classroom setting. The next section answers the question:

How is literacy and numeracy in mathematics enacted in a classroom setting?

The author had already identified the Mathematical Quality of Instruction (MQI) (Harvard University 2018a) standardised framework for examining, analysing and assessing mathematical instruction, as significant in the Irish context (Section 3.4.2.2). Mathematical Quality of Instruction was developed specifically for elementary and middle school teachers in the U.S., therefore a validity argument needed to be made for it to be used in this study (Delaney 2012; Krauss *et al.* 2008). This was achieved in three ways:

Firstly, the Irish construct of mathematical competency as evidenced in the post-primary mathematics syllabuses objectives was mapped against the U.S. conceived mathematical proficiency construct of mathematical proficiency (Kilpatrick *et al.* 2001). As can be seen from Figure 4.2, they are identical.

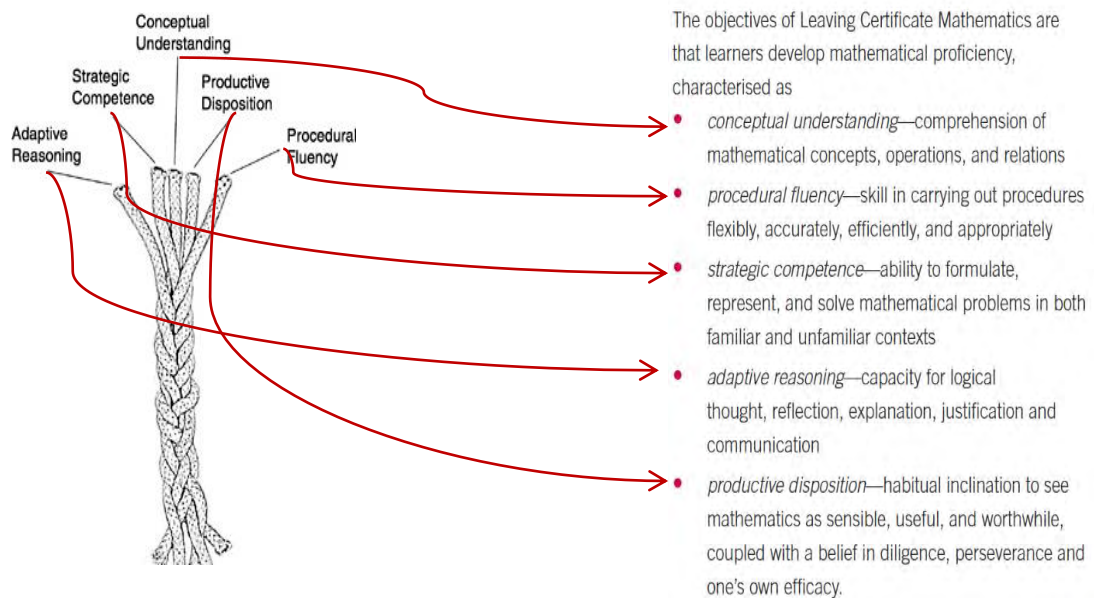


Figure 4.2 Kilpatrick et al.'s (2001, p.5) conception of mathematical proficiency mapped to learning objectives from Ireland's post-primary mathematics syllabuses

Secondly, the eight U.S. Common Core Standards were mapped against the synthesis and problem solving learning outcomes in the syllabuses and, as a corollary, with elements in the literacy and numeracy definitions (Common Core State Standards Initiative 2018). As Table 4.8 demonstrates, there is a strong association between the U.S. and Ireland's problem solving competencies.

Table 4.8 Common Core Standards for mathematical practice mapped to Ireland's problem solving learning outcomes

<i>U.S. Common Core Standards for Mathematical Practice</i>	<i>Irish Syllabuses: Synthesis and Problem Solving Learning Outcomes</i>
Construct viable arguments and critique the reasoning of others (3) Attend to precision (6)	Explain findings Justify conclusions Communicate mathematics verbally and in written form
Look for and make use of structure (7) Look for and express regularity in repeated reasoning (8) Reason abstractly and quantitatively (2)	Explore patterns and formulate conjectures
Use appropriate tools strategically (5) Model with mathematics (4)	Analyse information presented verbally and translate it into mathematical form Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions
Make sense of problems and persevere in solving problems (1)	Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts

Thirdly, following the identification of common ground between the U.S. and Irish context, the dimensions in the Mathematical Quality of Instruction framework were assessed for relevance and suitability. As previously illustrated in Chapter 2 and Chapter 3 there are five dimensions in Mathematical Quality of Instruction (Harvard University Center for Education Policy Research 2018a):

- (1) Classroom Work is Connected to Mathematics,
- (2) Richness of the Mathematics,
- (3) Working with Students and Mathematics,
- (4) Errors and Imprecision,
- (5) Common Core Aligned Student Practices.

However, the elements in the Mathematical Quality of Instruction (MQI) 'richness of mathematics' and 'working with students and mathematics' dimensions were chosen to represent content domains for teaching mathematics as they capture the approach taken by the Department of Education Inspectorate, National Council for Curriculum

and Assessment (NCCA), and the State Examinations Commission to Continuous Professional Development for mathematics teachers rolled out by Maths Development Team from 2008-2015 (Ireland, Department of Education and Skills 2013, Maths Development Team n.d.). For example, ‘linking between representations’, an element in the richness of mathematics dimension featured in Maths Development Team, workshop 3, by representing fractions verbally, numerically and visually. Similarly, Maths Development Team, workshop 4, introduced the multi-representational approach to Algebra through story, table, graph and mathematical models. This aligns with the ‘representations’ element of the Unifying Strand that is a new feature of the Junior Cycle Mathematics specification (Ireland, National Council for Curriculum and Assessment (NCCA) 2017). Also, each dimension represents a literacy and/or numeracy aspect, for example: mathematical language (literacy) and mathematical sense-making (numeracy). Table 4.9 gives a description of selected content areas, *Richness of Mathematics* and *Working with Students and Mathematics*, from the Mathematical Quality of Instruction protocol (Harvard Graduate School of Education 2018a) which have been combined for this study and labelled as one domain: *MQI for LNMTI*. Note, the author has extended the Mathematical Quality of Instruction definition of ‘linking between representations’ to include literacy elements relevant to the Irish context marked with an asterisk (*) in the table.

Table 4.9 MQI for LNMTI domain

<i>MQI for LNMTI Content</i>	<i>Explanation</i>
1. Linking between representations	This is an explicit link between representations of mathematical ideas. This can be visual (using graphs or physical models), numerical, algebraic, verbal* (printed text or spoken language).
2. Explanations	These illustrate which answer or why a mathematics procedure, solution etc. works.
3. Mathematical Sense-Making	This focuses on number sense, reasonableness of a solution, geometry definition.
4. Multiple Procedures or solution methods	These take different mathematical approaches to solving a problem.
5. Patterns and generalisations	This describes the examination of an example and its development into a generalisation.
6. Mathematical language	This focuses on the fluency of the teacher and the support given by the teacher to develop mathematical language use in the students.
7. Remediation of student errors and difficulties	This captures the way in which a teacher deals with a student misconception and difficulty with an area of mathematics.
8. Teacher uses student mathematical contributions	This describes how the teacher manages student answers/responses/work to advance the mathematics under instruction.

4.4.3 Literacy and Numeracy for Mathematics Teaching Definition

Following this investigation into the possible synthesis of literacy, numeracy, mathematics, and teaching, the definition for LNMTI was constructed by the author:

Literacy and numeracy for mathematics teaching in Ireland (LNMTI) encompasses the ability to understand, use and critically appreciate number, algebra and functions, geometry and trigonometry, and statistics and probability in various forms including spoken language, print, broadcast media, and digital media as well as the enactment of rich instruction while working with students and mathematics.

As a conclusion to the domain and taxonomic process, the following table, illustrated in Figure 4.3, was constructed that identifies the major domains in LNMTI as well as a list of specific features for each domain. This functioned as the LNMTI framework.

As previously illustrated, the content domain of numeracy in mathematics arose out of the one-to-one correspondence between numeracy elements and mathematics syllabus strands. The Mathematical Quality of Instruction for Literacy and Numeracy category represented the pedagogical instruments. The literacy processes of 'use', 'understand' and 'critically appreciate' were also mirrored in the numeracy definition. Finally, by regarding numeracy as a form of communication, the elements of numeracy in mathematics could be expressed in the various forms such as printed text or broadcast media, specified in the literacy definition.

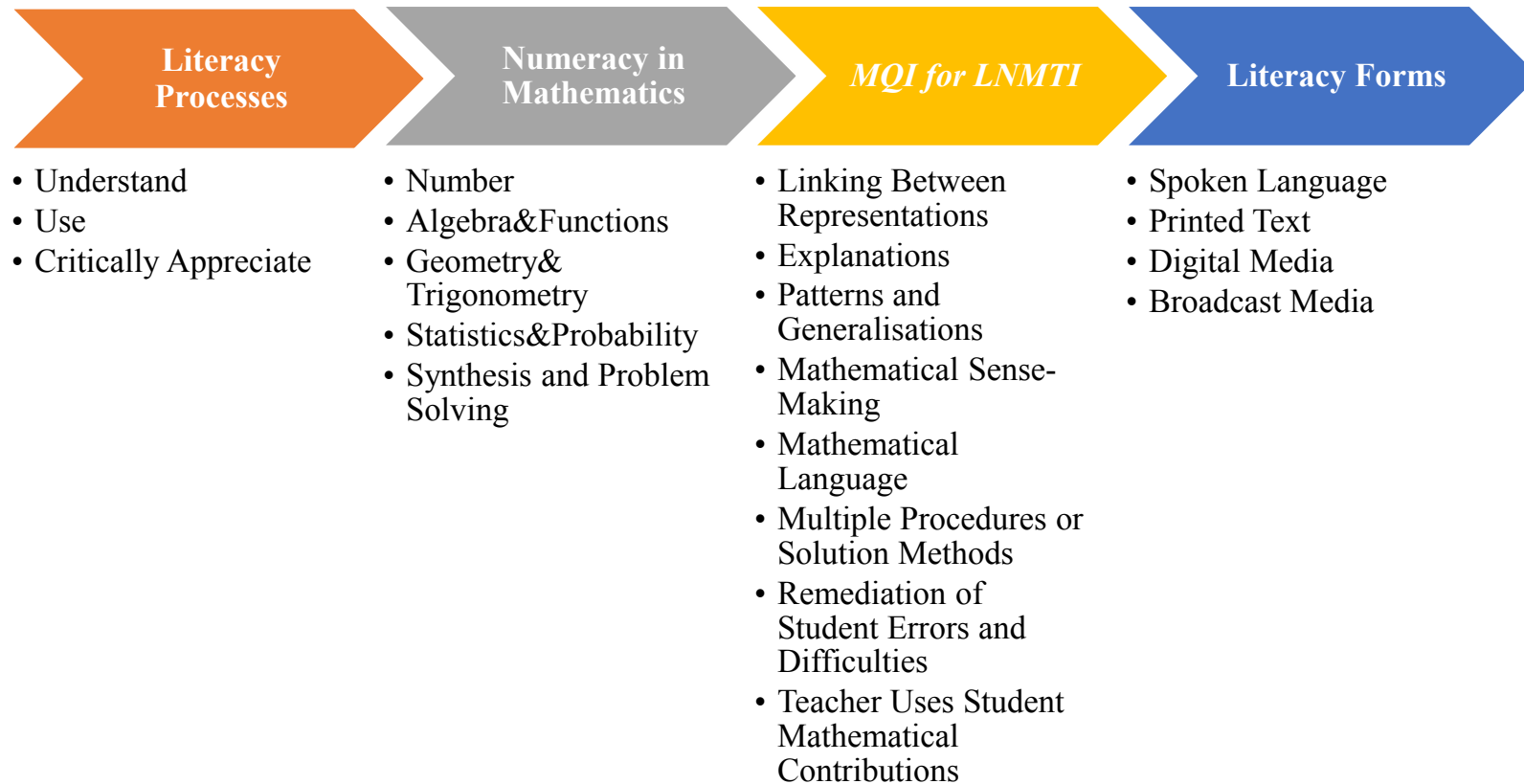


Figure 4.3 Author's conception of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI)

4.5 Formulating the LNMTI Survey

To formulate the LNMTI survey, the author researched established, validated and reliable tests that assessed teacher knowledge for mathematics teaching. The University of Michigan group developed ‘Mathematical Knowledge for Teaching’ (MKT) for elementary teachers (Learning Mathematics for Teaching (LMT) 2008); Mathematics Teaching for the 21st century (MT21) did a comparative analysis on teacher education programmes for secondary school mathematics teaching in six different countries (Schmidt *et al.* 2007); and COACTIV measured German teachers’ competence ‘COgnitively ACTIVating instruction and the development of student mathematical literacy’ (Krauss *et al.* 2008, p.874). The COACTIV study was aligned with PISA 2003/4 in so far as it focussed on teachers while PISA assessed the students being taught mathematics by those same teachers. Finally, the Teacher Education and Development Study in Mathematics (TEDS-M), a follow-on study from MT21, aimed to describe the knowledge pre-service primary and secondary school teachers acquire during their education training programmes (International Association for the Evaluation of Educational Achievement (IEA) 2009). The technical reports from MT21 (Schmidt *et al.* 2007) and TEDS-M (Tatto 2013) were employed as guides for the LNMTI survey because of the teacher education focus, which was compatible with this study. Also, the Programme for International Assessment of Adult Competencies (PIAAC) technical report was consulted because it assessed skills in literacy and numeracy (OECD 2013c).

Twenty items were chosen to reflect the numeracy in mathematics content domains from the LNMTI framework of:

- (1) Number,
- (2) Algebra and Functions,
- (3) Geometry and Trigonometry,
- (4) Statistics and Probability

Each of the four strand classifications contained five items. Each item was assigned a distinguishing title such as ‘Place Value’, a distinct syllabus reference from Junior Certificate and/or Leaving Certificate and was categorised under one or more of the literacy forms. From a practicality point of view, the survey was pen and paper therefore items that required a spoken language response such as *Explain the reason for each step as you would to a student?* were assessed in written text format.

To adhere to the enactment of literacy and numeracy in mathematics each item had a Mathematics Quality of Instruction for Literacy and Numeracy dimension taken from content domains of *Richness of Mathematics* and *Working with Students and Mathematics* as previously described. The author labelled this domain *MQI for LNMTI*. This dimension simulated situated classroom context features because ‘how teachers encounter mathematics in their teaching directly shapes the nature of the mathematical knowledge that is needed’ (Ball and Bass 2002 cited in Phelps and Howell 2016, p.53). As a consequence, the test consisted of constructed response items to allow the participants to:

- provide explanations of mathematical thinking,
- demonstrate situation specific skills of interpretation, decision making and perception in the domain of literacy and numeracy in mathematics teaching,
- demonstrate knowledge of the syllabuses for secondary level mathematics in Ireland.

Also, task difficulty was categorised under the literacy processes domain of ‘use’, ‘understand’ and ‘critically appreciate’.

Moreover, individual items were selected or adapted from the following resource banks:

- Continuous Professional Development materials from the Project Maths website,
- Published items that assessed student learning from the Project Maths syllabus managed by the State Examinations Commission of Ireland,

- Resource materials provided by Maths Eyes website, an Irish numeracy initiative to promote the visualisation of mathematics in real life contexts,
- Irish publications on international assessments such as: *PISA mathematics: A Teacher's Guide* (Shiel *et al.* 2007),
- International Association for the Evaluation of Educational Achievement (IEA), who managed international assessments such as TIMMS and TEDS-M,
- Learning Mathematics for Teaching (2008) project from the University of Michigan who produced items for MKT, mathematical knowledge for teaching,
- Zazkis' (2017) teacher education methodology: 'lesson play in mathematics education' that requires pre-service teachers to script dialogue in response to a prompt as preparation for the dynamic environment of the mathematics classroom.

Permission was sought from organisations such as the International Association for the Evaluation of Educational Achievement (IEA) who are responsible for producing TEDS-M and TIMSS items (email, 24/09/2016, Appendix W). Also, individuals such as Phelps for Learning Mathematics for Teaching project (email, 11/07/2016) and Zazkis (email, 16/10/2016) were contacted to seek permission to use or reference materials from their work (Appendix X). Table 4.10 provides a full listing of all twenty items. The next section gives a detailed account of two of the items that were included in the survey bank.

Table 4.10 Full list of LNMTI survey bank items

<i>Item</i>	<i>Label</i>	<i>Literacy Process</i>	<i>Numeracy in Mathematics</i>	<i>MQI for LNMTI</i>	<i>Literacy Form</i>
A1	Irrational Numbers and Calculator	Understand	Number with connections to Algebra&Functions J.C: 3.1/3.2&L.C: 3.1/4.1/5.2	Teacher uses student mathematical contributions	Spoken language/ Digital Media
A2	Order of Operations and Calculator	Understand	Number JC: 3.1 and LC: 3.1	Remediation of Student Errors and Difficulties	Digital Media
A3	Multiplication of Real Numbers between 0 and 1	Use	Number JC: 3.1/3.6 and LC: 3.1	Mathematical Sense-Making	Spoken Language/ Printed Text
A4	Place Value	Use	Number JC: 3.1 and LC: 3.1	Mathematical Explanations	Spoken Language/ Printed Text
A5	Irrational Numbers in Context	Critically Appreciate	Number with connections to Geometry&Trigonometry JC: 2.1/2.3/3.1/3.4: and LC: 2.1/2.3/3.1/3.4	Mathematical Sense-Making	Spoken Language/ Printed Text
B1	Geometry Definitions	Understand	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Mathematical Sense-Making	Digital Media
B2	Multiple methods	Critically Appreciate	Geometry&Trigonometry JC: 2.1/2.2 and LC: 2.1/2.2	Multiple Procedures and Solution Methods	Printed text

Table 4.10 continued

<i>Item</i>	<i>Label</i>	<i>Literacy Process</i>	<i>Numeracy in Mathematics</i>	<i>MQI for LNMTI</i>	<i>Literacy Form</i>
B3	Similar Triangles	Understand Use	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Mathematical Language	Printed text
B4	Theorem	Understand Understand	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Linking Between Representations	Spoken Language/ Printed Text
B5	Symmetry	Understand Critically Appreciate	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Mathematical Sense-Making	Printed Text
C1	Linear pattern	Understand Critically Appreciate	Number, Algebra&Functions JC: 3.1/4.1/4.2/5.2 and LC: 3.1/4.1/5.1	Remediation of Student Errors and Difficulties	Spoken Language/ Printed Text
C2	Exponential Equation	Critically Appreciate	Algebra&Functions JC: 4.2/5.2 and LC: 5.1	Patterns and Generalisations	Broadcast Media
C3	Simplifying Algebraic Fractions	Understand Use	Algebra&Functions JC: 4.6 and LC: 4.2	Mathematical Explanations	Spoken Language/ Printed Text
C4	Ratios/Rates and Proportions	Use Critically Appreciate	Number, Algebra&Functions JC: 3.1/4/4 and LC: 3.1/3.3/4.1/5.2	Teacher uses Student Mathematical Contributions	Spoken Language/ Printed Text

Table 4.10 continued

<i>Item</i>	<i>Label</i>	<i>Literacy Process</i>	<i>Numeracy in Mathematics</i>	<i>MQI for LNMTI</i>	<i>Literacy Form</i>
C5	Simultaneous Equations	Use Critically Appreciate	Number, Algebra&Functions JC: 4.3/4.4/4.8 and LC: 4.1/4.2	Mathematical Making/Mathematical Language	Sense- Printed Text
D1	Reading Graphs	Understand Use	Statistics&Probability connections to Number JC: 1.6/3.1 and LC: 1.6/3.1	with Linking representations	between Broadcast Media
D2	Probability Concepts	Critically Appreciate	Statistics&Probability connections to Number JC: 1.1/3.1 and LC: 1.1/3.1	with Remediation of Student Errors and Difficulties	Printed Text
D3	Number Systems and Probability Concepts	Understand Use	Statistics&Probability connections to Number JC: 1.2/3.1 and LC: 1.2/3.1	with Remediation of Student Errors and Difficulties	Printed Text
D4	Graphs and Frequency	Understand Use	Statistics&Probability connections to Number JC: 1.6/3.1 and LC: 1.6/3.1	with Remediation of Student Errors and Difficulties	Spoken Language/ Printed Text
D5	Statistics and Probability in the media	Critically Appreciate	Statistics&Probability JC: 1.2/1.5/1.7 and LC: 1.2/1.3/1.5/1.7	Mathematical Sense-Making	Broadcast Media

4.5.1 Rationale for LNMTI Items

This section will describe in detail for the reader why two items, C3, *Simplifying Algebraic Fractions* and D4, *Graphs and Frequency* were included in the LNMTI survey bank. The description of the other eighteen items can be found in Appendix D.

Item C3, *Simplifying Algebraic Fractions* presented in Figure 4.4 was adapted by the author from materials produced from a lesson study on simplifying algebraic fractions and the misuse of cancelling in an Irish post-primary school (Mathematics Development Team n.d.c).

Ciara is a second year Junior Certificate student with good mathematical ability. She simplifies the following rational algebraic expression correctly:

$$\frac{2x - 1}{2x^2 + 5x - 3}$$

(a) Show how she does this.

(b) Explain the reason for each step in simplifying the above expression as you would to a student preparing for Junior Certificate Higher Level.

Figure 4.4 Item C3, Simplifying Algebraic Fractions from the bank of survey items

This item was chosen based on pervasive issues that exist with the conceptual understanding of fractions (English and Halford 1995; Hodgen 2011; Speiser and Walter 2015). In one CPD course/meeting provided for post-primary mathematics teachers, samples of students' misconceptions of simplifying algebraic fractions and the misuse of cancelling from an Irish pilot school used to trial Project Maths materials were highlighted (Maths Development Team n.d.c). In answer to the question: 'what changes would I make in the future, based on what I have learned in my teaching, to address students' misconceptions?' (Slide 54), teachers from this school referenced literacy and numeracy strategies:

The topic of the creation of equivalent fractions and the verbalisation of the operations used to be emphasised in first year (and every year) with a view to its impact on the simplification of algebraic fractions later on.

(Maths Development Team n.d.c, slide 54)

The key mathematical concept underpinning these algebraic errors was the basic numerical concept of equivalent fractions. Moreover, Hodgen (2011) described a group of prospective mathematics teachers, several of whom had doctorates in mathematics, encountering difficulties in explaining the concepts underpinning the procedures in the addition and multiplication of fractions.

The statistics item, D4, *Graphs and Frequency*, is taken from TEDS-M (International Association for the Evaluation of Educational Achievement (IEA) 2009) and focuses on the remediation of a student's error. Kahan *et al.* (2003) recognise a strong mathematical background as a key factor in a teacher's ability to identify 'teachable moments' (p.245), however it does not always translate into rich mathematical instructional experiences for students. Fitzmaurice *et al.* (2014) from an Irish context, found pre-service teachers' abilities in statistics and conceptual knowledge did not positively correlate. The ability of these teachers to describe aspects of statistical knowledge was found to be a weakness. The model for statistical literacy conceived by Gal (2002) defines the knowledge and dispositional domains necessary to acquire statistical literacy. One knowledge element is 'how to decode data' but a dispositional element describes a person having knowledge and experiences that influence, for example, the interpretation of a graph. Francois *et al.* (2008) argue that in order to achieve statistical literacy, instructional methods need to be changed. This theme has been advanced by the Maths Development Team in Ireland (MDT), producing three workshops aimed to improve the statistical knowledge of teachers and their instructional strategies (Maths Development Team n.d.c, workshops 1, 5, 10).

Following the compilation of the LNMTI survey item bank, three experts in the field of mathematics education at post-primary level were contacted to review the questions. Details of the review process is contained in the following section.

4.6 Expert Panel

The LNMTI expert panel comprised of a senior mathematics inspector of post-primary mathematics (E1) and two Regional Development Officers (RDO) of post-primary mathematics education in Ireland. One RDO, E2, developed CPD materials for literacy and numeracy and also works as a teaching supervisor tutor for pre-service teachers of mathematics. The second RDO, E3, worked on the pilot phase of Project Maths, the CPD national roll-out and was a Project Maths representative at national syllabus committee level. The experts were asked to independently read the LNMTI framework and the survey items document and then complete the following review without peer consultation:

- (1) Choose two items from each classification,
- (2) Rate the two items as easy/moderate/difficult,
- (3) Estimate how much time it should take to complete the two items,
- (4) Give a reason(s) for choosing these two items.

Two experts (E2, E3) chose A1, 'Irrational numbers and Calculator'; the decision to include A3 'Multiplication of Real Numbers between 0 and 1' was unanimous. The algebra strand was equally streamlined: the three experts decided on C3, 'Simplifying Algebraic Fractions' and although two (E1, E2), opted for C1, 'Linear Patterns' and E3 chose C2, 'Exponential Equation', it was noted by E3 that C1 was a good question to help teachers reflect on the importance of enabling students to think at a deeper level about the answers they offer. E3's choice of C2 was justified by the broadcast media format that would enable teachers to engage all students in algebraic thinking and multiple representations. The Statistics and Probability strand choices were similarly congruent with the three experts choosing D3, 'Number Systems and Probability Concepts' and two experts (E1, E2) selecting D1, 'Reading Graphs', while E3 chose D2. However, the Geometry and Trigonometry strand generated more varied responses with one expert (E1) opting for B1 'Geometry Definitions', two (E2, E3) selected B2 'Multiple Methods', two (E1, E3) selected B3 'Similar Triangles' and one

(E2) chose B5 ‘Symmetry’. See Table 4.11 for Expert Panel item selections and the final survey items.

Table 4.11 Expert panel choices from LNMTI survey item bank

<i>Item</i>	<i>Label</i>	<i>Expert Approval</i>	<i>Survey</i>
A1	<i>Irrational Numbers and Calculator</i>	E2,E3	✓
A2	<i>Order of Operations and Calculator</i>		
A3	<i>Multiplication of Real Numbers between 0 and 1</i>	E1,E2,E3	✓
A4	<i>Place Value</i>	E1	
A5	<i>Irrational Numbers in Context</i>		
B1	<i>Geometry Definitions</i>	E1	✓
B2	<i>Multiple methods</i>	E2,E3	
B3	<i>Similar Triangles</i>	E1,E3	✓
B4	<i>Theorem</i>		
B5	<i>Symmetry</i>	E2	
C1	<i>Linear pattern</i>	E1,E2	✓
C2	<i>Exponential Equation</i>	E3	
C3	<i>Simplifying Algebraic Fractions</i>	E1,E2,E3	✓
C4	<i>Ratios/Rates and Proportions</i>		
C5	<i>Simultaneous Equations</i>		
D1	<i>Reading Graphs</i>	E1,E2	✓
D2	<i>Probability Concepts</i>	E3	✓
D3	<i>Number Systems and Probability Concepts</i>	E1,E2,E3	
D4	<i>Graphs and Frequency</i>		
D5	<i>Statistics and Probability in the Media</i>		

The final selection of eight questions, two from each strand, from the expert panel bank was made based on the majority decision from the experts, the fifty minute time constraint, and the difficulty level. All of the unanimous expert selections were included in the final draft of the LNMTI survey except D3 *Number Systems and Probability Concepts*. Item D3, was taken from the Mathematics Teaching in the 21st century project (MT21) (Schmidt 2013). These items were written to examine participants’ knowledge of probability for teaching (Stohl 2005). However, the author took the decision to replace this item with D2, *Probability Concepts*. This was adapted by the author from a Leaving Certificate Ordinary Level question (State Examinations Commission 2013) because it specifically examined probability knowledge for teaching basic probability concepts in the Irish context. The final draft survey was

piloted in early November 2016. Details of the pilot testing can be found in Sections 3.5.1 and 3.7.3. Following the trial, no changes were required. The survey was taken by the participants on 28th November 2016.

4.7 Survey Rubric

A four point scoring rubric modelled on the Mathematical Quality of Instruction protocol (Harvard Graduate School of Education 2018b; Learning Mathematics for Teaching (LMT)/Hill 2014) and Mayring's (2015) deductive coding manual was written to code participant responses to the survey. The author sought permission to use aspects of the Mathematical Quality of Instruction rubric for this study from Heather Hill, who was an original member of the Learning Mathematics for Teaching (LMT) (2008) project that developed the Mathematical Knowledge for Teaching survey and Mathematical Quality for Instruction protocols. Hill and her team in the Harvard Graduate School of Education have developed the Mathematical Quality for Instruction instrument further and provide on-line training for the Mathematical Quality of Instruction instrument that the author completed. In an email to the author dated 10/10/2016, Hill granted permission for use of the materials that were relevant to the study (Appendix X).

The four scales focused on mathematical content knowledge and/or teaching of mathematics. For the teaching of mathematics, an item related to knowledge of the syllabus, a teaching interaction and dealing with student misconceptions was included. As the pedagogical knowledge was dependent on mathematical content knowledge (Kahan *et al.* 2003; Schmidt *et al.* 2007) a separate coding system was not required. The coding guideline contained a value: 'Not Present', 'Low', 'Mid' and 'High'. These ordinal categories were assigned numbers 0,1,2,3 and an explicit description of competency attainment for each category was written (Jonsson and Svingby 2007).

To illustrate this process the following algebra item C3, titled, *Linear Patterns* is presented. This item, adapted from Zazkis' (2016) work on lesson play is a teacher education methodology conceived following an examination of pre-service teachers'

lesson planning where a distinct avoidance of problematic learning situations and contexts was noted. Lesson play is a strategy to script and act out situations where student misconceptions and errors arise in a fictional classroom setting. This item presents one such scenario on the numeracy domain of patterns and sequences (Ireland, Department of Education and Skills 2011). The scripting of the dialogue is a strategy to focus on verbal and written forms of communication, key elements in the Ireland's literacy definition (*ibid*, p.8). (See Figure 4.5).

A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is green, and the fifth is red and sixth is blue, and so on.

What is the colour of the 39th car?

The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student's work.

T: Why is the 39th car yellow?

S: Because the 3rd car is yellow and 39 is a multiple of 3.

(a) Identify the student error/misconception in this instance

(b) Outline how you would help the student correct the error/misconception.

Figure 4.5 Item C5, Linear Patterns adapted from Zazkis (2016)

The Mathematical Quality of Instruction four point scale rubric (Harvard Graduate School of Education 2018b) define four types of remediation: simple corrections, conceptual, procedural, and pre-remediation. Table 4.12 presents examples of remediation for this question.

Table 4.12 Examples of remediation for Linear Patterns Question from the LNMTI survey

<i>Simple corrections</i>	The student difficulty is not addressed but a correct step is given for example: <i>it's multiples of 4 in this pattern</i>
<i>Conceptual</i>	Identifies the source of the misconception for example: yellow is the third colour and 39 is a multiple of three. The student thinks this is a proportional relationship and three is the constant of proportionality.
<i>Procedural</i>	Corrects students' problems and procedures
<i>Pre-remediation</i>	Calls students' attention to a common error

Following these guidelines, the deductive coding manual, as presented in Table 4.13, was developed for this question.

Table 4.13 Coding manual for Linear Patterns Question from the LNMTI scoring rubric

<i>Value</i>	<i>Definition</i>	<i>Anchor example for High</i>
Not Present	1. remediation does not go beyond correcting the student answer 2. remediation is confusing	The student's reasoning is incorrect. The reason the 39 th car is yellow is because the yellow cars are in this sequence
Low	brief conceptual remediation occurs or brief procedural remediation	3, 7, 11, 15, 19, 23, 27, 31, 35, 39 and it is described by this rule:
Mid	1. moderate conceptual remediation or extensive procedural remediation 2. use of counter example	$T_n = 4n - 1$ <p>I would ask him/her to find the second yellow car in this sequence by drawing the pattern and/or listing the terms in the sequence</p> <p>I would ask him/her to write out the multiples of 3</p> <p>I would ask him/her to examine the position of green cars in the sequence.</p> <p>Compare the position of the green cars with the yellow cars.</p>

As outlined in Chapter 3, judgements were collected from two raters: the author and an Education Expert who was deliberately chosen for his senior role in the design and construction of Project Maths reforms. The author viewed this as an important

collaboration in the community of practice which for this study is the mathematics education community (Lave and Wenger 1991).

To begin, the Education Expert and researcher independently assessed the pilot surveys using the rubric. Communication was conducted mainly via e-mail. The author organised the responses by content, that is, instead of assessing all eight questions in succession, the participant responses were separated in the four content areas and sent in four separate emails. This approach was adopted because of the author's experience as an examiner for the State Examinations Commission where the successive marking of individual questions on examination papers was a promoted practice to ensure marking reliability. By reliability, the author means observer/marketing consistency in assigning judgements to a piece of work (Cohen *et al.* 2011). The expert replied with a rating and a comment. Initially, for the 16 judgements from the pilot, there was 75% agreement. This figure was calculated using Cohen *et al.*'s (2011) 'simple level' (p.210) inter-rater agreement where the number of actual agreements was calculated as a percentage of the number of possible agreements. Following a review of the evaluations with an expert in the field of mathematics education, it was noted that two of the discrepancies were as a result of human error in the reading of the marking rubric and human error was also involved in evaluating a participant response. Finally, one difference emerged in a Statistics and Probability item where incorrect student work on a probability concept was presented to the participants and the task was to evaluate this work. The rubric specified that an acknowledgement of the core misconception was required to score above 'Not Present'. The author opted to give this participant a 'Mid' (= 2) score but the Education Expert awarded the attempt with 'Not Present' (= 0). Following a re-check of the work, the author adjusted her score in line with the Education Expert accepting the participant's presentation of ideas as ambiguous.

4.7.1 Exploring Inter-Rater Reliability

Despite consistency in the pilot test rating agreements between author and Education Expert more variability was witnessed in the actual participant assessments. Table

4.14 presents the frequency of agreements for each of the ordinal categories from the author as rater and the Education Expert. For example the number 23 in the vertical column labelled ‘Low’ intersects with the row also labelled ‘Low’. This means the author and Education Expert agreed 23 times in awarding a ‘Low’ grade to a participant response. However in the same row, the number 14 represents the number of times the author awarded a ‘Low’ grade and the Education Expert gave a ‘Mid’.

Table 4.14 Frequency of agreed/disagreed judgements between author and Education Expert

<i>Raters</i>	<i>Education Expert</i>					<i>Total</i>
	4-point scale	High	Mid	Low	Not Present	
<i>Author</i>	High	4	0	0	0	4
	Mid	4	13	2	0	19
	Low	3	14	23	5	45
	Not Present	0	2	1	17	20
	<i>Total</i>	11	29	26	22	88

Out of 88 decisions under consideration, there were 57 perfect agreements. The author used Cohen’s kappa k , a statistic that measures the agreement between raters of categorical data. The following three assumptions were fulfilled (Sun 2011):

- The judgements that were made were measured on a nominal scale (with an ordinal variable for the LNMTI rubric) with mutually exclusive categories.
- The participants were individuals who independently completed the survey.
- The surveys were rated independently by the author and the Education Expert.

The kappa coefficient 0.5075 for these judgements was calculated manually (Appendix H) and corresponds to a moderate agreement. The 95% confidence interval for k was [0.3679, 0.6471].

However as can be seen from Table 4.14, fourteen disagreements were recorded at the ‘Mid-Low’ range, that is, where the Educational Expert awarded a grade in the ‘Mid’ category and the author awarded a low. Over a half of these disagreements (57%) were generated by the algebra questions on *Linear Patterns* and *Simplifying Algebraic Fractions* (Appendix F). The author was harsher than the Education Expert and it appeared there were different interpretations for the rubric. For example the ‘High’ response or point 3 on the scale for the *Simplifying Algebraic Fractions* question was as follows:

$$\frac{2x - 1}{2x^2 + 5x - 3} = \frac{2x - 1}{(2x - 1)(x + 3)} = \frac{1}{x + 3}$$

A reference must be made to A. Participants may reference B-D to support A.

- A. simplifying a fraction using the concept of equivalent fractions
- B. Factorise the quadratic expression in the denominator
- C. Divide the numerator and the denominator by the highest common factor of both numerator and denominator
- D. When the highest common factor of the numerator and denominator is 1, then the fraction is simplified

This scoring anomaly is explained by the Education Expert rewarding the participant for a slight or implied reference to equivalent fractions (A) whereas the author required a more explicit reference. As a result, the author decided to apply a linear weighted kappa, k_l which is used when the difference in the judgments between two raters is examined. This tool can only be used in instances where the categories are ordinal. For instance, for the same response, if one rater awarded a ‘Not Present’ grade and another gave it a ‘Low’, this is a one point difference. However, if for the same response, one rater awarded a ‘Not Present’ and another a ‘High’, this is a three point difference. Therefore, the weights take into account the proximity of agreements between each category (Warrens 2013). For four categories, the linear scheme and relative linear weight is shown in Table 4.15.

Table 4.15 Linear scheme and linear weights for a four point ordinal scale

		Not Present	Low	Mid	High			Not Present	Low	Mid	High
<i>Linear Scheme</i>	Not Present	0	1	2	3	<i>Linear Weight</i>	Not Present	1	0.6	0.3	0
	Low	1	0	1	2		Low	0.6	1	0.6	0.3
	Mid	2	1	0	1		Mid	0.3	0.6	1	0.6
	High	3	2	1	0		High	0	0.3	0.6	1

The relative linear weight w_i is calculated by the formula:

$$w_i = 1 - \frac{i}{c-1}$$

where i is the difference between the categories and c is the number of categories (Lowry 2018). Using the weighted values in the calculations (Appendix H), the kappa coefficient was 0.5924 with a 95% confidence interval of [0.4890, 0.7058], which translates to a moderate to substantial agreement (Viera and Garrett 2005).

To conclude this process, the author made the decision to defer to the Education Expert's judgements, for the following reasons:

- The Education Expert's senior role in Project Maths reform.
- The Education Expert's direct experience working with pre-service teachers of mathematics in Ireland as a mathematics methods tutor and teaching practice supervisor.
- The current research on poor performance of algebra manipulation by Irish university students (Prendergast and Treacy 2017).

This action is supported by the study's post-positivist nature of enquiry that facilitates multiple perspectives on the nature of reality and the theoretical framework where the author in her role as novice researcher is learning from more experienced others (Lave and Wenger 1991).

4.8 Conclusion

This chapter described the derivation of the definition, survey framework, survey, and survey rubric for literacy and numeracy for mathematics teaching in Ireland (LNMTI). Content analysis provided the vehicle to begin the process of combining and connecting ideas from Ireland's post-primary mathematics syllabuses, literacy and numeracy definitions, and protocols for quality mathematical instruction to generate these Educational Design Research products. The chapter also considered issues of validity and reliability throughout the phase. The following chapter, Chapter 5, will present the results and findings from the LNMTI survey.

Chapter 5: Discussion of Findings of Pre-Service Teachers' Prior Knowledge of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI)

5.1 Introduction

This chapter reports on the findings from the Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI) survey that was taken by pre-service teachers of mathematics ($n = 11$) in the final semester of teacher preparation. The reader will also gain insights into the analysis of a focus group interview ($n = 6$) that was conducted post survey. This phase was guided by the second Research Question:

***RQ2:** Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms?*

To answer this question, four sub-questions emerged:

***RQ2(a):** Do participants have adequate knowledge of LNMTI?*

***RQ2(b):** Are the participants who studied Project Maths at post-primary level better or less prepared for LNMTI?*

***RQ2(c):** What are the specific LNMTI knowledge areas of strength and difficulty demonstrated?*

***RQ2(d):** What is the perspective of participants on their knowledge and understanding of LNMTI?*

Together, the findings from these questions provide important insights into teacher preparation for mathematics reform in Ireland such as:

- Participants' overestimation of their basic knowledge of mathematics for teaching.
- The performance of participants who completed their Leaving Certificate studies, in mathematics pre-2012, in the LNMTI survey was not significantly different to those who took the final examination during the reform period in 2012.

- Participants' superior performance on abstract mathematics knowledge for teaching items in comparison to items situated in a real life context.
- Participants' difficulties in developing the formulation of generalisations in a mathematics teaching context.
- Participants' lack of clarity about the relationship between numeracy and mathematics for the post-primary mathematics classroom.

5.2 Participants

The participants in this phase of the study consisted of 12 pre-service teachers (9 females and 3 males) taking an Initial Teacher Education programme in a university in Ireland and were enrolled in the same mathematics methods module. Twelve pre-service teachers of mathematics were expected to participate in the LNMTI survey but one participant was unavailable on that day. Participation was voluntary. As previously described (Section 3.7.1) the participation sample was representative of the population of Professional Master of Education pre-service teachers of post-primary mathematics in Ireland.

5.3 Procedure

On 28th November 2016, the participants completed the LNMTI survey, developed by the author, designed to provide baseline data of their LNMTI knowledge. The pen and paper survey was given to the participants at the mathematics methods lecture and they had fifty minutes to complete it. Calculator use was allowed. The survey was organised in two sections: Section 1 collected demographic data about the participants that consisted of four variables: the year the participants sat the Leaving Certificate examination, the level and grade they achieved in Leaving Certificate mathematics and whether they completed a mathematics degree or not. The Leaving Certificate mathematics data was deemed important by the author because of the phased implementation of Project Maths and the possible varied classroom experiences of participants as students of mathematics in the post-primary system (O'Connell 2009).

Leaving Certificate candidates study an integrated mathematics course that students, to cater for ability level, can take at three levels, Higher, Ordinary and Foundation. Both Higher and Ordinary levels are examined by two papers: paper 1 predominantly assesses Number, Algebra, Functions, and paper 2 examines knowledge of Statistics and Probability and Geometry and Trigonometry. Foundation level has one paper that assesses all five strands. Following the reform in mathematics at post-primary level, in 2012, changes were made to the Leaving Certificate Higher and Ordinary paper 2 assessment to reflect the new syllabus and teaching and learning methodologies. Furthermore, for Irish Leaving Certificate students entering tertiary education, if the number of applicants for a university course exceeds the number of places on the course then course offers are based on a points system, ubiquitously known as CAO (Central Application Office) points. These points are calculated on grades awarded at Leaving Certificate Level (Central Applications Office 2018). For example, the highest award is H_1 representing the percentage range 90%-100%. If a candidate opts to take the Higher Level paper and achieve this grade, they are awarded 100 CAO points. In 2012, for the first time, candidates were awarded an extra 25 CAO points as an incentive to study the more difficult and challenging Higher Level mathematics course. Therefore, these two mutually exclusive groups, 'pre 2012' ($n = 6$) and '2012' ($n = 5$) had the potential to generate insightful findings regarding mathematics knowledge for teaching (Hill and Ball 2009), defined for this study as the literacy and numeracy knowledge needed to teach mathematics at post-primary level. Table 5.1, summarises the demographic data from Section 1 of the survey:

Table 5.1 Summary of participants' demographic data

<i>Variable</i>	<i>Category</i>	<i>Number</i>	<i>Percentage</i>
Leaving Certificate Year	2012	5	45%
	Pre-2012	6	55%
LC Mathematics Level	Higher	10	91%
	Ordinary	1	9%
LC Mathematics Grade (HL)	A	2	18.18%
	B	4	36.36%
	C	4	36.36%
LC Mathematics Grade (OL)	A	1	9.09%
Mathematics Degree	Yes	10	91%
	No*	1	9%

*required mathematics modules stipulated by the Teaching Council (2013) were completed by this participant.

The second section comprised of eight questions in each of the four LNMTI knowledge domains: Number, Algebra and Functions, Geometry and Trigonometry and Statistics and Probability. Table 5.2 provides an overview of the questions in the survey that were aligned with the LNMTI framework.

Table 5.2 Survey Questions aligned with LNMTI Framework

<i>Question</i>	<i>Label</i>	<i>Literacy Process</i>	<i>Numeracy in Mathematics</i>	<i>MQI for LNMTI</i>	<i>Literacy Form</i>
1	Irrational Numbers and Calculator	Understand	Number with connections to Algebra and Functions J.C: 3.1/3.2&L.C: 3.1/4.1/5.2	Teacher uses student mathematical contributions	Spoken language/ Digital Media
2	Multiplication of Real Numbers between 0 and 1	Use	Number JC: 3.1/3.6 and LC: 3.1	Mathematical Sense-Making	Spoken Language/ Printed Text
3	Geometry Definitions	Understand	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Mathematical Sense-Making	Digital Media
4	Similar Triangles	Understand Use	Geometry&Trigonometry JC: 2.1 and LC: 2.1	Mathematical Language	Printed text
5	Linear pattern	Understand Critically Appreciate	Number, Algebra&Functions JC: 3.1/4.1/4.2/5.2 and LC: 3.1/4.1/5.1	Remediation of Student Errors and Difficulties	Spoken Language/ Printed Text
6	Simplifying Algebraic Fractions	Understand Use	Algebra&Functions JC: 4.6 and LC: 4.2	Mathematical Explanations	Spoken Language/ Printed Text
7	Reading Graphs	Understand Use	Statistics&Probability with connections to Number JC: 1.6/3.1 and LC: 1.6/3.1	Linking between representations	Broadcast Media
8	Probability Concepts	Critically Appreciate	Statistics&Probability with connections to Number JC: 1.1/3.1 and LC: 1.1/3.1	Remediation of Student Errors and Difficulties	Printed Text

For example, Question 1 in the survey ‘Irrational numbers and calculator’ was to demonstrate understanding of the concept of rational and irrational numbers. The next column titled ‘Numeracy in Mathematics domain’ presents the Junior Certificate and Leaving Certificate syllabus reference for this question. The *MQI for LNMTI* element, an acronym for Mathematical Quality for Instruction for Literacy and Numeracy dimension, which characterised the work of the teacher in facilitating literacy and numeracy skills in the mathematics classroom, was: ‘Teachers use student mathematical contributions’. Finally, the ‘Literacy Form’ contained two elements: spoken language because it was a student contribution and digital media as information was conveyed by a calculator display. Negative attitudes to calculator use persist in the Irish secondary school context (NCCA 2006; Close *et al.* 2008) however, their capability to enhance mathematical proficiency is pervasive (Brown *et al.* 2007; Guin *et al.* 2005). Close *et al.* (2008) also note the aims and objectives of Project Maths reflect the rationale for calculator use in the mathematics classroom in the facilitation of problem solving and investigative pedagogies, working with real life data as well as improved computation.

The responses provided by the participants were judged using the LNMTI scoring rubric (Appendix G). The responses were awarded a category from the four point scale: ‘Not Present’, ‘Low’, ‘Mid’, ‘High’. See Section 4.7 for a detailed discussion on the scoring rubric. The results of this process were entered into Microsoft Excel. The frequency/percentage for each of the four categories was calculated for all participants and represented graphically. The frequency of awards for each individual participant was calculated and visually displayed. This was followed by an analysis of the four Numeracy in Mathematics content domains: Algebra and Functions; Number; Geometry and Trigonometry and Statistics and Probability. The next section describes the process in more detail and reports on the findings from the survey.

5.4 Overall Performance

This section presents findings on participants’ overall poor performance on the LNMTI survey. The performance of participants who completed the first Project

Maths assessment in 2012 compared to those who took the final examination pre-2012 is also discussed. To answer Research Question 2(a):

RQ2(a): Do participants have adequate knowledge of LNMTI?

frequency counts and percentages were used to summarise the graded responses from the participants. To capture an overall evaluation of participants' LNMTI knowledge, the 88 graded responses from the four point scale ('Not Present', 'Low', 'Mid', 'High') were assigned the dichotomous variables: 'LNMTI Adequate' and 'LNMTI Inadequate'. These categories were generated from grouping the 'Not Present to Low' observations to represent participant responses that demonstrated inadequate knowledge of LNMTI and the 'Mid-High' responses that demonstrated adequate knowledge of LNMTI. As illustrated, in Figure 5.1, over half of the responses, 55%, from the participants were awarded a grade in the 'Not Present-Low' category, in other words, LNMTI Inadequate. This is a poor result given the participants were in the final semester of the teacher education programme having completed a twenty-four module and two assessments on literacy and numeracy for classroom implementation.

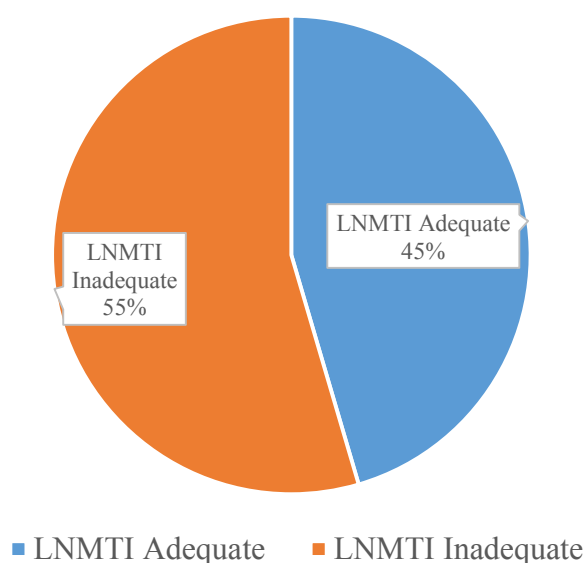


Figure 5.1 Percentage of responses awarded a grade in the LNMTI Adequate/Inadequate category

Figure 5.2 presents individual participants' performance in the LNMTI survey. As previously mentioned, data on P5 is not present as this participant was not in attendance on the day the survey was administered. Guided by general principles of assessment, the author took the decision not to enable this participant to take the survey at another time as the consistency of the data might have been compromised (University of Sheffield, n.d.). The following example demonstrates how each bar in the bar graph was constructed for each participant. Table 5.3 shows the frequency P1 was awarded a position on the four point scale for the eight responses:

Table 5.3 P1's responses on the four point scale

<i>Four Point Scale</i>	Not Present	Low	Mid	High
<i>Frequency</i>	1	5	1	1

These scores were grouped into the dichotomous variables: LNMTI Adequate and LNMTI Inadequate. Therefore, as Table 5.4 illustrates, for P1, only two (or 25%), of the responses were concordant with LNMTI Adequate indicating this participant's overall LNMTI knowledge was inadequate.

Table 5.4 P1's frequency count for LNMTI Adequate/Inadequate

	LNMTI Adequate	LNMTI Inadequate
<i>Frequency</i>	2	6

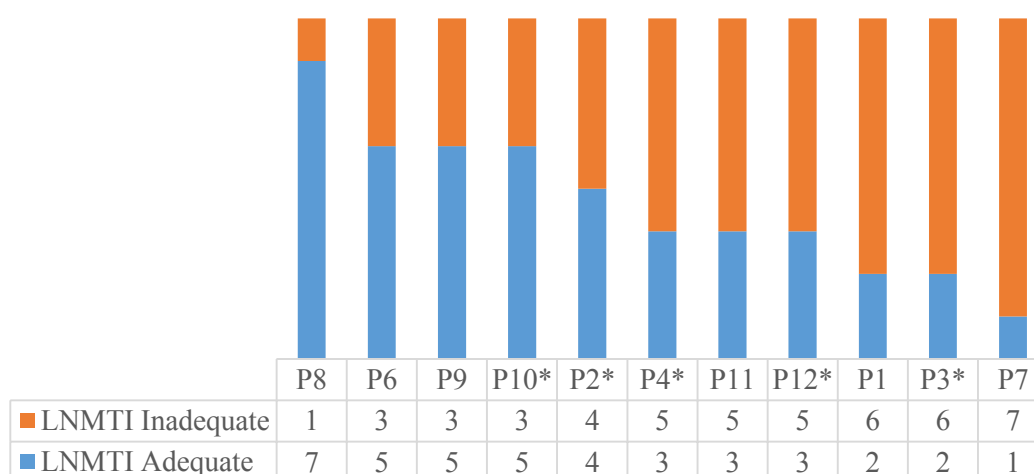


Figure 5.2 Participants' performance: LNMTI Adequate/Inadequate

As the bar chart indicates, five out of the eleven participants' responses scored 50% or more in the LNMTI Adequate range (P8, P6, P9, P10, P2). 87.5% of responses from P8 were assigned to the LNMTI Adequate category and 62.5% for P6, P9 and P10. Out of these top five scores, three of those participants sat the post-primary Leaving Certificate Mathematics pre-2012. As the demographic data demonstrated, five of the participants, marked with an asterisk in the chart, (P10*, P2*, P4*, P12*, P3*) were part of the cohort who sat the first Project Maths assessment in 2012. These observations led to investigating the next Research Question:

RQ2(b): Are the participants who studied Project Maths at post-primary level better or less prepared for LNMTI?

Overall, based on the data, there is no reason to believe that there is any statistically significant difference in the performance of the pre-2012 participants and those who took the first Project Maths assessment in 2012. This conclusion has been reached based on the performance of the Mann-Whitney U-test, designed for small samples, that tests statistical differences between two independent groups based on rank. Calculations are presented in Appendix I where the conclusion arrived at is as follows:

$$U_{stat} = 13.5 > U_{crit} = 3, \text{ at } \alpha = 0.05$$

that is, there is no difference between the ranks of Pre-2012 and Project Maths 2012.

Secondly, P7 and P8, who completed the Leaving Certificate in 2012, accounted for the extreme results. P7 scored very poorly with one response out of eight awarded a grade in the LNMTI Adequate category. Two of P6's responses earned a 'High' grade, in comparison to P10* who acquired one 'High' and P9 whose five responses were awarded a grade in the 'Mid' category only. As illustrated in Figure 5.3, from those responses that were awarded a grade in the 'Mid' or 'High' categories, almost three quarter of the responses, 73%, were in the 'Mid' category. Four participants, P2*, P4*, P7, P9 did not have responses awarded to the 'High' category, two from each cohort, 2012 and pre-2012. However, it must be noted, that the pre-2012 group earned more 'High' awards (8 responses) in comparison to the 2012 group (3 responses).

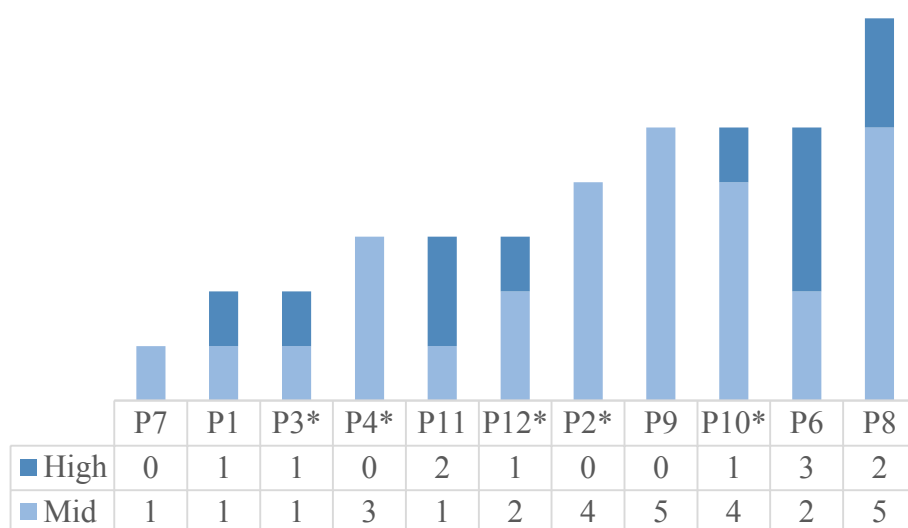


Figure 5.3 Participants' performance in the LNMTI Adequate category ('Mid-High')

This preliminary analysis indicates overall, participants' LNMTI knowledge is poor. Secondly, the performance of participants who completed their Leaving Certificate studies in mathematics pre 2012, in the LNMTI survey was not significantly different to those who took the final examination during the reform period in 2012. The next section will present findings on participants' strengths and limitations in the LNMTI knowledge domains.

5.5 Strengths and Weaknesses in LNMTI Knowledge Domains

This part of the analysis was guided by the second Research Question in this phase:

RQ2(b): *What are the specific knowledge areas of strength and difficulty demonstrated?*

As previously mentioned, the survey had eight questions representing each strand from the LNMTI framework referred to as LNMTI knowledge for Number, Algebra and Functions, Geometry and Trigonometry and Statistics and Probability. Scoring on the individual domains delivered interesting findings. Figure 5.4 provides a breakdown

of the responses from the four strand areas categorised as LNMTI Adequate or LNMTI Inadequate.

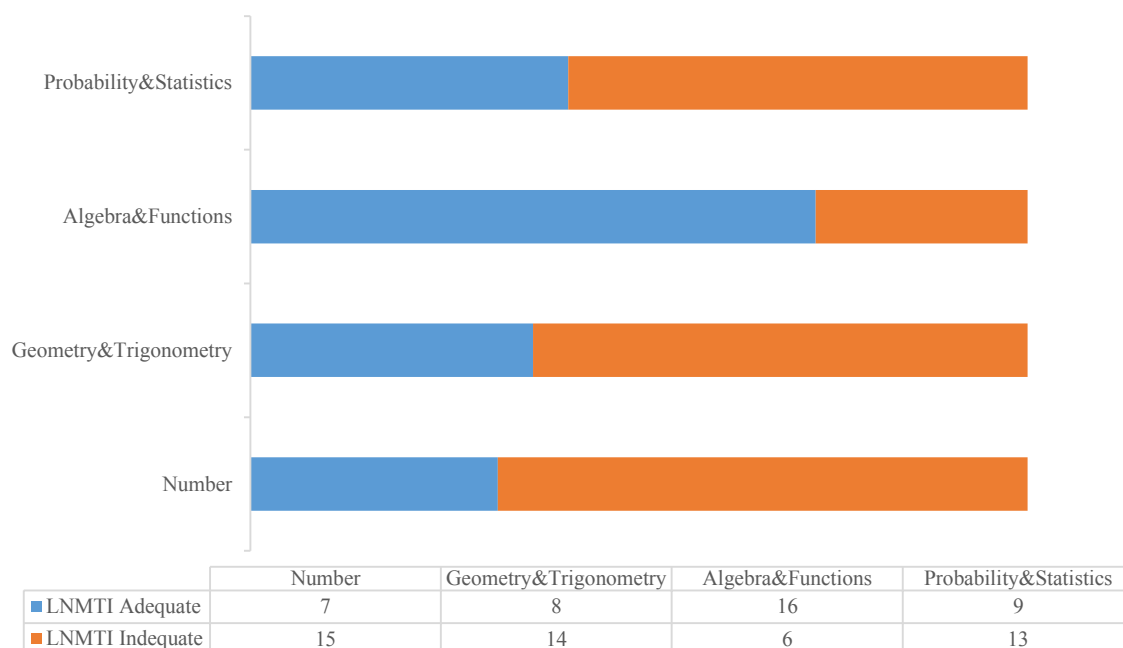


Figure 5.4 Frequency of LNMTI Adequate/Inadequate for each of the 4 domains

It is apparent from this bar chart the responses from the participants on LNMTI knowledge for Algebra and Functions were much stronger than the other domains. Almost three quarter of the responses (73%) were awarded a grade in the ‘Mid’ or ‘High’ category whereas almost the same proportion of responses (68%) for LNMTI knowledge for Number were awarded a grade in the ‘Not Present’ to ‘Low ‘ or LNMTI Inadequate category. The responses from the other two domains were only slightly better with 63% of responses in the ‘Inadequate’ category for LNMTI knowledge for Geometry and Trigonometry and 59% for Statistics and Probability. To explore specific areas of strengths and weaknesses in the responses in more detail, the author examined the distribution of grades awarded LNMTI Adequate/ Inadequate in individual questions. This produced findings that highlighted the affordances and deficits that existed in an LNMTI knowledge domain. Figure 5.5 shows the differences in grade allocation in the LNMTI Adequate category for the eight questions in the survey.

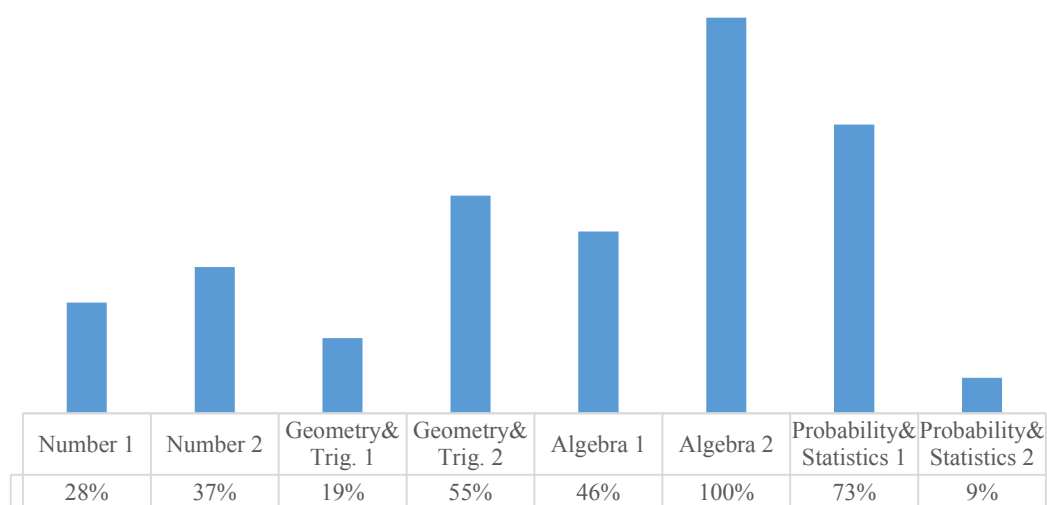


Figure 5.5 Frequency of LNMTI Adequate responses in the eight survey questions

As the graph illustrates, there was a 100% ‘Mid-High’ grades awarded to the ‘Algebra and Functions 2’ question on *Simplifying Algebraic Fractions*, in other words, each of the eleven participant responses was LNMTI Adequate. The ‘Statistics and Probability 1’ question on *Reading Graphs* yielded 73% while 55% of the responses from ‘Geometry and Trigonometry 2’ question on *Similar Triangles* were LNMTI Adequate. In contrast, the three questions that yielded overall weak responses and consequently categorised as LNMTI Inadequate were:

- ‘Statistics and Probability 2’ question on *Probability Concepts* (9% of responses were deemed LNMTI Adequate).
- ‘Geometry and Trigonometry 1’ question on *Geometry Definitions* (19% of responses were deemed LNMTI Adequate).
- ‘Number 1’ question on *Irrational Numbers and Calculator* (28% of responses were deemed LNMTI Adequate).

The next section moves on to discuss in more detail the strengths and weaknesses described above.

5.5.1 Question Analysis: Algebra and Functions

The question that generated the best overall LNMTI Adequate score, 100%, was the abstract algebra question on *Simplifying Algebraic Fractions* (Figure 5.6)

Ciara is a second year Junior Certificate student with good mathematical ability. She simplifies the following rational algebraic expression correctly:

$$\frac{2x - 1}{2x^2 + 5x - 3}$$

(a) *Show how she does this.*

(b) *Explain the reason for each step in simplifying the above expression as you would to a student preparing for Junior Certificate Higher Level.*

Figure 5.6 Simplifying Algebraic Fractions question from LNMTI survey

The question was adapted from materials produced from a Project Maths lesson study completed in one Irish post-primary school on simplifying algebraic fractions and the misuse of cancelling (Maths Development Team n.d.c., workshop 10). Aligning with the LNMTI framework, the key mathematical concept underpinning these algebraic errors is the numeracy outcome of equivalent fractions; effectively explaining the procedure is a literacy outcome. Participants from the author's study were more comfortable with the abstract algebra task from the LNMTI survey and given that most of the explanations were procedural in nature, the use or implied reference to equivalent fractions was present. Moreover, Procedural Fluency is still a learning objective in current mathematics syllabuses and specification. In addition, further emphasis has been put on Procedural Fluency by including it as an element in the Unifying strand under the title 'Building Blocks' (National Council for Curriculum and Assessment 2017, p.15).

Despite the incorrect use of the word 'root' in the explanation, P7, who attained the poorest overall score in the LNMTI survey with a LNMTI Adequate score of 12.5%, was awarded a grade in the 'Mid' category by showing competency in this area of mathematical pedagogical knowledge (Schmidt *et al.* 2007). Similar to the work of all

of the participants, conceptual understanding of a rote procedure in an explicit or implicit way was evident (Star 2007). See Figure 5.7:

Show how she does this:

$$\frac{2x-1}{2x^2+5x-3}$$

$$\frac{2x-1}{(x+3)(2x-1)} = \frac{1}{x+3}$$

$$\begin{array}{l} 2x^2+5x-3 \quad (5-1) \\ 2x^2+6x-x-3 \quad -6 \\ \hline 2x(x+3)-1(x+3) \quad -1 \times 6 \\ (2x-1)(x+3) \quad 2 \times 3 \\ \hline \quad \quad \quad 6 \times 1 \end{array}$$

Cannot simplify or factor any further so leave for time being

Can factorise this so now will factorise this using the guide number method

Need to watch out for signs here as the 6 is negative

Will then use the guide number method to find the roots when grouping terms and replace them where the quadratic was as they are factorised

There is now a HCF between the numerator and the denominator so these can cancel or divide and you are left with $\frac{1}{x+3}$

Figure 5.7 Participant response (P7) to Simplifying Algebraic Fractions question from LNMTI survey

Figure 5.8 presents the linear patterns question on the coloured toy trains (Zazkis 2016) that focussed on the pattern/functions based approach to algebra which is the promoted pedagogy for the teaching of algebra for Project Maths and the Junior Cycle Mathematics specification (Maths Development Team n.d.c., workshop 4). These type of problems aim to develop Strategic Competence in learners, defined as ‘the ability to formulate, represent and solve mathematical problems’ (Kilpatrick *et. al* 2001, p5) ‘in both familiar and unfamiliar contexts’ (National Council for Curriculum and Assessment, p.6).

A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is green, and the fifth is red and sixth is blue, and so on.

What is the colour of the 39th car?

The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student's work.

T: Why is the 39th car yellow?

S: Because the 3rd car is yellow and 39 is a multiple of 3.

- (a) Identify the student error/misconception in this instance
- (b) Outline how you would help the student correct the error/misconception.

Figure 5.8 Linear Patterns question from LNMTI survey

This question saw more participants in the LNMTI Inadequate category, 56%, than the abstract algebra question indicating a lack of familiarity and practice with this approach (Prendergast and Treacy 2017). P8, who was the top performer on the LNMTI survey with a LNMTI Adequate score of 87.5% was awarded a 'Low' for the following response as the approach to helping the student to correct the error/misconception had merit, but was confusing:

P8: Ask student what colour the 40th car would be. Expect Green because of multiple of 4. Point out 40 is also a multiple of 2 so surely should be blue and a multiple of 1 so should be red. Assert the multiplication idea is correct but we are looking for the remainder after looking for multiples of 6. Explain with a diagram after this.

The following response from P3, with a LNMTI Adequate score of 25% was also awarded a 'Low'. Apart from the confusing statement at the start, the participant attempted to use a counter example as a strategy to develop mathematical thinking (Guerin 2017; Klymchuk 2008). However, the overall approach is teacher directed:

P3: This is not wholly about multiples. It is about patterns. I would show the student an example. The 10th car is green but is 4 a multiple of 10? I would encourage the student to draw out the pattern.

In summary, the abstract algebra question with the emphasis on Procedural Fluency generated higher LNMTI Adequate scores in comparison to the question with the primary focus of developing Strategic Competence to support problem solving skills.

5.5.2 Question Analysis: Number

It is well established that success in algebra is dependent on knowledge of number (Ketterlin-Geller and Chard 2011), and specifically rational number knowledge is a predictor of algebra success in post-primary education settings (McMullen *et al.* 2017). However, this association was not replicated in participant responses on both of the LNMTI knowledge for Number questions where 28% and 37% of the responses were in the LNMTI Adequate category in comparison to 48% and 100% for LNMTI knowledge for Algebra and Functions. Figure 5.9 presents the first question on the survey which describes a classroom scenario where a teacher has to deal with a student contribution about rational and irrational numbers (Zazkis 2016). Using a student mathematical contribution, the question investigated participants' ability to support students' learning in the classroom. The question had a distinct digital media reference, the use of the calculator, an element of the literacy definition.

The class are discussing rational and irrational numbers. One student claims that $\frac{23}{43}$ is an irrational number because his calculator shows 0.53488372 when 23 is divided by 43 and there is no repeating pattern of digits. In the role of the teacher write the response you would make to this student's claim?

Figure 5.9 Irrational Numbers and Calculator question from the Number strand

However, as was previously mentioned, participant responses to this item were generally poor. 82% of the responses were awarded a grade in the 'Not Present-Low' category or LNMTI Inadequate. Table 5.5 presents the rubric from the LNMTI four point scoring rubric for this question. As illustrated in the 'Anchor Example for 'High'', to earn a 'High' grade, participants were required to give a definition of rational numbers, relate the number to the definition and reference the calculator display.

Table 5.5 LNMTI scoring rubric for Irrational Numbers and Calculator

<i>Value</i>	<i>Definition</i>	<i>Anchor example for High</i>
<i>Not Present</i>	Incorrect response; Response is confusing and lacks clarity; Response does not develop student understanding.	Ask the student to recall the definition of a rational number: Rational Numbers are any number of the form $\frac{p}{q}$, where $p \in \mathbb{Z}$ and $q \in \mathbb{Z}$ and $q \neq 0$.
<i>Low</i>	Responds in a pro-forma way.	
<i>Mid</i>	Response has some features listed under high.	
<i>High</i>	Response encourages the student to think about claim by asking an appropriate question(s) that develops student understanding. Mathematical definition of rational/irrational number given. References to digital media given, i.e., the limitation of the calculator display.	Ask the student to position this number in relation to the definition: 23 and 43 are both integers, therefore $\frac{23}{43}$ is a rational number. Eventually the digits would repeat but the calculator has a limited digit display.

The following response from P3 demonstrated no understanding of basic knowledge of rational and irrational numbers and was therefore awarded ‘Not Present’:

P3: *Rational numbers do not terminate i.e end. And this one does. Therefore it is an irrational number.*

Three participants, P2, P9 and P12 acknowledged the role of the calculator in their responses but only one, P2, merited anything higher than a ‘Mid’:

P2: *I would first ask them what the definition of an irrational number is or to explain it in their own words. I would prompt them appropriately. I would then ask them to compare $\frac{23}{43}$ with $\sqrt{2}$ by inputting them into the calculator and pressing the $S \rightarrow D$ or change button. I would hope they would see that $\frac{23}{43}$ is a fraction where as $\sqrt{2}$ is not. I would bring them back to the definition.*

This response attempts to show the contrast between rational and irrational numbers but the definition of either irrational or rational numbers is not given. A reference to digital media is given but the limitation of the calculator or the repeating decimal feature is not mentioned.

Similarly, the second number item *Multiplication of Real Numbers between 0 and 1* presented in Figure 5.10, had 7 responses assigned to the LNMTI Inadequate category.

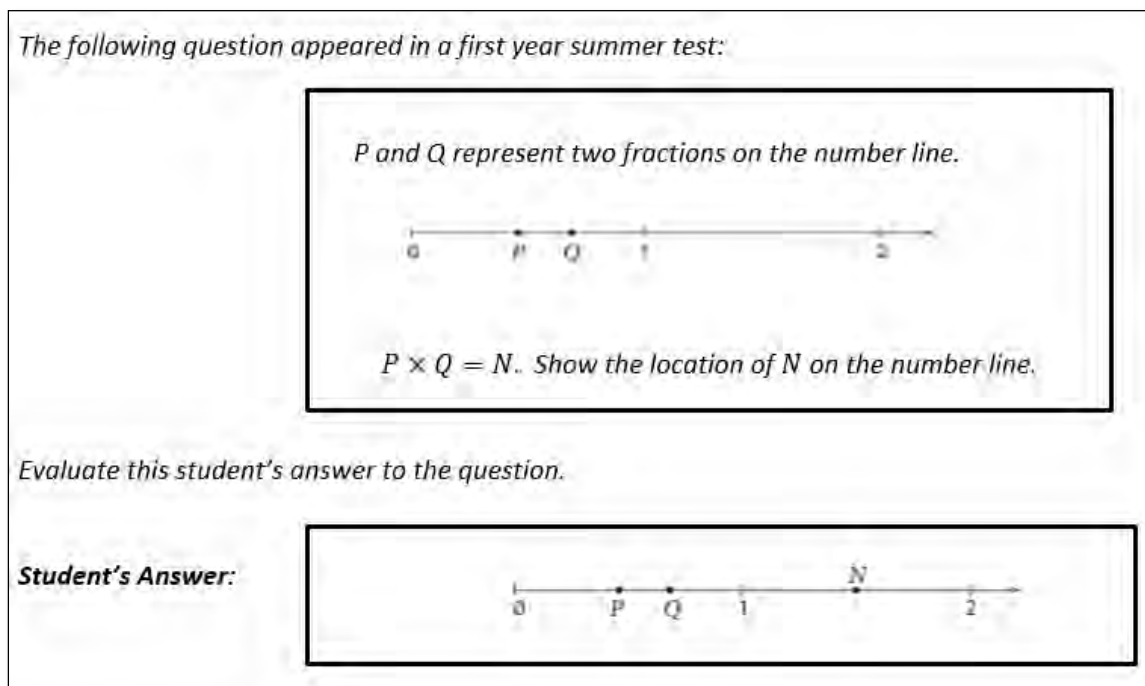


Figure 5.10 *Multiplication of Real Numbers Between 0 and 1* question from the Number strand

This question was taken from TIMSS 2011 Grade 8 number strand to capture reasoning with fractions and decimals (National Center for Education Statistics (NCES) 2015). ‘Mathematical Sense Making’ was the *MQI for LNMTI* element under scrutiny. This element focuses on number sense in mathematics (Harvard University Center for Education Policy Research 2018a). For this question, it referred to making sense of rational numbers and the operation of multiplication. The scoring rubric for this question is presented in Table 5.6.

Table 5.6 LNMTI scoring rubric for Multiplication of Real Numbers Between 0 and 1

Value	Definition	Anchor example for High
Not Present	Incorrect response; response is confusing and lacks clarity; response does not develop student understanding	The student is confused about the operations of addition and multiplication for real numbers between 0 and 1.
Low	Identifies possible student error/misconception, but does not address this	To remediate the misconception, use specific numerical/diagrammatic examples and:
Mid	Identifies possible student error/misconception and makes an effort to address this.	(1) multiply two fractions between 0 and 1 and represent the answer on the number line. (2) add two fractions between 0 and 1 and represent the answer on the number line
High	Giving meaning of numbers in symbolic form. Giving meaning to the number in the number line representation. Giving meaning of operations for real numbers between 0 and 1. Focus on the reasonableness of a solution.	Ask the student if they can identify any pattern (the answers lie between 0 and P for multiplication and greater than Q for addition)

The following response from P2, presented in Figure 5.11 was graded in the ‘Not Present’ category:

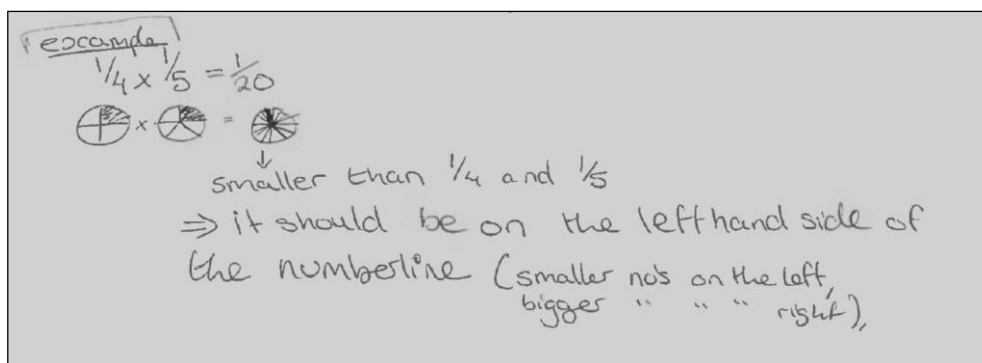


Figure 5.11 Participant response (P2) to Multiplication of Real Numbers Between 0 and 1 question from LNMTI survey

The choice of visual display for this problem is not making sense of the operation of multiplication of rational numbers between 0 and 1. According to Rau *et al.* (2017) graphical displays are a fundamental strategy in building conceptual knowledge of

fractions. By representing the problem with two unit circles with shaded regions to generate the third unit circle does not adequately represent the problem. In this question, a quarter of one fifth of the unit is required which corresponds to one twentieth of the unit. Secondly, the participant is attempting to use a single example to establish a general point which does not align with the author's definition of LNMTI by not demonstrating 'rich instruction while working with students and mathematics' (Section 4.1).

In contrast, the following response from P6 was awarded a grade in the 'Mid' category or LNMTI Adequate. The student's misconception is addressed however it lacks the detail of the 'High' response as exemplified in Table 5.6.

I would ask the student to multiply a series of fractions of less than one together and look for a pattern.

I would write an example with numbers: $P = \frac{1}{2}$ and $Q = \frac{5}{8}$

$$P \times Q = \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$$


and ask the student to fill in $\frac{5}{16}$ and ask them for their own example and re-evaluate their answer.

However, this participant's response is more in line with the LNMTI definition by giving an example of good practise where students are encouraged to look for patterns in an attempt to establish a general deduction. Interestingly, Swan (n.d.) found teachers generally experience difficulties in attempting to facilitate this element in a classroom setting. This demands further investigation in an Irish context as Generalisation and Proof is an element of the Unifying strand in the Junior Cycle Mathematics specification (National Council for Curriculum and Assessment 2017). Pre-service teachers need to be prepared to facilitate a teaching and learning environment where mathematical thinking or sense-making is developed through the practice of generalising.

5.5.3 Question Analysis: Geometry and Trigonometry

There was variation in the responses to the LNMTI knowledge for the Geometry and Trigonometry questions. Both questions examined participant knowledge of geometry concepts based on Van Hiele's level 2 of geometric thinking (Mayberry 1983). In the first geometry question, *Geometry Definitions* presented in Figure 5.12, participants were asked to identify and define geometric shapes (Milgram 2005) in a photograph accessed from an Irish numeracy initiative, 'Have You Got Maths Eyes' (2018). The second question, titled *Similar Triangles*, required participants to solve a geometric problem using similar triangles (Maths Development Team n.d.d.). Unlike previous mathematics syllabuses in Ireland, the technical language of mathematics is specified in the current mathematics syllabus documents: 'students should be able to use the terms theorem, proof, axiom, corollary, converse and implies' (National Council for Curriculum and Assessment 2013a, p 18). Furthermore, 'communicating and expressing' (p.8) is identified as a required skill to develop mathematically. In addition, communication is one of the key elements of the Unifying strand in the Junior Cycle Mathematics specification.

Examine the photograph of this tiled floor and answer the following questions:



<http://www.haveyoutgotmathseyes.com/wp-content/gallery/resources/009-tiled-floor.jpg>

(a) Identify the shapes in this tiled floor
(b) Define each shape

Figure 5.12 *Geometry Definitions* question from *Geometry and Trigonometry* strand

Only four of the participants, P4, P6, P7, P8, defined a square correctly. Examples of other participants' attempts were:

P3: *Four sided shape*

P1&P2: *Four sided shape, all angles equal 90°*

P9&P10&P12: *Four equal sides in length*

Only two participants used the word 'quadrilateral' but the connection that it is the domain category for four sided polygons was not made. One other participant, P7, defined a triangle as a *three sided shape that comprises of 360°* and another, P1, stated a triangle was *three vertices*. P11 produced the following series of deconstructed polygons in response to the task presented in Figure 5.13:

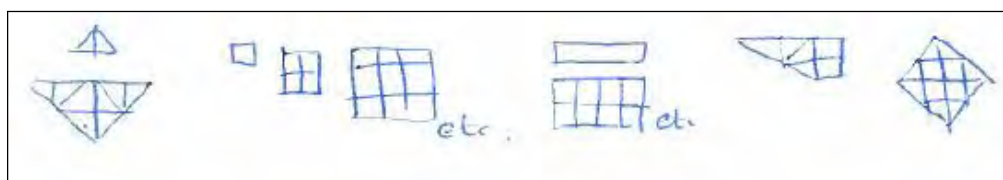


Figure 5.13 Participant response (P11) to the Geometry Definitions question from the LNMTI survey

These findings clearly demonstrate the participants' basic knowledge of geometry is extremely poor despite the current mathematics teaching regulatory environment where all but one participant studied pure mathematics in third level. This raises serious issues about their preparedness to deliver 'rich instruction' (LNMTI definition) from the Geometry and Trigonometry strand in post-primary mathematics. The literature confirms Irish students' poor performance in geometry from TIMSS 1995 and 2011 assessments where fourth class primary students' (10 year-olds) geometry knowledge was described as a 'national weakness' (Eivers and Clerkin 2012, p.29-30). Similarly, the 2015 chief examiner's report on Junior Certificate mathematics confirmed geometry as an area of difficulty (State Examinations Commission 2016a, p.13, p.21, p.29). This trend is set to continue for Irish students if interventions at third level are not addressed. However, basic mathematical knowledge has been identified

as an issue amongst pre-service teachers internationally despite having a third level qualification in the subject (Ma 2010; Goos 2017).

The second Geometry and Trigonometry question, *Similar Triangles* presented in Figure 5.14, was taken from a problem solving set aimed at Junior Certificate Higher Level students, aged 15, with an aptitude for mathematics.

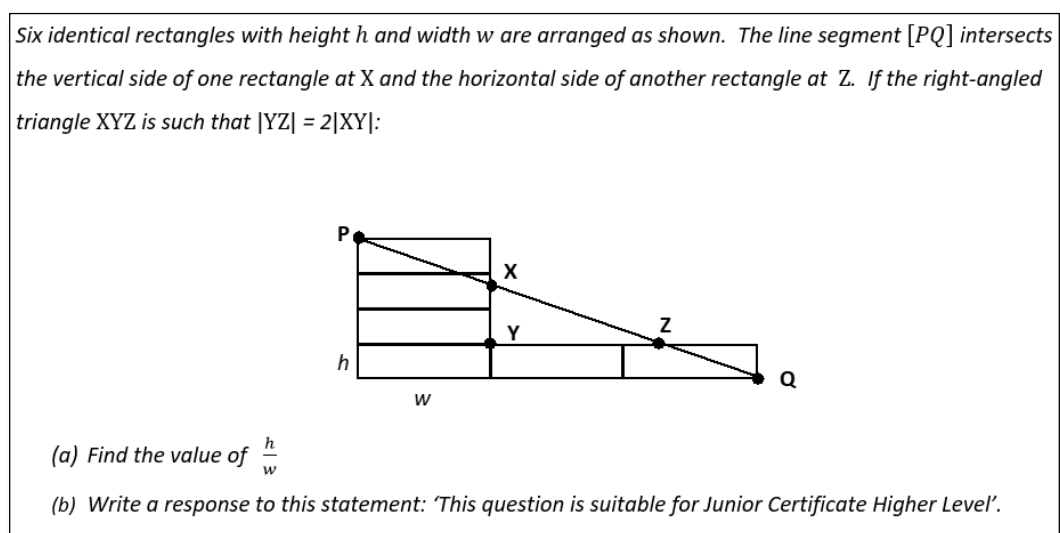


Figure 5.14 *Similar Triangles* item from the Geometry and Trigonometry Strand

Participants were assessed on their ability, in a Geometry and Trigonometry context, to solve a problem and think and communicate quantitatively as well as communicate knowledge of the syllabus. Despite all the participants rating this question as 'Difficult', the work presented was awarded a higher rating. In fact, as shown in Figure 5.15, the work presented for the *Similar Triangles* question acquired 55% in the LNMTI Adequate range in comparison to 18% in Geometry Definitions.

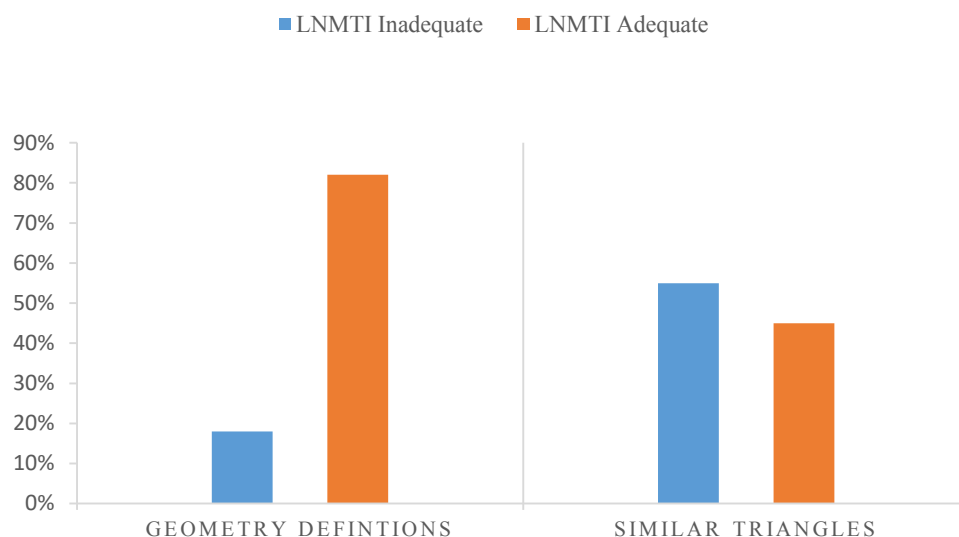


Figure 5.15 Comparison of overall participant awards in the Geometry and Trigonometry questions

From a literacy perspective for both Geometry and Trigonometry questions, participants had to read visual images to gather information about the tasks. As can be seen from Figure 5.12, the first question is a digital media image but the image for the second task, presented in Figure 5.14, is consistent with the type of imagery found in mathematics textbooks and materials studied at post-primary and tertiary level (Walsh 2015). The literature describes the learning of mathematics as an interplay between text, symbol and visual with leading researchers in language and mathematics education such as Sfard and O'Halloran (2008, 2005 cited in Hammill 2010) acknowledging the primacy of the visual. Furthermore, Northcut (2007) reports on the use of visual images in a classroom setting:

Two lessons tend to emerge: the students lack a vocabulary for describing images, and they jump to evaluations and aesthetic commentary rather than describing. If students are assisted in developing the vocabulary to describe and interrogate images, they can begin to evaluate critically (p.259).

These findings suggest participants should be given opportunities to work with visual imagery and develop a fundamental understanding of geometric ideas in order to progress their understanding of LNMTI as defined in the LNMTI framework.

To conclude, participants in this study had inadequate geometry vocabulary to describe the image yet for the most part, they made a better attempt at translating an abstract Geometry and Trigonometry context into numerical quantities to solve a problem, demonstrating strategic competence.

5.5.4 Question Analysis: Statistics and Probability

As with the other domains two items were presented to examine LNMTI knowledge for statistics and probability, titled *Reading Graphs* and *Probability Concepts*. The *Reading Graphs* question, shown in Figure 5.16, with a predominantly statistics focus, was taken from PISA 2003 (Shiel *et al.* 2007). It assessed participant understanding of translating from a graphical to a numerical representation and using this association to explain their reasoning. Irish 15 year-olds in 2003 found this item difficult scoring below the OECD average (Shiel *et al.* 2007).

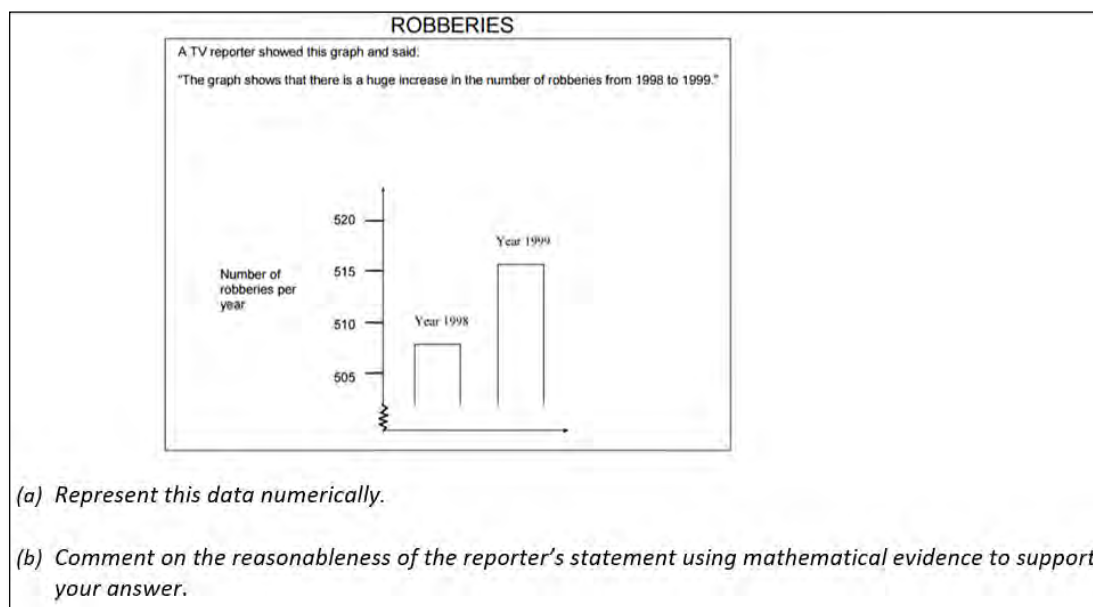


Figure 5.16 Reading Graphs question from the LNMTI survey

As can be seen from the text in the Figure 5.16, participants were asked to represent the graphical data numerically and then to comment on the reasonableness of the reporter's statement using mathematical evidence to support the answer. Table 5.7 presents the scoring rubric for this question.

Table 5.7 LNMTI scoring rubric for Reading Graphs

Value	Definition	Anchor example for High		
Not Present	response is incorrect or poorly executed	Year	1998	1999
Low	response has 1 feature listed under High	No. of robberies	508	516
Mid	response has 2 features listed under High	$\frac{8}{508} \times 100 \approx 1.6\%$		
High	Response has the following: (1) Correct Numerical representation/ Numerical representation in tabular form and/or displays a broader understanding of ‘numerical representation’ vis-à-vis JC syllabus, (2) Identifies increase/relative size, (3) References the exaggerated use of the word ‘huge’.	The reporter’s statement is exaggerated. There are 8 more robberies reported which is a percentage increase of 1.6%		

Despite 8 out of 11 participants scoring at least in the ‘mid’ category from the LNMTI scoring rubric, 73% of the participants misread the graph, with the majority of misreads accounting for the ‘Year 1999’ bar as 515 robberies per year. This demonstrates a lack of rigour and accuracy by the participants in a basic skill which may lead to a weakness in transferring or enabling these skills in their students. Secondly, these findings also indicate the LNMTI scoring rubric for this question requires revision.

Findings from responses to the *Probability Concepts* demonstrated that the participants’ basic probability content knowledge was extremely weak. All of the participants, except P8 who scored a ‘Mid’, were awarded a grade in the ‘Not Present’ category. In an email written to the author in February 2017, the Educational Expert who rated the participant responses from the LNMTI survey remarked: ‘I’d be concerned at the quality of teaching/learning [of Statistics and Probability] that school students would experience under many of these respondents’ (Appendix Y).

As Figure 5.17 presents the question that was adapted by the author from a summative assessment question sat by Irish post-primary students in 2013 in a national state examination (State Examinations Commission 2013). This question was written to examine participants' knowledge of probability for teaching (Stohl 2005) by evaluating a student's response to a probability question that contained multiple errors.

The following problem appears in a Leaving Certificate Ordinary Level Examination Paper, 2013:

Katie tossed a coin 200 times and threw 109 heads. Joe tossed the same coin 400 times and threw 238 heads. Lucy tossed the same coin 500 times and threw 291 heads. Lucy uses all the above data and calculates that the best estimate of the probability of throwing a head with this coin is 0.58. Show how Lucy might have calculated this probability

One student works through the problem in the following way:

Step 1	Step 2	Step 3
Katie: $\frac{109}{200} = 0.545$ Joe: $\frac{238}{400} = 0.595$ Lucy: $\frac{291}{500} = 0.582$	$0.545 + 0.595 + 0.582 = 1.722$	$0.1722 \div 3 = 0.574$ <i>0.58 is an estimate of 0.574</i>

Evaluate this student's method and final answer:

Figure 5.17 Probability Concepts item from the Statistics and Probability strand

As mentioned earlier, 45.5% of participants sat the first Project Maths Leaving Certificate examination in 2012 that provided assessment questions on Statistics and Probability to reflect the reform. Nevertheless, probability was a popular and relatively well-answered topic amongst Higher Level students' pre 2012 (State Examinations Commission 2005a). In addition, the participants had completed all of the required modules in probability and statistics required for Teaching Council registration regulations for post-primary mathematics teaching in Ireland (2013). Despite this, with 91% of responses assigned to the LNMTI Inadequate category, it shows that participants' have a particularly poor understanding of LNMTI knowledge for probability (Gómez-Torres *et al.* 2016).

The anchor example for the ‘High’ response taken from the LNMTI scoring rubric was as follows:

(1) To get more accurate results for the experiment, the student should have counted up the total number of trials (1100) from Kate, Joe and Lucy and the total number of successes (638) and then calculated the probability: $\frac{628}{1100} = 0.58$

The student performed the calculation on the left hand side which is not equivalent to the required calculation on the right hand side.

$$\left(\frac{a}{b} + \frac{c}{d} + \frac{e}{f}\right) \div 3 \neq \frac{a + c + e}{b + d + f}$$

(2) The student made a slip by incorrectly changing 1.722 to 0.1722.

(3) The student rounded incorrectly.

P2 noticed the error or slip made as a result of carelessness (Gardee and Brodie 2015) and P6, P9 and P10 identified the rounding error. However, 91% of the participants did not identify the core misconception outlined in (1) above. 45.5% of the participants believed the student’s approach to the problem was correct with one participant P4 remarking:

P4: I would have solved this sum in exactly the same way.

In summary, while the question with a primary focus on statistics generated strong responses from the participants, skills that support problem solving competencies such as interpreting graphical displays was weak. Secondly, basic mathematical knowledge of probability was very poor.

The following section will discuss how the focus group findings provided further insights into the performance of participants in the LNMTI survey (Morgan 1988).

5.6 Focus Group Data: Introduction

The Educational Design Research methodology promotes the ‘perception poll strategy’, defined as an approach to gather information on participants’ attitudes, needs, and wishes to the problem (McKenney and Reeves 2012, p.94). Consequently,

a focus group was employed to uncover the pre-service teachers of mathematics perspective, knowledge and understanding of LNMTI (Krueger and Casey 2009). It operated on three levels: to extend findings in the survey, compare participant knowledge of LNMTI with the LNMTI framework, and to provide recommendations for an LNMTI intervention. Finally, this method also functioned as a form of data triangulation for the survey findings (Cohen *et al.* 2011; Morgan 1988).

Out of the eleven respondents who completed the survey, six females, identified as P1, P2, P3, P4, P5 and P6 volunteered to participate in the focus group. It took place on January 30th 2017, two months after the participants took the LNMTI survey, following a mathematics methods lecture. All of the participants who volunteered to participate in the interview had completed the LNMTI survey, except P5. The author structured the interview in five parts to gather information on the following themes:

- Response to the Survey,
- Participant understanding of literacy and numeracy for mathematics teaching,
- Evidence based practice of literacy and numeracy in their classrooms,
- Training provided by the university's Initial Teacher Education programme for literacy and numeracy,
- Suggestions from participants on what they would like to be covered in the LNMTI module.

PowerPoint slides of the five individual questions were presented to the focus group participants. This kept the discussion focussed and enhanced the author's neutrality during the interview by enabling the participants to read and respond to the questions on the slide (Litosseliti 2003; Patton 2002). The focus group interview was transcribed by the author (Appendix K). To improve the accuracy and the validity of the transcription, the participants were contacted by e-mail in June 2017 and were invited to verify and/or make changes to the transcribed interview (Birt *et al.* 2016). No changes or amendments were requested (Appendix V). For the content analysis process, the unit of analysis was a response. This took the form of an individual response or a group response. The coding procedure applied to this data was

‘Evaluation Coding’ which is a useful process to describe, compare and predict (Saldana 2009). The rationale for employing an evaluative perspective to the focus group data was three fold:

- to describe and categorise participant verbal responses to questions in the LNMTI survey as a data triangulation method to the pen and paper LNMTI survey,
- to compare participant literacy and numeracy practices in the mathematics classroom with the standards set out in the LNMTI framework,
- to predict a change or recommendation to the provision of literacy and numeracy education for mathematics teaching at post-primary level.

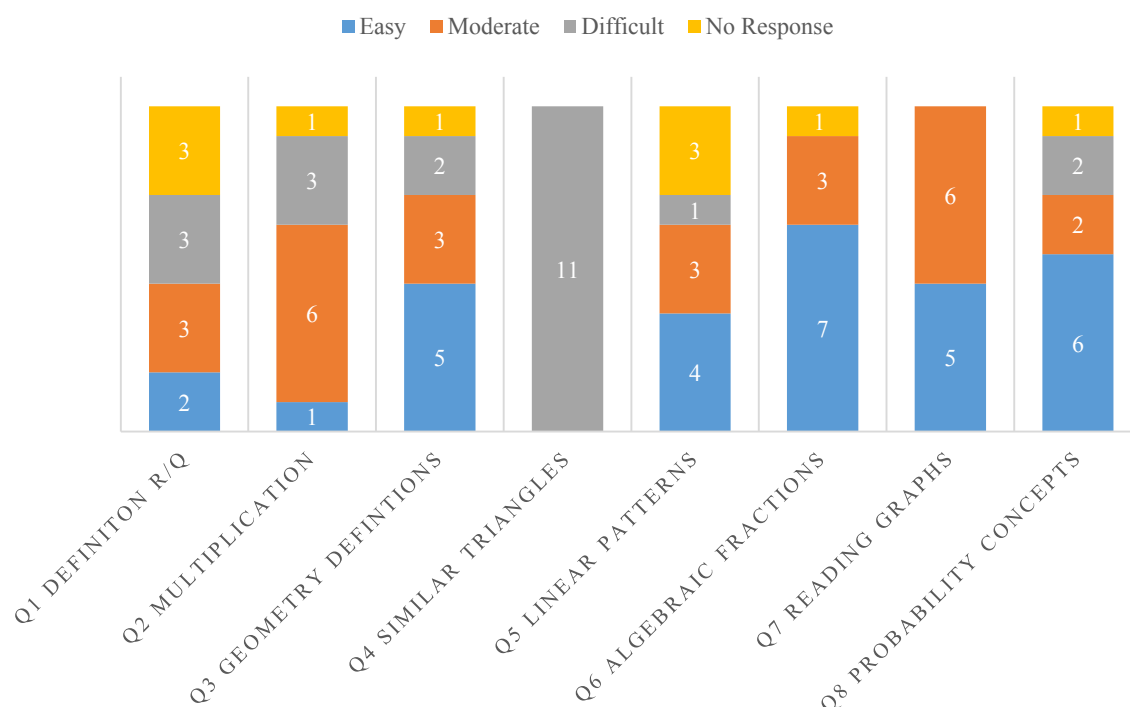
The first coding procedure was done by hand and then the data was entered into QSR NVivo 11 for further analysis. Secondly, a word frequency query was performed to explore what words were used most often in the focus group interview to support the author’s analysis of the coding process. The following section describes the results of the evaluation coding method, as applied to the participants’ responses.

5.6.1 Participant Response to the LNMTI Survey

For the first question, a copy of the survey was given to the participants and they were given time to read over the questions and recall as best they could, given the two month time lapse between the LNMTI survey and focus group interview, whether they rated the questions as easy/moderate/difficult. They were invited to expand on their decision and that response was compared to the rating they had given the question on the day of the survey. Out of 88 responses from the paper and pencil survey, 34% of the responses rated the questions as easy, 30% moderate and 25% difficult. Some participants did not rate individual questions therefore, a fourth category emerged titled: ‘no response’. As Figure 5.18 reveals, half of the ‘difficult’ category responses (11 responses) were generated by the *Similar Triangles* question where, as previously

discussed in Section 5.3, participants did well with 55% of participant responses scoring in LNMTI Adequate category.

Figure 5.18 Participants' rating of questions in the LNMTI survey



Based on the participants' varied performance in the survey, responses from the focus group participants were coded according to whether they had overestimated their LNMTI knowledge. By overestimation in this context, the author means when a difference exists between a verbal response on a difficult rating in the focus group interview and the actual graded response from the LNMTI survey. For instance, P3 described the *Geometry Definitions* question where participants were asked to identify and define shapes from a photograph, as *an easy enough question*. The real life context as well as its accessibility to all learners was noted. The same participant also rated the question 'Easy' in the LNMTI survey. However the response to this question by P3 was awarded a grade in the LNMTI Inadequate category. Out of the five participants who were involved in the focus group interview, three rated the question as 'Easy' and two as 'moderate' but only one response was awarded a grade in the LNMTI Adequate range. In addition the participants agreed the difficulty rating was from the perspective of a post-primary student in the classroom (Appendix R).

Similarly, an overestimation of LNMTI knowledge was evident for the *Probability Concepts* question, where participants were asked to evaluate a student response that contained multiple errors. All of the participants except one, P8, were awarded ‘Not Present’ or LNMTI Inadequate for the work presented for this question in the LNMTI survey. In addition, as the bar chart in Figure 5.18 reveals, over half of the participants rated the question as ‘Easy’ in the LNMTI survey. Only one participant at the focus group interview rated the question as ‘Difficult’. However P4, who rated the question ‘Easy’ in the survey, remarked:

P4: you can do it in the class throwing coins. They can see when they put all their data together that they will get to close to 0.5. They would be able to appreciate the concept. A real life activity.

As previously mentioned, this participant failed to identify the core conceptual error in the student’s work in the LNMTI survey and endorsed the flawed work of the student (Section 5.5.4). Yet this response from the focus group interview demonstrates understanding of the core concept.

In contrast, participant reaction to the *Simplifying Algebraic Fraction* question in the focus group interview aligned with their performance in the LNMTI survey. This question earned ratings of ‘Easy’ or ‘Moderate’ in the LNMTI survey by all of the participants, except one, who gave no response. As previously mentioned all of the respondents scored in the LNMTI Adequate range for this question.

When the author looked for clarification on the motivation for ranking each question, the participants all agreed it was based on the point of view of a student in the post-primary classroom. The findings in the survey results, supplemented by data from the focus group interview, suggest participants had difficulty with the questions while also demonstrating a lack of awareness of the gaps in their knowledge (Kominsky and Keil 2014).

5.6.2 Participant understanding of LNMTI

The second question directed to the focus group dealt with the participants’ understanding of LNMTI (Appendix J). Responses were coded ‘Positive LNMTI’ if

participants demonstrated a understanding of both literacy and numeracy in mathematics teaching by making a reference to the domains of ‘literacy’, ‘numeracy’ and ‘teaching’ in a response. The other evaluative code, ‘Negative LNMTI’ was employed if one or more of the three elements were not included in the response. This decision was motivated by the overall aim of the study which was to investigate if pre-service teachers of mathematics were prepared for LNMTI which encompassed all of the previously mentioned elements. All of the six focus group members were provided with an opportunity to respond, and each of their responses was coded as ‘Negative LNMTI’. Three of the respondents, P1, P3, P5, referenced ‘literacy’ exclusively such as:

P5: understanding terminology and being able to define it.

Only the ‘numeracy’ element was referenced by P2

P2: Looking at numeracy and seeing patterns within questions and to help themselves.

The only participant who made an attempt to encompass both ‘literacy’ and ‘numeracy’ was P4.

P4: There’s a whole context part to the paper now so they would need to apply what they know to unfamiliar problems and decipher it. They need to be familiar with the terms etc.

P6, who had the highest LNMTI Adequate survey score of those participating in the interview did not contribute to the discussion and when the author probed for this participant to comment, she replied:

P6: It’s an area I struggle with a lot myself so I need to develop a better understanding of it.

Question three asked the participants to give a specific example of literacy and numeracy in their classroom practice. These responses were coded according to categories in the four domains of the LNMTI framework: Literacy Processes, Numeracy in Mathematics, Literacy Mathematical Quality of Instruction for Literacy

and Numeracy for Mathematics Teaching in Ireland or *MQI for LNMTI* and Literacy Form. For example, the following response from P1:

P1: For literacy I remember when I was doing algebra with the second years, linear equations came up and we just wrote the word 'linear' down. I said, 'what does that mean?' They said 'it is a line from linear'. It kind of made more sense, linear equations, that was highlighting to them to make sense of words.

was coded as 'understand' for Literacy Processes as the participant was enabling student understanding of 'linear'. The Numeracy in Mathematics content was taken from Algebra and Functions. *MQI for LNMTI* was demonstrated with the focus on Mathematical Language and an implied use of Teacher uses Student Mathematical Contributions. The Literacy Forms used were Spoken Language with the use of questioning and verbal responses from the students coupled with Printed Text as the participant and students 'wrote the word linear down'. This coding procedure is summarised in Table 5.8.

Table 5.8 Coding procedure for participant (P1) vignettes for LNMTI

<i>Literary Processes</i>	<i>Numeracy in Mathematics</i>	<i>MQI for LNMTI</i>	<i>Literacy Forms</i>
Understand	Algebra and Functions	Mathematical Language Teacher uses student Mathematical Contributions	Spoken Language Printed Text

The most striking feature from the coded responses was the references to mathematical language (5 out of 6 participants) where most of the participants described their literacy practices in terms of enabling student understanding of mathematical words (Appendix Z). In contrast, one participant, P4, described numeracy in terms of mathematical symbols as a form of communication:

P4: Then numeracy, I'm relying on that more than ever this year because I've got three Spanish students in my TY class with very little English. So I find myself drawing up symbols on the board and drawing visuals for them while I am talking.

This participant expressed literacy practices in terms of facilitating a lesson on problem solving:

P4: Literacy wise, last year, I had a really weak class that were daunted by problem solving in trigonometry. They didn't even put pen to paper. They weren't sure of where it was going to go.

In the unravelling of the problem, P4 describes: *I gave them visuals....In pairs they looked at the visual, they created their own problems where they had to include given key words.*

This participant generated more codes from the LNMTI framework than any other participant.

P6's vignette which described an activity based, problem-solving class was second to P4 in generating codes from the LNMTI framework, even though this participant admitted she had difficulty with this domain. The Statistics and Probability based lesson was coded under 'Understand, Use and Critically Appreciate'. From the *MQI for LNMTI* domain that contains eight elements, this participant applied four: Mathematical Language, Patterns and Generalisations, Teacher uses student mathematical contributions, Mathematical Sense-making:

P6: We started probability. So there is a load of literacy. I did the water bottle challenge with them and they flipped it at different heights. For literacy they had to discuss, describe each event, whether it was 'likely', 'unlikely', 'impossible'. That actually worked really well because they figured out 'impossible' meant it definitely can't happen whereas they were saying 'impossible' means 'it's just really hard'. For numeracy they had to make up a table. They had to make sense of the data. They had to use tallies and they were able to come up with the relative frequency themselves so as to apply the formula themselves.

This response contrasts with P2 that generated only one code in the *MQI for LNMTI* domain: mathematical language

P2: We made a maths wall so when we learn new a word, we write it on the maths wall and put the symbol opposite it. We associate the symbol with the word for literacy and they read the word and they take it down in their vocab copies. So it's reading, writing and vocab.

Combining the data from question 2 and question 3 indicates participants have deficits in identifying specific numeracy elements in their lessons; literacy moments for most participants are limited to identifying key words. However, participant vignettes, as in the case of P6 and P4, suggest good LNMTI practices are stimulated by the facilitation of problem solving lessons.

5.6.3 Participant Input for LNMTI Intervention

The fourth question: ‘Is there anything you think should be covered in a module entitled ‘Literacy and Numeracy for Mathematics Teaching?’ concentrated on the training in Literacy and Numeracy the participants experienced in the university’s Initial Teacher Education programme. These responses were categorised under the node, ‘Recommendation’. This was subdivided into two child nodes: ‘Balance’ and ‘Clarity’ (Appendix AA). The following paragraphs will describe the relevance of these themes to the focus group discussion.

Firstly, the group spoke about the imbalance that existed between lecture time allocation and assessment weighting between literacy and numeracy (6 references). A sample of the references included:

P5: It was more literacy than numeracy.

P6: Two thirds versus one third.

P4: In the assessment literacy is worth more as well.

The ‘Clarity’ node was developed because the discussion centred on participants’ lack of understanding in the domain of numeracy. For example, out of the six focus group participants present, four took the decision not to complete a numeracy assignment in mathematics:

P4: I actually did my numeracy assignment in [other subject] instead of in maths because I just looked at it in maths and I like I didn’t know where to start. I just kind of moved on because I feel like, I don’t know.

P3: I was like P4. I did [other subject] because I thought the maths was too broad. I was like, ‘I wouldn’t even know how to start’.

In contrast, participant assessment work on literacy generated positive comments from the participants. The literacy module focused on the ‘five pillars of adolescent literacy’: oral language (and classroom dialogue), the teaching of vocabulary, the teaching of reading, the teaching of comprehension and the teaching of writing (Murphy 2017).

P4: *We learned a load from actually taking the pillars and applying it to our subject and then getting up and speaking about it. For me my lesson plan literacy makes so much more sense. But then in terms of numeracy, I’m still a bit unsure to be honest.*

Group: *Signs of agreement*

Following the coding of responses from participant recommendations for an intervention on LNMTI, a word cloud analysis was applied as a visual representation to identify the focal points of the focus group discussion (Krueger and Casey 2009). The word cloud is a cluster of the most commonly used words in the discussion represented by their relative size in the cluster. The more frequently used words are the most prominent (Atenstaedt 2012). As can be seen in Figure 5.19, the two most noticeable words in the visual are ‘maths’ and ‘numeracy’.

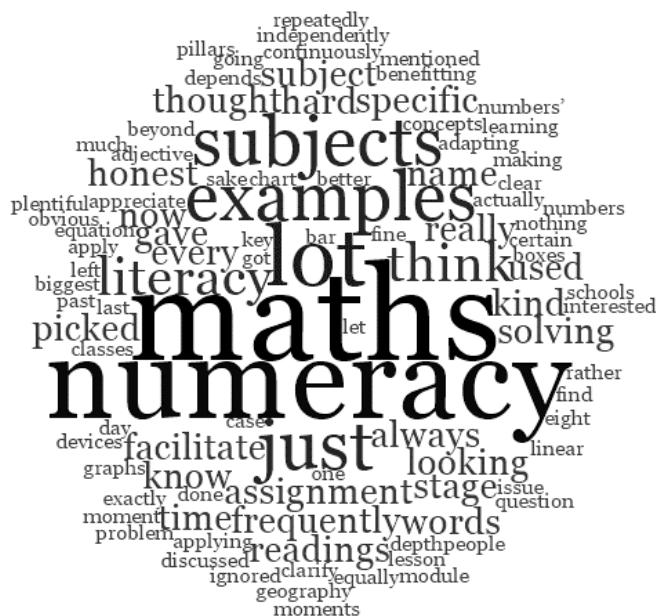


Figure 5.19 Word cloud from participant recommendations for LNMTI intervention

These words depict participants' need for clarity regarding the relationship between numeracy and mathematics in the post-primary mathematics classroom. This was the primary recommendation. A sample of the comments follows:

P3: Is numeracy going beyond the obvious? Let's say when you are solving a linear equation in maths. Is that what numeracy is in maths?

P4: I think it was so clear it wasn't discussed. If someone could just clarify it in a sentence we'd be fine. Is it just the numbers and the symbols or is there equally a lot of depth to it.

P6: I know I am doing it every day but to name exactly what it is.

The words 'subject' and 'examples' are also prominent conveying participants comments regarding the imbalance that existed in the education programme between identifying numeracy examples in other curricular subjects and mathematics:

P4: We were independently adapting them for maths continuously.

P1: We spent a lot of time looking for numeracy moments in all the other subjects.

Therefore, a second recommendation from the participants for the LNMTI intervention focused on presenting/generating specific examples of numeracy in mathematics.

Following on from these comments, the author questioned the participants' use of the mathematics syllabuses. The group as a whole responded that the textbook was the primary source for mathematics content and lesson planning with only one participant, P6, using the syllabus 'to get key words'. Content knowledge for mathematics teaching requires more than subject matter knowledge and includes knowledge of the mathematics syllabus and the structure and connections of mathematics being taught (Krauss *et al.* 2008).

5.7 Conclusion

This chapter reported on data generated from two main sources, the LNMTI survey and the Focus Group interview, to answer the research question:

Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms?

The survey demonstrated that participants showed weaknesses in basic knowledge for mathematics teaching that had a direct impact on LNMTI knowledge. For instance, Building Block knowledge such as defining a square or using precise mathematical language when referring to types of number was poor. Although the National Literacy and Numeracy Strategy (2011) indicates developing numeracy skills in students is not just the remit of mathematics teachers but the responsibility of all subject teachers, the most interesting finding, contrary to expectations, was that pre-service teachers of mathematics lacked confidence and conviction in their numeracy knowledge. These results functioned as the foundation on which the Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI) intervention was developed. Chapter 6 will describe in detail the next meso-cycle in the Educational Design Research methodology for this study which produced the LNMTI intervention.

Chapter 6: The Design, Construction and Evaluation of the LNMTI Module

6.1 Introduction

Phase three of the Educational Design Research process centred on the design and construction of the LNMTI intervention to address the third Research Question:

***RQ3:** What content and characteristics should a teaching and learning intervention have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?*

The reader will gain insights about the design process and the product resulting from this phase which took the form of a university based teaching and learning module, written and delivered by the author to enhance pre-service teachers' knowledge of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI). From Bernstein's (1990) theory of Social Construction of Pedagogic Discourse, the intervention was the medium whereby formal messages from policy and curriculum were recontextualised and translated into teacher knowledge for implementation in the Irish mathematics classroom. The chapter is divided into three main sections, each of which presents the results relating to one of the following research questions:

***RQ3(a):** What are the design requirements and design constraints for the LNMTI intervention?*

***RQ3(b):** What type of intervention should be employed to support participants to gain the relevant expertise for LNMTI?*

***RQ3(c):** How should the LNMTI module be structured to enhance participants' understanding of LNMTI?*

Findings show design requirements for the LNMTI intervention centred on issues related to numeracy; the type of intervention with a primary focus on authentic tasks and contexts, such as exploring syllabus documents, contributed to a deeper learning experience for the participants of this study.

6.2 Design Requirements and Design Constraints

This aspect of the Educational Design Research cycle was guided by the Research Question:

***RQ3(a):** What are the design requirements and design constraints for the LNMTI intervention?*

The sources employed to answer this question came from empirical data from the LNMTI survey that assessed participant prior knowledge of LNMTI and context analysis. By context, the author means understanding the physical, cultural and academic environment of the participants (Patton 2002).

First, critical elements of participant understanding of LMNTI were analysed from the LNMTI survey and the following list outlines priority issues (McKenney and Reeves 2012):

- Address confusion about the relationship between numeracy and mathematics;
- Provide guidance on facilitating problem solving sessions,
- Enable knowledge of basic geometry definitions and basic probability concepts,
- Enable a familiarity and practice with abstract mathematical ideas in real life contexts,
- Implement precise use of mathematical language,
- Model the pedagogical practice of generating patterns and generalisations in the mathematics classroom.

These priority issues were the foundation of the main learning goal of the intervention which was to situate literacy and numeracy as constructs to facilitate richer mathematical instruction. The specific learning targets for participants to enable this goal were as follows:

- To understand the interpretation of numeracy and mathematical proficiency in the mathematics syllabuses and mathematics assessment items,

- To understand elements of Mathematical Quality of Instruction for LNMTI (*MQI for LNMTI*) and how it can be operationalised in a classroom setting,
- To appreciate problem solving as a central element of literacy and numeracy,
- To expand the repertoire of communication formats such as digital media and broadcast media to explore mathematics.

Secondly, the author researched the context where the intervention was due to take place because organisational structures influence educational design (Park and Zhang 2011). This was achieved by accessing details of the university's Initial Teacher Education (ITE) programme provision for literacy and numeracy in the second year of the Professional Master of Education (PME 2) course. Additionally, a meeting was organised with the Professional Master of Education programme course coordinator to elicit a more detailed perspective on the literacy and numeracy programme delivered to the participants (Appendix BB).

The programme for literacy and numeracy in terms of module provision, credit and assessment allocation exceeded the other ITE programmes in Ireland (Section 3.7.1). For the participants in this study, the course requirements included mandatory participation in a literacy and numeracy module that consisted of twelve two hour lectures. Five credits out of a total of sixty were assigned to the module. The module mainly focused on the cross-curricular context and the concepts encountered in a generic way were disseminated by ensuring literacy and numeracy was accounted for in lesson planning. As well as completing two research papers on literacy and numeracy for assessment purposes, two of the participants, representing the mathematics module class, gave an oral presentation on literacy in mathematics. The focus group interview established that participants' understanding of literacy was accomplished but comprehension of the numeracy dimension was at a lower level (Section 5.4.3). This imbalance was reflected in twice as many lecture hours given over to literacy in the ITE programme. Secondly, 65 marks were allocated for literacy assessments compared with 35 marks for numeracy. Therefore one of the goals of the LNMTI intervention was to address this imbalance while at the same time integrate

with participants' learning from the university's ITE programme. In the next section, the evolution of the LNMTI intervention will be described.

6.3 LNMTI Intervention

On professional learning McKee and Eraut (2012) describe three general attributes formulated as questions:

- (1) *What is it that professionals need to know?*
- (2) *What must they be able to do?*
- (3) *How best can they acquire the relevant expertise? (p.1)*

The LNMTI definition, framework and survey answered the first two questions and the next step in Phase 3 of the study had the aim of answering the third question which was phrased as follows:

RQ3(b): *What type of intervention should be employed to support participants to gain the relevant expertise for LNMTI?*

O'Meara (2011) describes the traditional approach to Continuous Professional Development (CPD) whereby an individual with expertise on a specific topic would present to a large group and share ideas and practices. This is followed by an outline of the various drawbacks and inefficiencies of the model resulting from the autonomous instructor led approach, time allocation for the intervention and the limited time participants have to engage because of other commitments (p. 272). Heeding the contextual opportunity and constraint whereby the university provided six one-hour lectures hours for the LNMTI module to be held on the university campus during the participants' second semester, the author employed the traditional approach to the delivery of the programme. The author took this decision for two reasons:

- (1) Roesken-Winter *et al.* (2015) present an international overview of Continuous Professional Development over the last forty years and report on the continued dominance of the traditional approach in most jurisdictions. In addition, Hattie's (2009) research which ranked influences in student learning from 800 meta-analyses in order of effectiveness placed teacher training programmes at

124. Therefore, within the constraints of the traditional model, the author through the LNMTI module aimed to *build lenses and conceptions that can lead to teachers being prepared for the rigors of the classroom* (Hattie 2009, p.109).

- (2) The Continuous Professional Development for Project Maths followed this particular model where teachers were contracted to attend ten workshops and one seminar at regional education centres (Ireland, Department of Education and Skills n.d.). This was run over a five year period from 2009-2014 to reflect the phased implementation of the new mathematics programme where the reform focused on changed pedagogical practices. Content knowledge difficulties were addressed by providing night time courses. Attendance at these courses was voluntary. However, the LNMTI module integrated content and pedagogical knowledge (Ball *et al.* 2008) as dependent constructs.

6.3.1 Prototype of LNMTI Intervention

Although formal pilot testing did not occur, the author took the opportunity to empirically test ideas when she was approached to facilitate a local cluster meeting for The North South Underachievement Practitioners Engagement Project, under the auspices of Cooperation Ireland, a charity formed to generate cooperating projects between Northern Ireland and the Republic of Ireland (Appendix CC). To assess the relevancy of the intervention for professional learning, the author used this opportunity to produce a prototype or mock up (Kali and Ronen-Fuhrmann 2011) of the workshops. One of the aims of this project was to create a professional development context to generate a collaborative culture of discussion and sharing of best practice between schools in both jurisdictions in the teaching and learning of literacy and numeracy (Co-operation Ireland n.d.). This was also an opportunity to explore the relevancy of the LNMTI concept from the local context to a more global viewpoint (Dierdorf 2013). The author facilitated a three and half hour workshop for four practising mathematics teachers from the Republic of Ireland. The attendance of an Education Training Inspector (ETI) in Northern Ireland who also co-authored the joint report on improving numeracy at post-primary level (Ireland, Department of Education

and Skills 2015b) participated by a live video link. The workshop was structured into six episodes (Appendix L):

- (1) Defining Literacy and Numeracy for Mathematics Teaching in Ireland,
- (2) Literacy and Numeracy tasks in Geometry and Trigonometry,
- (3) Productive Disposition in Number,
- (4) Literacy and Numeracy moments across the curriculum,
- (5) Multi-representational approach to number facilitating literacy and numeracy development,
- (6) Look for a pattern to generate a disposition for problem solving and algebraic thinking.

In a letter to the author (Nov 2016), the manager of the project summarised the evaluations from the cluster participants who described the overall workshop as a ‘great resource’ and the ‘practical approach being noted as a particular highlight’ (Appendix DD). This validated the relevance of the educational materials and also defined the sequence of workshops for the intervention which are presented in the next section.

6.3.2 LNMTI Intervention: General Overview

The intervention comprised of a multimedia module on LNMTI with a direct focus on implementation conditions for the classroom. Six one-hour workshops were facilitated by the author whose classroom experience as a post-primary mathematics teacher spans almost three decades with nine years Continuous Professional Development facilitator experience. The author acknowledges this experience as a limitation and further research involving other initial teacher education lecturer/tutors delivering a similar module would be necessary to make more specific generalisations about the intervention’s effectiveness and its contribution to instruction theory globally (Dierdorff 2013).

Twelve final-year Professional Master in Education pre-service teachers participated in the intervention. As previously mentioned, all of these participants completed the

LNMTI survey except P5 who participated in the focus group interview only. The intervention took place during January - March, 2017. Participants were given access to all materials, PowerPoint presentations, and instructional tasks developed for the workshops through the university's online learning system. Table 6.1 outlines the schedule of workshops included in the module:

Table 6.1: LNMTI workshop schedule

<i>Workshop</i>	<i>Date</i>	<i>Workshop Title</i>
1	30 th January 2017	Literacy and Numeracy for Mathematics Teaching in Ireland
2	6 th February 2017	Literacy and Numeracy for <i>Geometry and Trigonometry</i> Teaching in Ireland
3	13 th February 2017	Literacy and Numeracy for <i>Number</i> Teaching in Ireland
4	27 th February 2017	Literacy and Numeracy for <i>Algebra and Functions</i> Teaching in Ireland
5	6 th March 2017	Literacy and Numeracy for <i>Statistics and Probability</i> Teaching in Ireland
6	13 th March 2017	Literacy and Numeracy for the Teaching of <i>Problem Solving</i> in Ireland

6.3.3 Overall Evaluation of LNMTI Intervention

To get a sense of the effectiveness of the type of intervention and participant learning during the module, data was collected by means of a pencil and paper workshop evaluation questionnaire that consisted of three questions (Appendix M). One Likert-style question asked the participants to rate the workshops on an ordinal scale using the descriptors: excellent, very good, good, poor, very poor. To implement the situated learning principle of learning and collaboration (Lave and Wenger 1991; Herrington and Oliver 2000), participants' perceptions of the intervention following each workshop were sought by means of two open construct response questions:

- What aspects of the workshop did you find most useful?
- What aspects of the workshop did you find least useful?

The uptake of the workshop evaluation questionnaire was voluntary and anonymous. Overall, the response rate was over 90% with returned evaluation surveys reaching the maximum number, twelve, for workshop 2 and the minimum was 9 for workshop 6. The relatively low take up for the final workshop was as a result of two of the participants having prior school placement obligations on that day. The following chart, Figure 6.1, displays participant responses to question 1 which shows, out of 65 responses, 100% rated the workshops as excellent, very good or good with 62% in the excellent category. These results indicate the type of intervention had a positive impact on participant learning. Chapter 7 will provide a more detailed discussion on the overall effectiveness of the LNMTI module which will also include observational data written by a qualified teacher/researcher who attended the workshops.

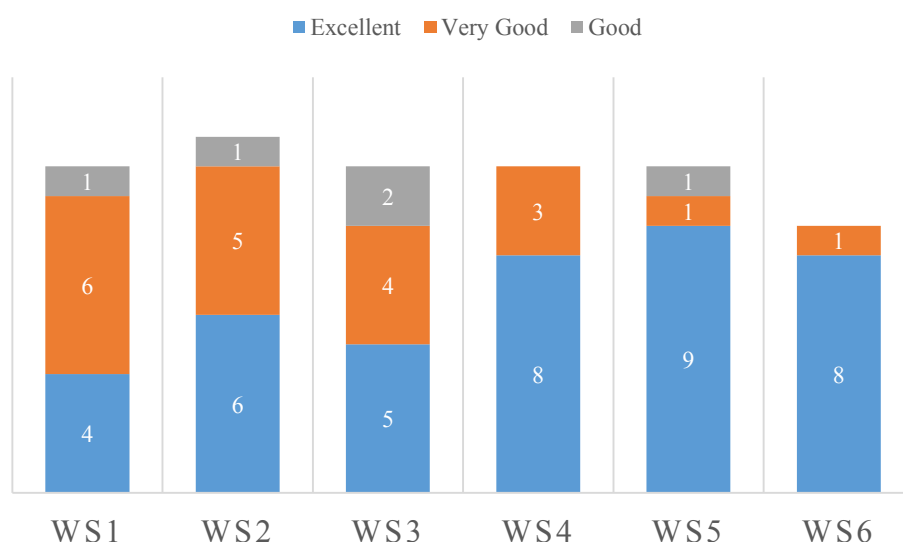


Figure 6.1 Evaluation of individual LNMTI workshops

Throughout the course of this chapter references will be made to participant workshop evaluations responses, which will be presented in the following way: *WX* represents the workshop number. For example, *W4* represents workshop 4 on Literacy and Numeracy for *Algebra* Teaching in Ireland. This will be followed by a point and a number to indicate an individual participant. For example, *W4.3*, indicates the third participant evaluation returned after the fourth workshop was completed. However,

the 3 does not always represent responses from the same person as the evaluation surveys were voluntary and anonymous. The survey evaluations were placed on a table as the participants exited the room which were then collected and numbered by the author.

6.4 Framing the Design and Construction of the LNMTI Intervention

This section describes the framing of the design and the construction of the LNMTI workshops to answer the Research Question:

RQ3(c): *How should the LNMTI module be structured to enhance participants' understanding of LNMTI?*

Three structural frameworks were used to guide the construction and content of the LNMTI workshops. Two of the frameworks, LNMTI framework and Instructional Design Framework, were derived from the domain specific frame from the study's theoretical framework as illustrated in Figure 6.2.

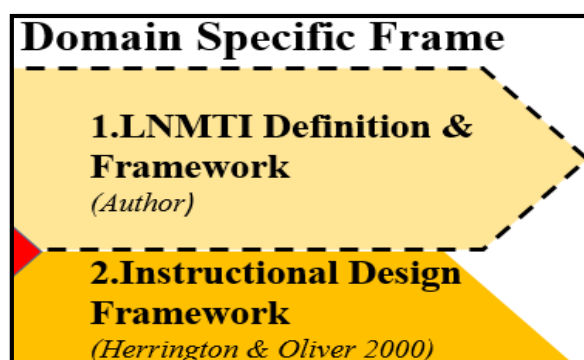


Figure 6.2 Domain specific frame from the Theoretical framework

The third, Hypothetical Learning Trajectories (HLTs), emerged from the study's Educational Design Research methodology. The author used the construct of hypothetical learning trajectories (Simon 1995), a version of a thought experiment advocated by Freudenthal in the Real Mathematics Education framework (Gravemeijer 2004), to chart the sequence of learning, instructional tasks and learning processes (Bakker and Van Eerde 2015). The heuristics in the domain specific theories, LNMTI framework and the instructional design framework guided the overall

design of the module. The central role of these three elements will be addressed in the following paragraphs.

6.4.1 LNTMI framework

Domain-specific theories in Educational Design Research guide practical solutions relevant to the dynamic classroom environment (Bakker and Van Eerde 2015; Dede 2004). The LNMTI intervention was driven and supported by the LNMTI framework that contained the four elements: Literacy processes; Numeracy in Mathematics Content domain; Mathematical Quality for Instruction for Literacy and Numeracy; and Literacy forms. Table 6.2, identifies the elements that had a primary focus in each workshop from the LNMTI framework.

Table 6.2 Workshop elements from LNMTI framework

<i>LNMTI Framework</i>	<i>Workshop 2</i>	<i>Workshop 3</i>	<i>Workshop 4</i>	<i>Workshop 5</i>	<i>Workshop 6</i>
Literacy Processes	✓ Understand ✓ Use ✓ Critically Appreciate	✓ Understand ✓ Use ✓ Critically Appreciate	✓ Understand ✓ Use ✓ Critically Appreciate	✓ Understand	✓ Understand ✓ Use ✓ Critically Appreciate
Numeracy in Mathematics	✓ Geometry & Trigonometry	✓ Number	✓ Algebra & Functions	✓ Statistics & Probability	✓ Synthesis and Problem Solving
<i>MQI for LNMTI</i>	✓ Mathematical language ✓ Patterns and Generalisations ✓ Linking between representations	✓ Mathematical Sense-making	✓ Multiple procedures and solution methods ✓ Explanations	✓ Remediation of student errors and difficulties	✓ Teacher uses student mathematical contributions
Literacy Forms	✓ Spoken Language ✓ Printed Text ✓ Digital Media ✓ Broadcast Media	✓ Spoken Language ✓ Printed Text	✓ Spoken Language ✓ Printed Text	✓ Spoken Language ✓ Printed Text ✓ Digital Media ✓ Broadcast Media	✓ Spoken Language ✓ Printed Text ✓ Digital Media ✓ Broadcast Media

Workshop 1 is not included in this display as the primary focus of this workshop was to establish the definition of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI) within the context of the prior learning experiences provided by the university. Using the LNMTI framework directly, for example, workshop 3 enabled participants to understand, use and critically appreciate content from *Number* by modelling mathematical sense-making communicated through the medium of spoken language and printed text. Moreover, Literacy Forms of spoken language and printed text featured in all workshops. Broadcast media concerned with the use of video, film, (Chandler and Munday 2016) newspapers, television and radio and digital media characterised as the use of PowerPoint and digital representational systems for mathematics were specifically addressed in workshop 2, workshop 5 and workshop 6. However, all of the workshop materials and content were displayed on a quality digital projector screen that projected text and imagery on PowerPoint slides to facilitate a shared learning experience. Also, participants had access to all of the module materials in digital format on personal computers or mobile devices to reinforce the learning. Similarly, cognitive/literacy processes of ‘understand, use and critically appreciate’ were interwoven into all of the workshops except workshop 5 because participant content knowledge in the areas of statistics and probability was poor. The next section will demonstrate how the LNMTI framework was utilised in LNMTI workshop 2 on Geometry and Trigonometry. However, in the design of the LNMTI workshop 5 on Statistics and Probability, an additional framework, Mathematical Discourse for Instruction, (Adler 2017) was referenced because of, as previously mentioned, the poor content knowledge of the participants in the basic principle of probability.

6.4.1.1 LNMTI Framework: Workshop 2 (Geometry and Trigonometry)

The focus of this workshop was driven by findings from the LNMTI survey and focus group. Participants overestimated their knowledge of basic geometry facts. By overestimation the author means where participants categorised a LNMTI survey question as ‘easy’ or ‘moderate’ but the work presented on the question was poor. For instance, 73% of the participants rated the *Geometry Definitions* question as ‘easy’ or ‘Moderate’ yet, only 18% of the participant responses were rated as LNMTI Adequate.

Secondly, weaknesses in the facilitation of generating patterns and generalisations was evident. However, research in the area of geometry in an Irish context demonstrates this knowledge domain as weak. For example, Perkins and Shiels (2016) who report on the poor performance of Irish students in the domain of shape and space make a particular reference to mathematical language: 34% of Irish students said they were familiar with the term ‘polygon’ in comparison to 62% in other OECD countries.

To address the knowledge gaps, a quiz as a recommended formative assessment tool to highlight basic knowledge or low level learning in a collaborative and non-threatening atmosphere was demonstrated for basic figures in geometry (Jones *et al.* 2018; Niu and Henderson 2015). Point, line and plane, their properties and notation were presented to participants to facilitate an ‘understanding’ process. Thirdly, a completed classification task to promote understanding of quadrilaterals was also given to the participants and working collaboratively, they had to reason and explain correspondences and relations. Secondly, linking theorem statements with real life applications was intended to accommodate a ‘use’ process (Maths Development Team n.d.c).

In addition, Ball (2012) in a keynote address to Irish educators and policy makers answered the question: ‘how do I develop my learners’ broad skills using numeracy’ by making the following recommendations:

- (1) Developing care with language,
 - (2) Focused on disciplined education, representation and reasoning,
 - (3) Using mathematical ideas and skills to work on problems in other domains.
- (p.46)

Consequently, three elements from the Literacy and Numeracy Mathematical Quality of Instruction domain that mirror these advocacies were showcased in the design of the task shown in Figure 6.3.




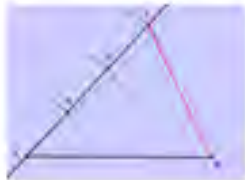
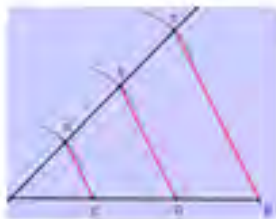
Division of a Line Segment into 3 Equal Parts	
Construction Steps in Words	Construction Diagram
	
	
	
	
	

Figure 6.3 Instructional task using the LNMTI framework

Furthermore, they are also specific components in the Unifying strand of the Junior Cycle specification under the headings: representation, generalisation and proof and communication (NCCA 2017). The task used the division of a line segment into n equal parts construction to enable these three elements in the following way:

- Mathematical Language to describe the construction steps in words,
- Linking between Representations was demonstrated by linking the geometric representation of rational numbers to the numerical representation,
- Patterns and Generalisations was presented by an informal visual exploration, connecting this construction to the geometric proof: *Let ABC be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $m:n$, then it also cuts $[AC]$ in the same ratio* (National Council for Curriculum and Assessment 2013).

The participant responses to the evaluation of the workshop demonstrate two important themes related to the LNMTI framework:

- (1) Content knowledge,
- (2) Making connections.

Content knowledge in the area of geometry was an issue for two participants. **W2.3** regarded *identifying what a square, parallelogram, etc. actually is defined as* as the most valuable aspect of the workshop. **W2.12** noted the most valuable aspect of the workshop was the work done on theorems. Secondly, the following word cloud (Figure 6.4) shows the most common word used in the evaluation was ‘link’:

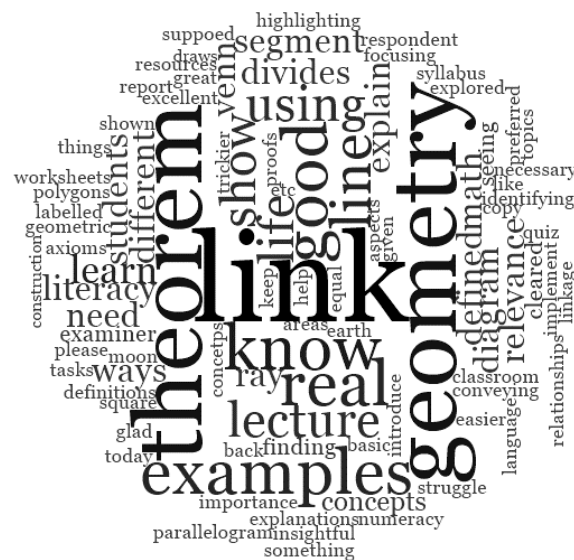


Figure 6.4 Word cloud illustrating the most common used words in the evaluation of workshop 2

This represents how most of the responses focused on learning the importance of making connections within the subject of mathematics as well as linking geometry to real life contexts (10 responses). For example:

***W2.10:** How to link different aspects of the maths syllabus with geometry eg. (respondent draws a line segment to represent a unit and divides it into 2 equal parts, labelled 1/2). Using real life examples to explain concepts eg. Line segment, rays.*

***W2.2:** Linking geometry to real life. Finding easier ways to introduce theorems. Relating literacy + numeracy to classroom. Basic examples of how to implement literacy.*

These responses endorse the applicability of the LNMTI framework for the Irish mathematics classroom context.

6.4.1.2 LNMTI Framework: Workshop 5 (Statistics and Probability)

From the LNMTI framework, the literacy processes of ‘use and critically appreciate’ were not addressed in the design and construction of the Statistics and Probability workshop as a result of participants’ extremely poor content knowledge in this domain as evidenced in the LNMTI survey. Therefore, to support the design, the author incorporated Adler’s (2017) Mathematical Discourses of Instruction (MDI) framework, as a supporting mechanism for the *MQI for LNMTI* domain, see Figure 6.5.

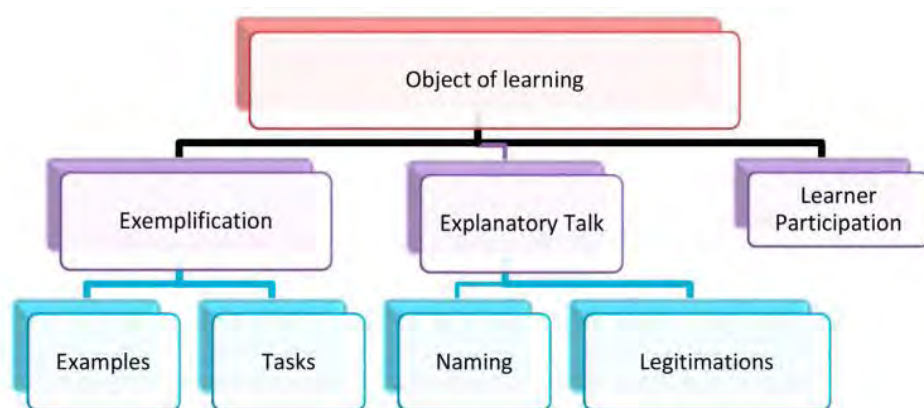


Figure 6.5 Mathematical Discourses of Instruction (MDI) framework (Adler and Ronda 2015)

This framework emerged from Adler's observation of teaching episodes in socially disadvantaged South African classrooms, where teacher-led instruction predominated. The framework was a structure to identify differences in instructional practices that would all have been characterised as 'rote and procedural' by the more sophisticated Mathematical Quality of Instruction (MQI) framework (Adler and Ronda 2015). The framework has four primary components: 'exemplification', 'explanatory talk' and 'learner participation', practices that enable the 'object of learning'. In mathematics, abstract ideas require exemplification with examples and tasks; teacher talk involves naming and legitimising the mathematics under instruction in an effort to achieve the learning goal and learner participation describes the 'interaction between teacher and learners and amongst learners' (p.237). The Mathematical Discourses of Instruction perspective suited the Statistics and Probability domain as ten of the eleven participants who completed the survey were awarded 'Not Present' from the four point scoring rubric for the work presented (Section 5.5.4). This instructional framework was generated to guide, assess, plan and predict Adler's research in mathematics instruction in a particular cultural context and the author employed the structure to plan and guide the LNMTI workshop for Statistics and Probability.

As previously mentioned, the framework begins with the object of learning and this object is mediated by three elements: exemplification, explanatory talk and learner participation. The first instructional task focused on conceptual understanding of the mean as an indication of 'fair share', the object of learning. Participants were asked to explain 'what does the 'mean' mean'? The responses returned synonyms of the noun 'mean' such as 'average' and 'standard' or a description of the algorithm to calculate the summary statistic. This talk around the concept required legitimising and was achieved by using unifix cubes to represent a visualisation of an 'unfair share' followed by movement of unifix cubes to generate a fair share. The episode prompted five out of eleven participants who positively evaluated the workshop to reference this task:

W5.1: Serious food for thought and useful activities. What is the mean?

W5.2: Practical resources eg mean - using blocks to visual represent mean.

W5.3: Made me more aware of the language I should use - fair/unfair.

W5.4: I'll use the unifix cubes for the mean.

W5.5: I loved the example of making more sense of the mean - fairness.

The second task began by asking the question: 'what is probability?' Following limited participation, the author displayed a definition followed by a learning outcome from the syllabus that focused on 'the language and concepts of probability'. Keywords from probability concepts were displayed and meanings were clarified. Two participants, W5.7 and W5.8, found this aspect to be the most useful from the workshop:

W5.7: Definition of probability.

W5.8: How to explain the different probability terms rather than just assuming they know what they mean.

Using the exemplification component as a guide, a copy of the probability syllabus for Junior Certificate along with an example of probability questions from Junior Certificate Ordinary level examination were given to the participants. The task was to identify the learning outcome in the summative assessment question. Out of the six participants who contributed to the focus group interview, only one admitted to using the syllabus and this participant's use was limited to finding key words. However, this task proved meaningful and useful for participants' growth in learning:

W5.2: I found linking exam questions to the syllabus very useful. It made me think of learning outcomes.

W5.10: Linkage to curriculum.

W5.11: Looking at syllabus to see what is expected of the treatment of statistics.

Finally, the reference to digital media as a literacy form for communicating mathematically was represented by the scientific calculator as a hand held technology. As yet, graphing calculators are not in use in Irish classrooms. Participants were shown the random number function on the calculator to simulate experiments such as

coin tossing and rolling a die. Given the reaction of participants (4 responses) on the usefulness of this basic technology, a more detailed exploration of the scientific calculator in working with students and mathematics should be facilitated in Initial Teacher Education programmes for mathematics teaching.

The next section will discuss how Herrington and Oliver's (2000) instructional design principles were used in the design and construction of the LNMTI module.

6.4.2 Instructional Design Principles

The Cognition and Technology Group at Vanderbilt (1990) attempted to formalise design principles to characterise situated learning ideals into practical approaches but acknowledged more work needed to be done in this area. Herrington and Oliver (2000) took up the challenge and designed a robust framework with nine elements (Section 3.4.4). To demonstrate an overall enactment of these principles, detailed illustrations from workshop 1 on Literacy and Numeracy for Mathematics Teaching in Ireland will be used. This will be followed by a description of workshop 6, Literacy and Numeracy for the Teaching of *Problem Solving* in Ireland, to emphasise specific design principles from the Instructional Design framework such as 'access to expert performances' and 'the modelling of processes'.

Table 6.3 lists eight of the nine design principles for instruction aligned with a description of content and method for workshop 1. As previously discussed in Chapter 3, design principle 9, 'authentic assessment of learning within the tasks' would not be fully achieved in the LNMTI module because the enactment of the skills in a classroom setting was the context for authentic assessment. This assessment was carried out using the LNMTI classroom observation instrument which will be discussed in detail in Section 7.3.3. Secondly, design principle 8 'coaching and scaffolding by the teacher at critical times' was not specifically mentioned by the participants, but was implied by references to the collaborative learning heuristic, design principle 5. In using the first design principle, 'authentic tasks', the method of concept mapping and a sorting task was employed in WS1. These instructional tasks are described in detail in the next section. The third column in the table shows participant reactions to the workshop

that included a specific reference to a design principle, therefore validating the existence of the heuristic in the enactment of the workshop. For example, the design principle: *authentic contexts that reflect the way the knowledge will be used in real life* was acknowledged by the following comment from W1.8:

W1.8: *Made it very relevant to today/the present. Where we are with maths in terms of numeracy and literacy presently.*

Table 6.3 Design principles in workshop1 with participant validation

	<i>Design Principles (Herrington and Oliver 2000)</i>	<i>LNMTI Workshop 1</i>	<i>Participant Evaluation</i>
1	authentic contexts that reflect the way the knowledge will be used in real life	✓Current Reform in Education expressed in Action Plan for Education 2016-2019	W1.8, W1.10, W.11
2	authentic activities	✓Concept Mapping, Sorting Task	W1.1,W1.2, W1.11
3	access to expert performances and the modelling of processes	✓Modelling concept mapping from L&N definition to Strand Structure	W1.3,W1.4
4	multiple roles and perspectives	✓Literacy and Numeracy in Strand Structure/Problem Solving Learning Outcomes/Numeracy Model/State Examination Assessment Items	W1.1,W1.2,W1.3 W1.5, W1.10
5	collaborative construction of knowledge	✓Groups of Four to engage with tasks	W1.2,W1.5, W.11
6	reflection to enable abstractions to be formed	✓Evaluation at the end of each workshop	ALL
7	articulation to enable tacit knowledge to be made explicit	✓Seeing numeracy moments in SEC assessment items	W1.1, W1.2
8	coaching and scaffolding by the teacher at critical times	Predominant role of the author throughout the module	

6.4.2.1 Instructional Design Principles: Workshop 1 (Defining LNMTI)

The learning intentions of workshop 1, titled Literacy and Numeracy for Mathematics in Ireland was to outline the characteristics of LNMTI including definitions, theories, processes and discourse and to establish the context for mathematics teaching in Ireland. The priority issue for the participants was an understanding of numeracy and its relationship with mathematics. From the participants' point of view expressed in the focus group interview, in the numeracy module there was an understanding that numeracy was subsumed by mathematics and a more pressing demand was to make numeracy moments visible in other subjects (Section 5.4.3). Guided by the design principle of 'multiple roles and perspectives', it was important to allow the participants to:

- (1) Examine the mathematics syllabus documents and the literacy and numeracy definitions where the 'message' regarding the content and approach to teaching and learning of mathematics originated;
- (2) Synthesise this message with Goos *et al.*'s (2012) model of numeracy advocated by the ITE university programme;
- (3) Audit State Examination Commission assessment items to explore how numeracy in mathematics was characterised for post-primary students.

Two instructional tasks were aimed at concept development between numeracy and mathematics. The first task used a concept mapping tool (McDaniels 2005) to enable the participants to find a content and cognitive relation between the syllabus objectives in the mathematics syllabuses and the numeracy model advocated by the university's Initial Teacher Education programme. The descriptions of each element were presented under the respective titles: 'Syllabus Objectives' and 'Numeracy Model'. The syllabus objectives have been discussed in Section 4.4.2. Goos *et al.*'s (2012) numeracy model described in Section 2.1.1, reflects the practicalities of the classroom environment for teacher planning and reflection as well as the changing nature of knowledge for the 21st century. In groups of four, participants were asked to draw a mapping relation between the two domains to demonstrate a synthesis of concepts, see

Figure 6.6. The results of this task validated the author's findings from phase 1 of the research study which explored the relevance of the mathematics syllabus to the domains of learning in literacy and numeracy (Section 4.4.1).

(a)

Mathematical Proficiency	Characterisation
<i>Conceptual Understanding</i>	Comprehension of mathematical concepts, operations and relations
<i>Procedural Fluency</i>	Skills in carrying out procedures flexibly, accurately, efficiently and appropriately
<i>Strategic Competence</i>	Ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
<i>Adaptive Reasoning</i>	Capacity for logical thought, reflection, explanation, justification and communication
<i>Productive Disposition</i>	Habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy

(b)

Element of model	Description of element
Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings.
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.

Figure 6.6 Relating (a) Irish mathematics syllabus/specification objectives and (b) Goos et al.'s Numeracy Model (2012)

The approach taken for the second instructional task was ‘sorting’ (Swan 2008). This task required participants to classify a Junior Certificate Higher Level summative assessment item as a learning objective. This classification was applied by the State Examinations Commission to the examination items for the year 2015, which was published in a report on the performance of candidates for that year. In this report, candidate performance was analysed using four out of the five learning objectives in the mathematics syllabus. ‘Productive disposition’, that describes a student’s belief system about mathematics, was not included as examination items did not specifically address this element although candidates engaged with more questions than previously recorded which, it was acknowledged, could be construed as a measure of this element (State Examinations Commission 2016a, p. 19). To facilitate this learning experience, the author assembled a montage of same category learning objective assessment questions representing all strands on the syllabus on four separate colour coded A4 pages. By exploring similarities and differences, the participants were asked to assign the learning objective title that best described the items on the A4 page. Each page could only be assigned one category (Appendix EE).

The evaluations from the participants illustrate that the design principles were adhered to. All of the eleven responses conveyed an engagement with the workshop content and tasks and the following response demonstrates the workshop’s success in providing multiple roles and perspectives, a collaborative learning environment, expert modelling and coaching:

W1.2: Being able to relate numeracy to the syllabus. Clearly explaining numeracy in maths. Been given examples of the different aspects of numeracy within the Junior Cert. Group work activity - could discuss and validate answers. Very enthusiastic. Clear explanations.

As a general summary of participants’ learning following the workshop, a word cloud was generated on the eleven responses. As illustrated in Figure 6.7, the most frequent words were ‘numeracy’ and ‘questions’ to convey the value and authenticity of instructional task 2 where participants had to identify syllabus objectives/numeracy moments in state examinations commission mathematics assessment items. However,

the respondents were very critical of showcasing the literacy and numeracy definitions at the start of the workshop (8 responses) although one participant acknowledged:

W1.8: Maybe reviewing the definitions. We are aware of them. Having that said the relevance of them and to the syllabi was interesting.

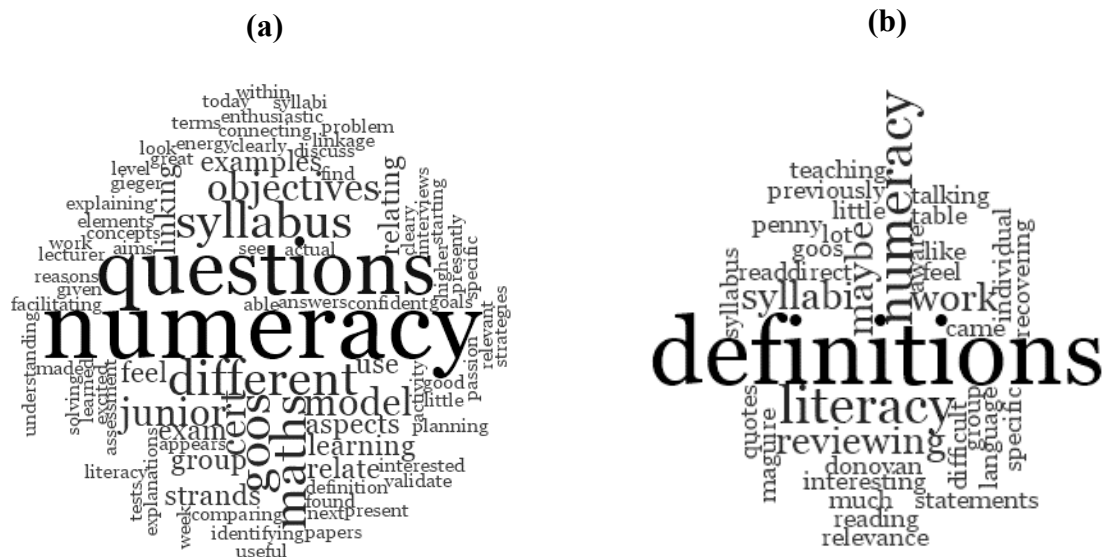


Figure 6.7 Word cloud on participant responses from (a) useful and (b) least useful aspects of workshop 1

6.4.2.2 Instructional Design Principles in Workshop 6 (Problem-Solving)

The final workshop focused on problem solving. The planning and design of this workshop was responding to the participants need to identify problem solving facilitation skills in the post-primary classroom, an authentic context:

P4: I'd be interested in learning how better to facilitate people problem solving. That's the biggest issue in my classes at the moment – I don't really have a set of steps to teach them. Every time it depends on the question and how I would solve it but I can't facilitate them to start (Post-survey focus group interview).

Collins, Brown and Holum (1991) research on theories of cognitive apprenticeship conclude ‘to make real differences in students’ skill, we need both to understand the nature of expert practice and to devise methods that are appropriate to learning that practice’ (p.2). Similarly, Vygotsky (1986) espoused the importance of simulating the actions of more experienced peers inferring that speech accompanied by these actions may be internalised by the learner and called on at a later time to solve similar problems. These ideas are synthesised in the third Instructional Design principle: ‘Access to expert performances and the modelling of processes’. Therefore, George Pólya, ‘the father of problem solving’ (Sury 2014) and Dan Meyer, who uses digital technology as a tool for teaching and learning (Computers in Education Society of Ireland 2013), were chosen as models of expert practice (Bandura 1977) because they both promote problem solving as a conduit for high quality mathematics instruction, which is directly relevant to this study and the concept of LNMTI. Secondly, both have recorded accounts of problem solving facilitation. Pólya’s influential book ‘How to Solve It’ is a collection of dialogues between student and teacher in the process of solving mathematical problems. Meyer has video recordings of a problem solving process for mathematics.

To identify specific characteristics of expert performances in problem solving facilitation, the author transcribed Pólya’s and Meyer’s spoken lines in an abstract mathematics problem on related rates (Pólya 1957) and a real life context problem titled: ‘Pyramid of Pennies’ (Nrich 2013) (Figure 6.8).

(a) Related Rates (Pólya, 1957, p.29)

Water flowing into a conical vessel at the rate r . The vessel has a shape of a right circular cone, with horizontal base, the vertex pointing downwards; the radius of the base is a , the altitude of the cone b . Find the rate at which the surface is rising when the depth of the water is y .

(b) Pyramid of Pennies (NRich Maths 2013)

Pyramid of Pennies: How many pennies are there?



Figure 6.8 (a) Pólya's Related Rates problem and (b) Meyer's Pyramid of Pennies problem

Content analysis was applied to Pólya and Meyer's discourse where the unit of analysis was a complete line of dialogue. Fey's (1970) patterns of verbal communication heuristics from the study's theoretical framework: 'structuring', 'soliciting', 'reacting', were the codes. 'Structuring' refers to the launch of the task and directing the process; 'soliciting' is eliciting verbal/cognitive or physical action; 'reacting' rates, clarifies, synthesises or expands on a student contribution. The author chose not to include the fourth heuristic, 'responding' into the analysing framework as it included a focus on student responses and the purpose of the exploration was to examine problem solving facilitator verbalisations. Units of analysis could be coded in either one or two or three of the categories. For example, the following line was coded in all three:

***Pólya:** Good. Introduce suitable notation. How would you write the ‘rate of change of y’ in mathematical symbols?*

It was coded ‘reacting’ because Pólya rated the student’s contribution as ‘good’. The line ‘introduce suitable notation’ was coded ‘structuring’ because Pólya is intentionally directing the process to advance progress. Finally, the line ‘how would you write ‘rate of change of y’ in mathematical symbols?’ is an example of ‘soliciting’ because Pólya is eliciting verbal/cognitive action (Appendix FF).

Despite the sixty year gap between Pólya and Meyer’s educational activities, the data, presented in Figure 6.9, shows cultural or mathematical (abstract or real-life) context does not impact on patterns of verbal communication for the mathematics classroom for the facilitation of problem solving.

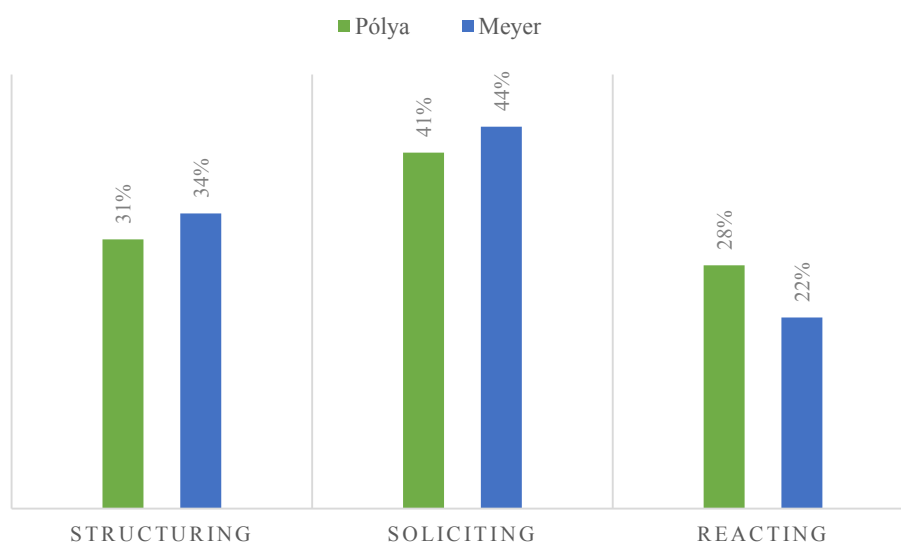


Figure 6.9 Comparison of Patterns of Verbal Communication: Pólya and Meyer

During the teaching and learning interaction, Pólya skilfully controls the questioning and responses of the student to facilitate the student’s translation of this calculus problem into abstract mathematical notation almost a third of the time, 31%. Meyer’s instances of ‘structuring’ is similar at 34%. There is an initial restating of the problem followed by Pólya’s question: ‘could you say it in other terms?’ and again he asks ‘could you restate it still differently?’ (p.30). These ‘solicitations’ accounted for 41%

in Pólya's facilitation and 44% in Meyer's demonstrating the prominent role of teachers eliciting student thinking in the problem solving process. The student is guided to unravel the context, identify the appropriate mathematical processes and represent it in abstract symbolic notation.

The author summarised Pólya's approach using process imagery from SmartArt as illustrated in Figure 6.10. Pólya's teaching and learning interaction is similar to the historical development of algebraic notation where a rhetorical expression of the problem is translated into a syncopated version and finally into mathematical notation (Sfard 1995).

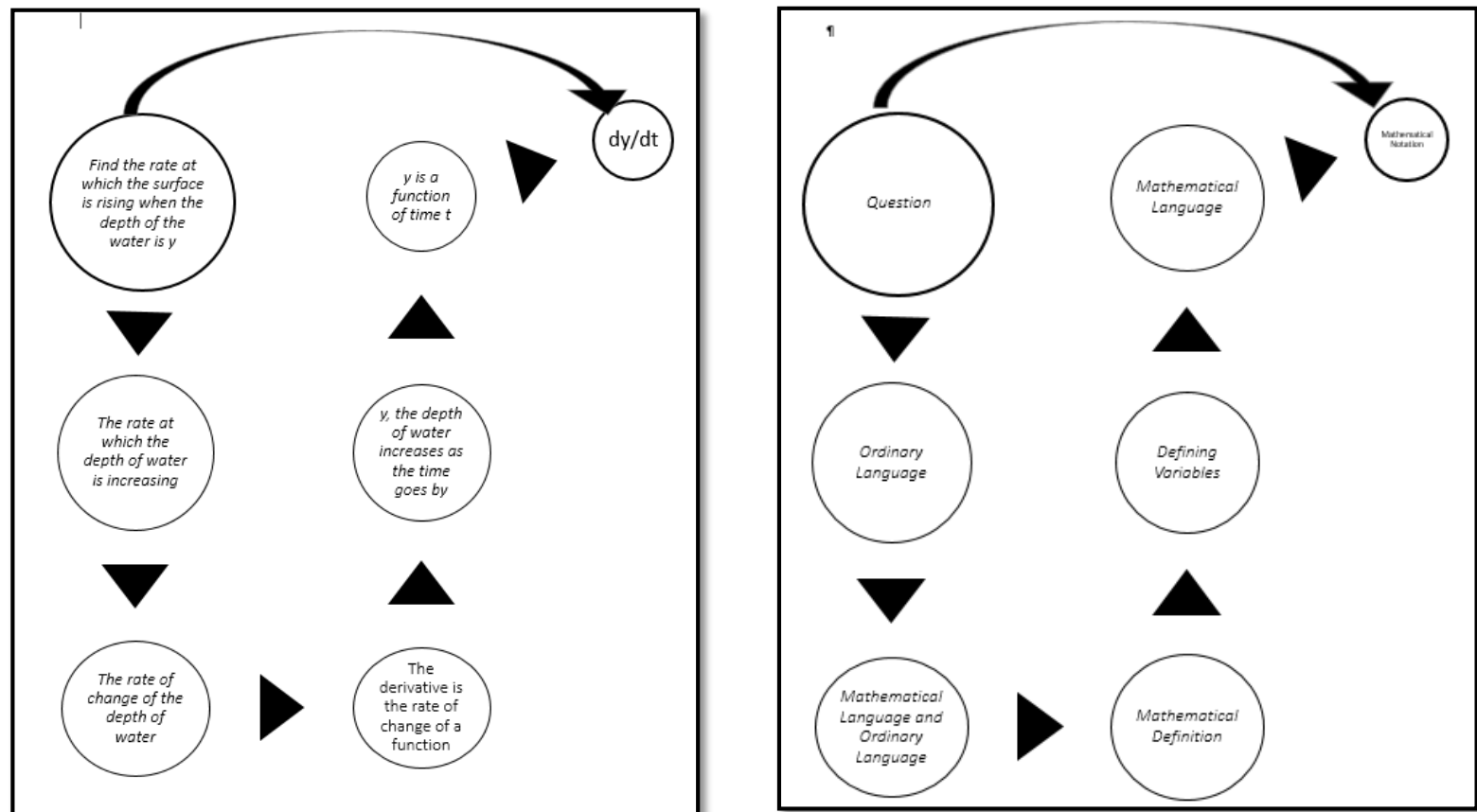


Figure 6.10 Pólya - Translating mathematical word problem into mathematical notation for solving

A similar process is evident in Dan Meyer's 'Three Acts of a Mathematical Story' construct. Pollak (2007), a pioneer of mathematical modelling in mathematics education, whose work influenced Dan Meyer (personal communication June 2016, Appendix GG), asserts knowing how to explain mathematical work is a necessary part of the learning process. Also, the motivation to learn mathematics influences pedagogical choices. Harel (2014) describes a meaningful situation for a learner who does not have sufficient knowledge to fully solve a problem and is motivated to fill that knowledge gap as an 'intellectual need'. Meyer informally represents this event as a 'headache' and it compares to Pólya's portrayal of the same notion as the 'best motivation'. Similar to Pólya's problem solving process, Harel describes the constituent and symbiotic parts of communication as 'formulating', the translation of the vernacular into mathematical language and 'formalising', the explicit description of the focal concept (p.34), i.e. the new mathematics to be learned. Meyer informally describes this episode as the 'aspirin' (formalising) to alleviate the 'headache' (intellectual need).

In 'Three Acts of a Mathematical Story' (Meyer 2011) activities, Meyer aims to create 'headaches' that focus on mathematics and the development of mathematical reasoning using digital technology and broadcast media. These literacy forms are employed as an alternative to traditional word problems that can have a high literacy demand and secondly produce the mathematical model for the problem instead of leaving that work to the student (Meyer 2010). As can be seen in Figure 6.11, Meyer mirrors the narrative structure in fiction or film where the first act introduces a conflict: 'how many pennies are there?' This is a structuring element. Using video footage of the object of interest situated in a real life setting, students are invited to answer the question with estimates that are too high or too low, an example of 'soliciting'. Act two is the phase where the mathematical modelling of the problem is enacted and information required to solve the conflict are identified where combined elements of 'structuring', 'soliciting' and 'reacting' are used. The final act is the resolution to the problem or the answer to the question.

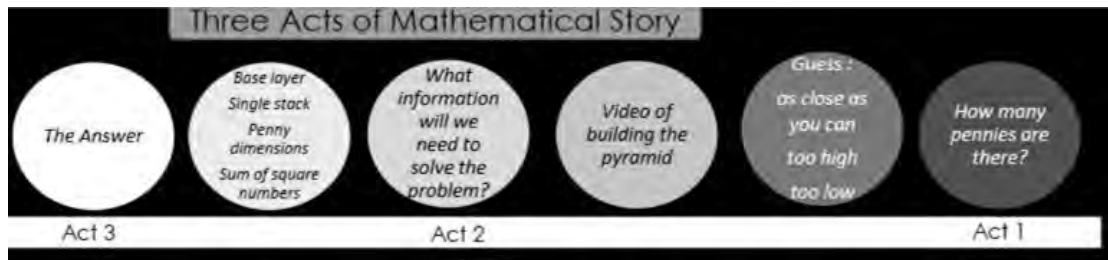


Figure 6.11 Meyer – structuring a problem solving task

In the construction phase of the workshop, the author chose three instructional tasks, presented in Figure 6.12, to represent Pólya and Meyer’s work. The first task used Meyer’s real life context *Sugar Packets* number problem that was structured in the ‘Three Acts of a Mathematical Story’ format (Meyer 2011). This was followed by the facilitation of an abstract geometry mathematics problem (States Examination Commission 2016c). Using broadcast media as a literacy format, the final task involved the discovery of an exponential pattern (Meyer 2009).

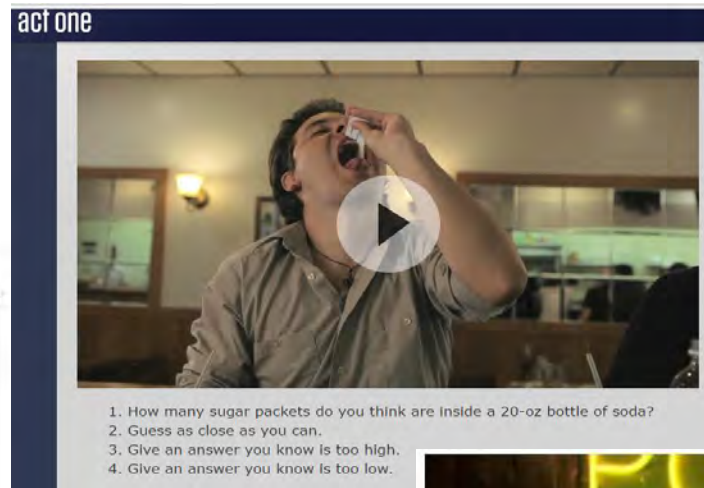
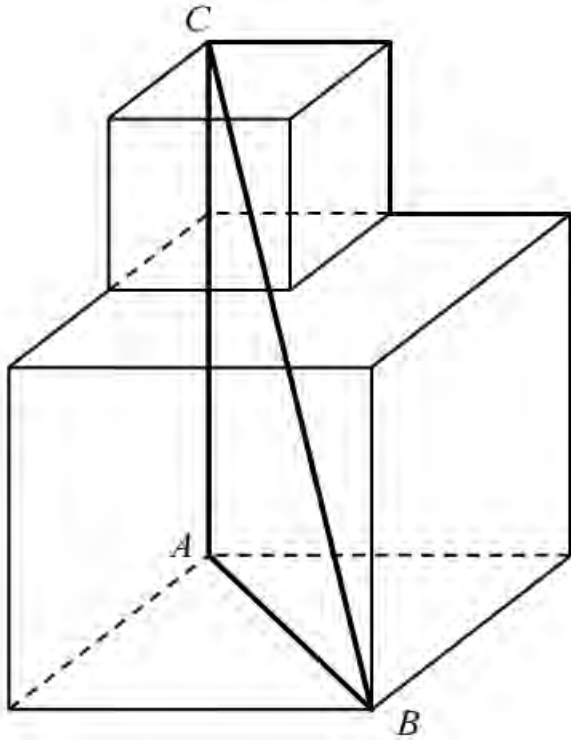


Figure 6.12 Three instructional tasks from workshop 6

The participant responses to workshop 6 were all positive except for one participant who declared the instructions before the videos were shown lacked clarity (**W6.5**). Three themes were specifically identified in the responses: facilitating problem solving, disposition and literacy and numeracy.

The first theme generated four responses and confirmed that the expert modelling as a design principle for this workshop had been achieved:

***W6.1:** I love how you explained how you would teach the task, giving examples of questions you would ask + possible answers the students would give,*

***W6.2:** Looking at problem solving in an investigation sort of way,*

***W6.5:** Representing problem solving in a visual way,*

***W6.6:** The discovery style teaching example was excellent to see.*

The second theme, ‘Student Disposition’, showed the participants valued ‘the ‘authentic contexts that reflect the way the knowledge will be used in real life’. It also demonstrated the element ‘dispositions’ in Goos *et al.*’s (2012) numeracy model and the mathematics syllabus:

***W6.3:** Showing practical examples of how to engage the students in problem solving activities,*

***W6.6:** I teach rotating TYs a number pattern module so will definitely use that Pratt video. Thanks,*

***W6.7:** Relating to material that students enjoy,*

***W6.8:** Ideas to try for a class,*

***W6.9:** Use of resources relevant to students’ interest.*

Furthermore, four responses applied to literacy and numeracy which also referenced the design principle: ‘articulation to enable tacit knowledge to be made explicit’. The response,

W6.2: The use of digital literacy was very good,

describes the importance of using other formats other than print media to communicate mathematics.

The next response,

W6.9: Linking words to maths,

identifies the translation process from exploratory to formalisation that happens in a problem solving activity. W6.3's response,

W6.3: Relating problem solving to the syllabus,

is a validation that the syllabus is not just a document that contains mathematics content to be learned but a specification of learning and pedagogy of which problem solving plays a central role. The reform in Junior Cycle acknowledges the changing role of documentation that conveys content knowledge and pedagogical messages to the education community by now referring to curriculum documents as 'specifications' instead of 'syllabuses' (National Council for Curriculum and Assessment 2017).

Finally, the comment from participant *W6.7* in response to the question 'what aspects of today's workshop on Literacy and Numeracy for the teaching of Problem Solving did you find useful?' was '*real life examples*'. This response directly acknowledges the implementation of the first design principle for the intervention: 'authentic contexts that reflect the way the knowledge will be used in real life' (Herrington and Oliver 2000). This authenticity has a dual purpose for the participants. It has a direct relevance for the teacher, the content and the classroom context as well as connecting with students to transfer the knowledge learned in the classroom to other contexts - in other words, enabling numerate behaviour (Goos *et al.* 2012).

6.4.3 Hypothetical Learning Trajectories

According to Simon (1995), the Hypothetical Learning Trajectory (HLT) has three elements: the learning goal, the instructional tasks and the learning processes. The use

of the word ‘hypothetical’ is important as it acknowledges the reality that exists between what is intended to be learned and what is learned can often differ. Consequently, Hypothetical Learning Trajectories are informally described as ‘best guesses of how learning might proceed’ (p. 135). In addition, ‘hypothetical’ also references the idea that ‘not one size fits all’ and adaptations are required to suit individual cultural contexts (Bakker and Van Eerde 2015). Nevertheless, patterns in participants’ reactions, strategies and reasoning do emerge which could eventually contribute to instructional theory in a localised or global context.

Simon’s research aimed to understand the implementation of constructivist theory in child student knowledge development in the classroom setting. Also, Gravemeijer’s (2004) definition of HLTs describes the teacher anticipating ‘how the thinking and learning, in which the students might engage as they participate in certain instructional activities, relate to the chosen learning goal’ (p. 8). In general, hypothetical learning trajectories have been used primarily in professional development for teaching with a student learning focus (Confrey *et al.* 2014). However, this intervention was targeted at pre-service teachers. Hence, like Bargagliotti and Rousseau Anderson (2017) in their research on a professional development intervention on statistics education for teachers, the author applied hypothetical learning trajectories that targeted teacher learning and knowledge. This decision was prompted by Ball, Thames and Phelps (2008) practice-based theory of mathematical knowledge for mathematics teaching, ubiquitously known as ‘the egg’ because of its direct relationship with elements of the Mathematical Quality of Instruction (MQI) protocols that are contained in this study’s theoretical framework (Harvard Graduate School of Education 2018b).

Ball *et al.* (2008) identified two distinct knowledge categories: subject matter knowledge and pedagogical content knowledge that contain three elements each. Subject matter knowledge is comprised of common content knowledge, specialised content knowledge, and horizon content knowledge. Common content knowledge is knowledge not unique to teaching such as 0 divided by 7 equals 0. Specialised content knowledge is particular to the teaching profession such as knowing multiple representations of operations on rational numbers, whereas horizon content knowledge

is where teachers understand the overview of the curriculum and see connections between topics. On the other hand, pedagogical content knowledge describes knowledge of content and students where the teacher must be able to predict difficulties students have engaging with a particular topic. While knowledge of content and teaching refers to the most appropriate sequence and/or approach to instruction to enable productive learning, the third element in pedagogical content knowledge is knowledge of content and the curriculum. According to Shulman (1986), this dimension has two aspects: lateral curriculum knowledge and vertical curriculum knowledge. Lateral knowledge describes making connections between other school subjects, the second element titled 'vertical knowledge' is applied to making connections within the same subject area. Both of these topics were explored in a Continuous Professional Development (CPD) workshop developed by Maths Development Team for Irish teachers during the national roll-out phase of Project Maths (Maths Development Team n.d.c, workshop 6 and 9). 'Connections' is also a component of the Unifying strand in the Junior Cycle Mathematics specification (National Council for Curriculum and Assessment 2017).

In addition, Whitehead (1929 cited in The Cognition and Technology Group at Vanderbilt 1990) described an issue in the enactment of knowledge known as inert knowledge. This knowledge can be recalled by an individual on demand but the same person can fail to apply it in a problem solving context. An example of this difficulty was manifest by the incorrect response of P4, to the ***Probability Concepts*** problem where the participants were required to evaluate incorrect student work on a basic probability concept, '*I would do the sum in exactly the same way*'. This was followed by the articulation of correct facts and appropriate pedagogical approach in the focus group interview:

P4: You can do it in the class throwing coins. They can see when they put all their data together that they will get to close to 0.5. They would be able to appreciate the concept. A real life activity.

Consequently, hypothetical learning trajectories developed for the LNMTI intervention attempted to promote participants' current understanding of literacy and

numeracy to a more sophisticated understanding of the presence of these constructs in the mathematics classroom (Appendix HH).

The next section describes how HLTs were operationalised in workshop 3 (Number) and workshop 4 (Algebra and Functions). Fey's (1970) heuristics, 'structuring' or setting the context and 'soliciting' or eliciting verbal, cognitive or physical action underpinned the process of learning the sequence of tasks. The discussion that follows indicates that while most of the hypothetical learning trajectories were successful, some problems did arise. By consulting the literature, the author dealt with these issues in two ways: extending a task or deciding, despite the difficulties, not to make any amendments.

6.4.3.1 HLTs for Workshop 3 (Number)

Students' inability to work with rational numbers and understand their usefulness in expressing important mathematical ideas has been an issue in Irish State examinations (State Examinations Commission 2016a; Maths Development Team n.d.c, workshop 3). However, limitations in the understanding of this concept amongst teachers and pre-service teachers internationally have also been highlighted in research (English and Halford 1995; Speiser and Walter 2015). The findings from the LNMTI survey indicated that participants had good knowledge of this concept in abstract contexts, (Section 5.3.2) but their use of mathematical language to describe this concept lacked precision. This formed the focus of the learning goal for the hypothetical learning trajectory.

Consequently, instructional tasks designed under the Number strand focused on describing ratios in words and notation and situating the ratio and similarity concept in a real life context that would address common student errors. As previously mentioned, the hypothetical learning trajectory was framed using two of Fey's (1970) heuristics from patterns of verbal communication from the study's theoretical framework, structuring and soliciting. The process began by presenting participants with a printed statement that anticipated students' error in reasoning absolutely instead of relatively. This was the structuring move with the pedagogical purpose of 'setting

a context’ as illustrated in Figure 6.13. The next pedagogical move comprised of two solicitations that each contained a literacy or numeracy focus. The first question focused on the everyday word ‘compare’ and its translation into the mathematical word ‘ratio’ in this context. The second question centred on the importance of order in ratio thinking and notation (Dougherty *et al.* 2016).

Move Type	Pedagogical Purpose	Examples
Structuring	Set a context	When we compare €60 to €20, using the idea of how many 'times' €60 is 'more' than €20 instead of the difference between €60 and €20, we call this the ratio 60:20
Soliciting	Elicit verbal information or cognitive/physical action	How does 60 compare to 20? The current standard pupil-teacher ratio is 1:19, what is wrong with this statement?

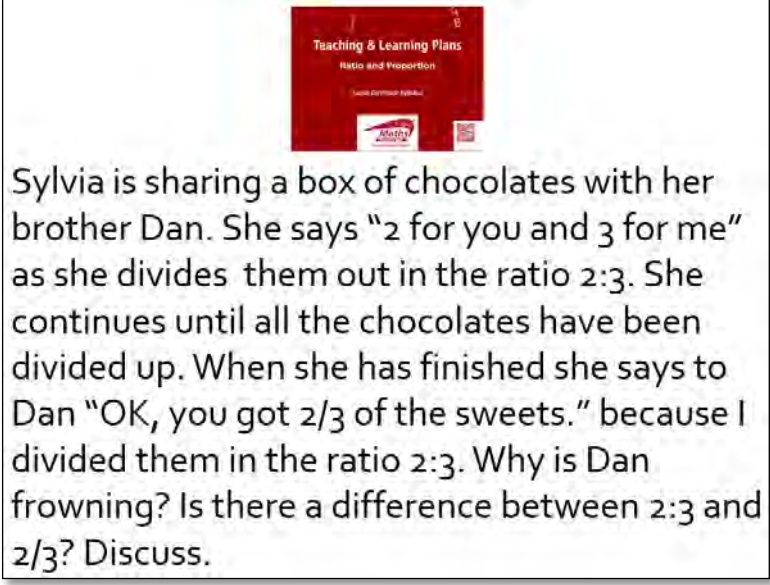
Figure 6.13 ‘Structuring’ and ‘soliciting’ moves for communicating ratios (Maths Development Team n.d.b)

Participants were then given the opportunity to script a series of questions modelled on the above exemplar using a context of their choice or an example from one of the tasks, facilitated by the author. This prompted positive participant reaction in the evaluation indicating this loop of the hypothetical learning trajectory had been achieved as illustrated by the following responses:

W3.4: *Very good in highlighting how ratio and proportion relates to similar triangles. This was really useful. Good resources - ideas for beginning lessons and posing problems.*

***W3.5:** Examples of questions + working through. I found it very beneficial seeing how to construct a question + how to facilitate students in answering - what questions to ask them. Also links numeracy with the objectives on strands we cover in the workshop.*

The second ‘Structuring’ task, presented in Figure 6.14, ‘Sylvia’s box of chocolates’ (Maths Development Team n.d.b) highlights the difference between part-part comparisons and part-whole comparisons. This task focused on mathematical sense-making whereby ‘Soliciting’, moves directed the participants to explore possible student misconceptions about the meaning of the following: 2:3 and $\frac{2}{3}$.



Sylvia is sharing a box of chocolates with her brother Dan. She says “2 for you and 3 for me” as she divides them out in the ratio 2:3. She continues until all the chocolates have been divided up. When she has finished she says to Dan “OK, you got $\frac{2}{3}$ of the sweets.” because I divided them in the ratio 2:3. Why is Dan frowning? Is there a difference between 2:3 and $\frac{2}{3}$? Discuss.

Figure 6.14 ‘Structuring’ and ‘soliciting’ moves for ‘Sylvia’s box of chocolates’ (Maths Development Team n.d.b)

However, this task generated mixed responses: *W3.8* listed it as the most useful aspect of the workshop:

***W3.8:** Showing students’ misconceptions about ratio especially 2:3 relating to $\frac{2}{3}$.*

This was in contrast to **W3.9** who ranked this task as least useful:

W3.9: Although I got what you meant with describing the difference between $\frac{2}{3}$ of the sweets and $\frac{2}{3}$ of her sweets, I'd be afraid highlighting [it as] that could confuse them.

This is evidence that the HLT was not fully achieved for all participants. In addition, Chinn and Ashcroft's (2007) research on specific learning difficulties in mathematics found a correlation between language and mathematics difficulties. However, despite student difficulties, they strongly caution against teaching approaches that would 'dumb down' the subject of mathematics:

Mathematics is a precise means of communication across the curriculum and in everyday life. It is important to resist the temptation to try and reduce it to a set of tricks (p.282).

Consequently, the author decided to extend the 'soliciting' component by including the Chinn and Ashcroft (2007) reference in the final version of the intervention to stimulate a discussion around issues of inclusion for all learners and the importance of maintaining mathematical integrity.

6.4.3.2 HLTs for Workshop 4 (Algebra and Functions)

Despite participants scoring well in the LNMTI survey on explaining abstract algebra to students, difficulties comprised of a misuse of mathematical vocabulary where the noun 'equation' was used incorrectly and all but three participants used the word 'expression' when describing the procedure. This finding is consistent with a recommendation from the State Examinations Commission arising out of overall candidate performance in the Junior Certificate Higher Level on algebraic procedures (State Examinations Commission 2016a):

It is recommended that teachers and candidates give due attention to distinguishing between equations and expressions, and understanding why some procedures may validly be applied to one and not the other (p.21).

Therefore, the HLT concentrated on algebraic literacy in relation to procedures. The 'Structuring' component took the form of presenting the participants with frequently used words in algebraic learning such as expression, equation, evaluate, simplify and solve. The 'soliciting' element elicited their current understanding of these nouns and

verbs. Returning to the ‘structuring component, correct definitions were then revealed to the participants followed by a series of concrete examples from Irish resources (Maths Development Team n.d.c, seminar; State Examinations Commission 2012, question 12(c)).

The ‘soliciting’ element elicited information from the participants on the role of the variable/unknown in expressions, equations, inequalities and functions in the following examples shown in Figure 6.15.

Question 20. In a hospital, there are 5 times as many nurses as doctors. If we let n equal the number of nurses and d equal the number of doctors, which one of the following is correct?

(a) $5n = d$ (b) $\frac{n}{d} = 5$ (c) $\frac{d}{n} = \frac{5}{1}$ (d) $n = 5 + d$

(c) Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when more than €35 is spent. She also has €50 cash. Write down an inequality in x to show the range of money she could spend in the shop.

$\leq x \leq$

Write down an inequality in y to show the price range of articles she could buy.

$\leq y \leq$

Figure 6.15 Identify the role of the variable/unknown

This prompted the following response from participant, W4.4, which indicated a lack of understanding of fundamental mathematics but the hypothetical learning trajectory had been achieved:

W4.4: Always thought an expression had variables and no equals and an equation had variables and an equals. Never knew equations characteristics was solving for an unknown.

In addition, Ellerton and Clements’ (1991 cited in Cummings 1996) research on language factors in mathematics teaching and learning include the use of imagery (p. 16). One of the most common metaphors employed by teachers and textbooks writers to teach the combining of like terms in algebra is the idea that $3a + 2b$ could be 3

apples and 2 bananas (Khan Academy 2015). Research demonstrates this approach as counterproductive in understanding the meaning of a numerical variable (Centre for Algebraic Thinking 2015). Two of the ten Continuous Professional Development days for Project Maths highlighted this issue (Maths Development Team n.d.c, workshops 4 and 5) and an extra day on algebraic reasoning that focused on student errors in algebra was also rolled out nationally (*ibid.*, seminar). A PowerPoint slide with the expression $3a + 2b$ was presented to participants and they were asked to identify the elements of an algebraic expression. An animated discussion that arose from this task is captured in the evaluation responses where participants were clearly enlightened demonstrating the Actual Learning Trajectory was achieved for these participants:

W4.2: *Going through what definitions mean - I would find putting them in words difficult. Also say x is a number.*

W4.6: *Linking that ' x ' is a number.*

W4.8: *Good explanation of why not to use fruit to teach algebra.*

W4.9: *Teaching algebra focusing on the number aspect.*

These comments indicate a conceptual change by participants from the letter as object conception of the variable. However, the following Figure 6.16, shows a negative comment from a respondent at the workshop. The incorrect labelling of the expression and the comment '*still confuses me*' demonstrates this task did not achieve the actual learning trajectory for the participant.

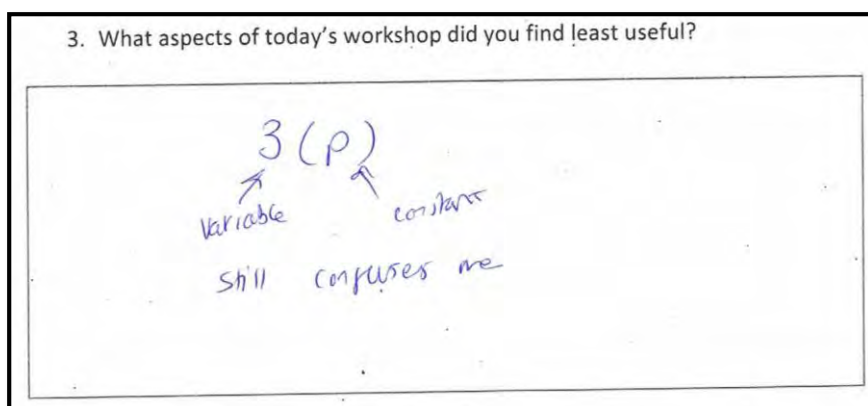


Figure 6.16 Participant response from workshop 4

However, Posner *et al.* (1982) outline the conditions when conceptual change in learners occur. When a learner comes into conflict with a core belief and that belief is challenged, the initial reaction is one of dissatisfaction but on further examination by the learner on the validity of the ‘new’ concept, it can replace incorrect knowledge. As a result, the author did not make any amendments to this task.

Finally, the concept of algebra as generalised number (Ketterlin-Geller and Chard 2011) was demonstrated by using a resource on the multiplication of integers that was originally developed by the Maths Development Team, the ‘Structuring’ component. This task was extended by the author to demonstrate the pattern and generalisation element from the LNMTI framework (Figure 6.17).

Table 1		
3 Times	Result	
3×4	$= 12$	Positive Answer
3×3	$= 9$	Positive Answer
3×2	$= 6$	+
3×1	$= 3$	+
3×0	$= 0$	0
$3 \times (-1)$	$= -3$	-
$3 \times (-2)$	$= -6$	-
$3 \times (-3)$	$= -9$	-
$3 \times (-4)$	$= -12$	-
$3 \times (-5)$	$= -15$	-

☐ Make a Conjecture about $n \in \mathbb{Z}$
☐ Try out other Examples
☐ Generalise in words
☐ Make a Rule

Figure 6.17 Algebra as generalised number

From a numeracy perspective, a sequence of instructions or ‘Solicitations’ was modelled to generate mathematical processes for problem solving such as looking for a pattern, conjectures and generalisations (Guerin 2017). In terms of literacy, illustrated in Figure 6.18, algebraic notation was derived from a starting position of verbalisation in words followed by a step-by-step introduction of mathematical notation. This reflects the development of algebraic expression through history that began with the rhetorical, then extended to a syncopated form and the latest development was the symbolic (Sfard 1995).

Table 3			
-2 Times	Result		
-2×4	$= -8$	Negative Answer	What can we say about -2 times an integer?
-2×3	$= -6$	Negative Answer	
-2×2	$= -4$		What can we say about $-2 \times$ integer?
-2×1	$=$		
-2×0	$= 0$	0	What can we say about $-2 \times n$?
$-2 \times (-1)$	$=$		
$-2 \times (-2)$	$=$		
$-2 \times (-3)$	$=$		
$-2 \times (-4)$	$=$		What can we say about $-2n, n \in \mathbb{Z}$?
$-2 \times (-5)$	$=$		

Figure 6.18 Rhetorical, syncopated and symbolic representation of an algebraic expression

The development of the knowledge of content and teaching domain is captured by *W4.1*'s reaction to the instructional task:

W4.1: *Never thought about teaching the rules in integers the way you showed. Fantastic.*

Three other participants, *W4.4*, *W4.6*, and *W4.10* specifically referenced this task being the most useful aspect of that workshop. Given the reaction to this method as an effective pedagogy, it validates the approach advocated in the Junior Cycle Mathematics specification (National Council for Curriculum and Assessment 2017) and the necessity of showcasing this pedagogical position to pre-service teachers earlier in their training.

6.5 Conclusion

This chapter described the design, construction, implementation and initial evaluation of the LNMTI module for second year Professional Master of Education students ($n = 12$), following a university Initial Teacher Education programme in Ireland during the last semester of their studies to answer the third Research Question:

***RQ3:** What content and characteristics should a teaching and learning intervention have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?*

The first phase of the process involved defining the design requirements that were generated by participant needs and context analysis. The type of intervention was chosen based on design constraints and traditional Continuous Professional Development models. The LNMTI framework and Instructional design principles guided the construction of the module that maintained the focus on authentic contexts, authentic tasks and models of best practice. Hypothesised Learning Trajectories (HLTs) contributed to the sequencing of the workshops and the workshop content. Workshop evaluations suggest the content and the characteristics experience of the LNMTI module was worthwhile in developing the participants' LNMTI knowledge and overall professional practice as mathematics teachers for the Irish classrooms. Also, one of the conceptual principles of the LNMTI module that emerged was the importance of exploring pedagogical messages from source, i.e. the mathematics syllabus. In the next chapter the reader will gain insights into the overall findings of the intervention which was measured by an evaluative survey, a researcher/teacher observation, focus group and classroom observation instrument.

Chapter 7: Post-Intervention - Evaluation and Reflection

7.1 Introduction

The purpose of this chapter is to describe for the reader the evaluation and reflective micro cycle of the Educational Design Research (EDR) methodology for this study (McKenney and Reeves 2012). The evaluative element determines the overall effectiveness of the intervention coupled with judgments about improvements and revisions. To investigate to what extent the goals of the intervention were achieved the fourth and final Research Question 4 was composed:

RQ4: What is the potential of a teaching and learning module based on authentic design principles to support pre-service teachers of mathematics develop an understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that implementation of the domain in the dynamic classroom environment is enabled?

To answer the question four new questions emerged:

RQ4(a) To what extent did the LNMTI intervention contribute to improving pre-service teachers' knowledge of literacy and numeracy for mathematics teaching in Ireland?

RQ4(b) To what extent did the enactment of the LNMTI intervention align with the design principles for authentic learning?

RQ4(c) How do we measure pre-service teachers' LNMTI skills in the mathematics classroom?

RQ4(d) To what extent were LNMTI skills enacted by pre-service teachers in the classroom setting?

The evaluation is both summative and formative in nature using the heuristics as performance indicators from the domain specific frame from the study's theoretical framework outlined in Figure 7.1.

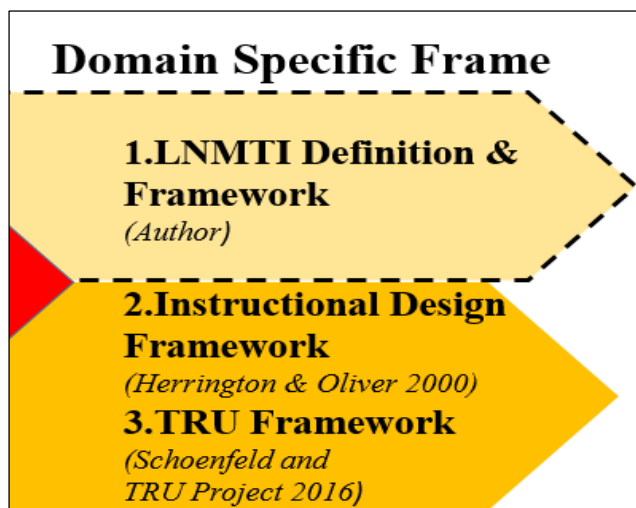


Figure 7.1 Domain specific frame from the study's Theoretical framework

The LNMTI definition and framework developed by the author was employed to assess the overall effectiveness of the LNMTI intervention. The Instructional Design Framework (Herrington and Oliver 2000) guided the assessment of the intervention's adherence to the principles of authentic learning. However, one of the design principles featured an 'authentic assessment' element. This inspired the design of a prototype of an LNMTI classroom observation instrument to authentically evaluate the enactment of LNMTI skills in the dynamic classroom environment. Schoenfeld and the Teaching for Robust Understanding Project (TRU) (2016a) provided the support and structure for this endeavour, which will be discussed in more detail in Section 7.3.3.

The following sections present the data sources, procedures and results of this evaluative and reflective process for the LNMTI intervention.

7.2 Data Sources and Procedures

Four main sources of data to evaluate the intervention were utilised: survey, focus group interview, expert appraisal and the LNMTI classroom observation sheet.

To perform an evaluation of a study using an EDR methodology, McKenney and Reeves (2012) advise to step back from the research and simply ask the question 'what do we need to know now'? (p.136). The answer to that question centred on the needs

of the participants who displayed weaknesses in their knowledge of literacy and numeracy for mathematics teaching. One of the key objectives of the intervention, guided by the LNMTI definition and framework, was to fill that knowledge gap as well as addressing priority issues such as participant confusion about the relationship between numeracy and mathematics (Section 6.2). Data was collected by a pen and paper survey. Six questions were asked to gain insights into participants’:

- Knowledge of LNMTI generally, in lesson planning, in teaching,
- Recommendations for programme continuation and/or for programme improvement.

A comment box was also provided if participants wanted to contribute a judgement that was not facilitated in the printed questions (Appendix N). How the intervention would be perceived by a wider audience (McKenney and Reeves 2012) was addressed by having an external expert observe and appraise the LNMTI workshops (Bopardikar *et al.* 2018).

The extent to which the learning experience adhered to the design principles was assessed by means of a focus group interview that took place in a university classroom following the conclusion of the LNMTI intervention. Participants identified as P2, P3, P4, P5 and P6 in this study took part and the interview was moderated by the author. P1 and P7 also volunteered but were unavailable to attend because of school placement commitments therefore a 1-1 telephone interview with these participants took place the same week. Selection of the participants for the focus group/1-1 interview was voluntary but not random. These participants were specifically targeted as they participated in the first focus group interview and they also volunteered to be observed teaching in a live classroom setting. No costs were incurred to administer the interviews and participants did not look for payment for their involvement (Marlowe 2008). To minimize moderator bias, the author stuck rigidly to a set of pre-prepared written questions and probes (Harrell and Bradley 2009). Five questions were asked that focussed on participants’ personal learning, learning for the classroom, relevance in terms of their professional context and in addressing the needs they had expressed

in an earlier interview and finally, suggestions for improvement were sought (Appendix Q). Both the telephone and focus group interviews were audio recorded and transcribed by the author (Appendix R and S). Transcripts of the interview were e-mailed to the participants (Appendix V) to authenticate the transcription (Birt *et al.* 2016). Participants were also invited to amend or add to the transcribed material. The participants were satisfied with the transcription. Content analysis was applied to the data to understand if the design principles for authentic learning were addressed and achieved (Ethell and McMeniman 2000).

As previously mentioned one of the design principles for authentic learning, *authentic assessment*, was excluded from this process and a new instrument, a classroom observation sheet for LNMTI was designed and developed. Details of the construction and implementation of this data source will be discussed in detail in Section 7.3.3. The findings from the data gathered from this instrument are summarised and compared with data from the LNMTI survey described in detail in Chapter 5. The limitations of the instrument will also be addressed (Cohen *et al.* 2011). In summary, Table 7.1 describes the sources of data, the number of participants, and a description of what the source evaluated.

Table 7.1 Data sources for evaluation

<i>Instrument and technique</i>	<i>Analysis</i>
Survey, $n = 12$	Effectiveness of the LNMTI intervention
Expert Appraisal (observation notes), $n = 1$	Effectiveness of LNMTI intervention for implementation and spread
Transcript of Focus Group Interview, $n = 5$	The extent to which design principles for authentic learning were implemented
Transcripts of 1-1 interview, $n = 2$	
LNMTI Classroom Observation Sheet, $n = 7$	The extent to which LNMTI was enacted in the dynamic classroom environment

The next section will describe the findings from these multiple data sources.

7.3 Findings

This section has two main components on the effectiveness of the intervention and the design and implementation of the LNMTI classroom observation sheet. The first part presents findings from the post intervention evaluation survey, expert appraisal and interviews (focus group and 1-1) to answer two Research Questions:

***RQ4(a)** To what extent did the LNMTI intervention contribute to improving pre-service teachers' knowledge of literacy and numeracy for mathematics teaching in Ireland?*

***RQ4(b)** To what extent did the enactment of the LNMTI intervention align with the design principles for authentic learning?*

The second part focuses on the design process for the LNMTI classroom observation sheet followed by an analysis of data gathered from this instrument to address the following Research Questions:

***RQ4(c)** How do we measure pre-service teachers' LNMTI skills in the mathematics classroom?*

***RQ4(d)** To what extent were LNMTI skills enacted by pre-service teachers in the classroom setting?*

7.3.1 LNMTI Intervention Effectiveness: Survey

The intended outcome of the LNMTI module was to contribute to participants' understanding of LNMTI. The extent to which that contribution was worthwhile was guided by the EDR evaluation process 'exploration of intervention attributes' (McKenney and Reeves 2012, p.179) summarised under the following three headings:

- **Overall effectiveness** defined as the extent to which the needs of the participants were addressed,
- **Social Validity** refers to the extent to which ownership of the knowledge and the enactment of the knowledge in a professional setting is facilitated,
- **Recommendations** considers the sustainability of the intervention as well as suggested improvements and revisions.

The following sections will review the results from the survey using these three headings as key themes.

7.3.1.1 Overall Effectiveness

The first three questions of the survey evaluated the validity, practicality and impact of the intervention from the point of view of the participants (McKenney and Van den Akker 2005).

- (1) Do you agree or disagree with the following statement and give a reason for your choice?

These workshops taught me what literacy and numeracy for mathematics teaching in Ireland was about and how it impacts on teaching and learning. Agree/Disagree.

- (2) Have these workshops enabled you to better describe literacy and numeracy learning outcomes in your lesson plans? Yes/No. Give a reason for your answer.

- (3) Have these workshops influenced your teaching in anyway? Yes/No. Give a reason for your answer.

As participant time was limited, the post-module evaluation questions were designed to gain an immediate dichotomous response to the three key learning outcomes of the module: understanding LNMTI, its use in lesson planning and teaching and learning in the classroom. This was followed by a comment box where participants were asked to give a reason for their choice. Figure 7.2 shows an overview of the results from the dichotomous rating for questions 1, 2 and 3.

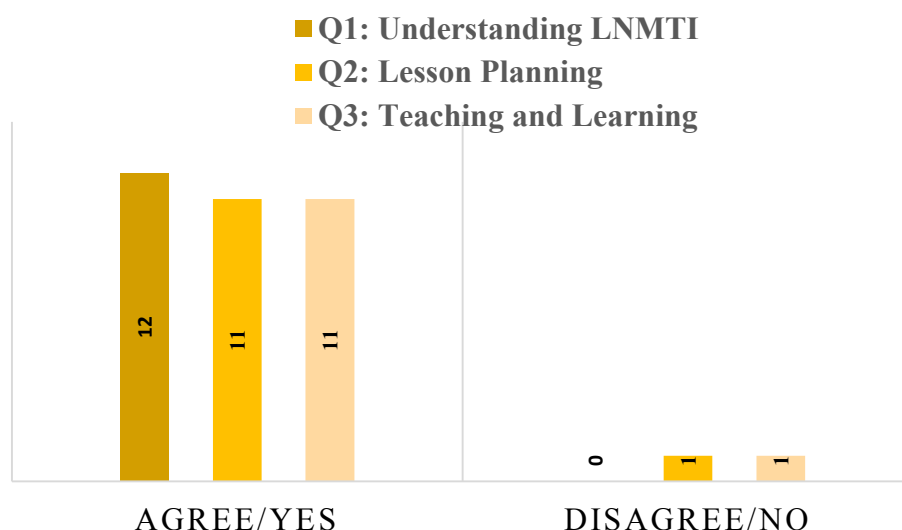


Figure 7.2 Participants' responses to survey questions 1, 2 and 3

As can be seen from the graph above, the twelve participants responded positively with a 100% agreement on the benefit of the workshops for their understanding of LNMTI. Three participants did not articulate a reason why they agreed and for the other nine, four mentioned engaging with the syllabus as a positive factor and the other responses referenced a deeper understanding of numeracy in mathematics (2 responses), an awareness of literacy to support learning (2 responses) or an enhanced understanding of literacy and numeracy (1 response). These positive results highlight the direct impact of the intervention on participant understanding of LNMTI contributing to teacher professional knowledge and expertise in a range of areas, particularly in relation to participants' knowledge of curriculum documents (Ball *et al.* 2008; Davis *et al.* 2014).

The second question came in response to participants' lack of knowledge and confidence in the area of planning for numeracy from the first focus group interview (Section 5.4). Eleven out of the twelve participants agreed the workshops helped them better describe literacy and numeracy learning outcomes in their lesson planning. Three respondents described a growth in confidence in this domain and seven responses referenced a greater understanding in linking mathematical content to numeracy as a result of showcasing specific examples:

P7: I have been able to relate it more to the syllabus and have therefore introduced this into learning outcomes,

P11: Some concrete examples given and this gives me a better starting position to develop my own.

Overall, a common view amongst the respondents was that the LNMTI intervention was supportive in preparing them for their professional role. A further study with more focus on pre-service teacher confidence/self-efficacy is recommended by the author as Charalambous' (2008) demonstrates teacher confidence in an area of mathematics facilitates richer and complex learning environments for students.

The third question aimed to explore the participants' perceived impact the workshops had on their teaching. The bar graph above shows eleven out of twelve participants believed the workshops had a positive impact. Three respondents commented on the usability of resources and teaching methodologies that were directly transferable to the classroom. Seven respondents commented on an ability to apply and critically appreciate the knowledge they gained in the workshops to the work of the classroom. The following responses by P6 and P8 illustrate this theme:

P6: While I had proportion taught prior to the workshops, I would now relate a lot of content of classes to this idea..... Also, I am more conscious to bind the strand together,

P8: I now use more concrete resources in my teaching than I did before.

However, P8 and P9 gave a negative response to Q2 and Q3 respectively but they could not articulate the reason why. P8 did not write a response and P9 remarked:

P9: They have not impacted in anyway. Do not know why?

This shows a negative outcome of the intervention. This response in particular provides evidence for involving a more structured reflective practice component in the workshops in addition to the paper evaluations that were provided at the end of each workshop (Section 6.3.2). This aspect will be discussed in more detail in Section 7.4.4.

7.3.1.2. Social Validity

The social validity of the workshops (Miramontes *et al.* 2011, O'Meara 2011) was captured in an open response question as recommended by Lindo and Ellemann (2010):

Do you feel better prepared for your first year of teaching mathematics professionally as a result of taking these workshops? Give a reason for your answer.

This questioned the level of importance and ownership the participants attach to the aims, methods and effects of the intervention after the participants leave university and enter the world of work. All of the participants affirmed the value of the intervention in preparing them for their professional role as mathematics teachers and all but one respondent gave a reason. Table 7.2 summarises the responses which were organised into three themes. Sample quotations are given to illustrate each theme (Bopardikar *et al.* 2018).

Table 7.2: Sample participant responses on the social validity of LNMTI intervention

Themes	# Responses	Sample Quotation
Enhanced knowledge for educational reform in mathematics	2	P1: Absolutely, I feel I have a better grasp of what project maths entails
Preparation for the world of work	5	P2: Definitely. I feel more prepared for both interviews this year as well as teaching - as I have examples + more familiar with the language I should be using.
Pedagogical knowledge	4	P10: Yes - although we have been introduced to methods of teaching, it was very beneficial to see a new perspective integrating the various strands.

This data demonstrates a meaningful recontextualisation of the macro-level pedagogic messages (Looi *et al.* 2011) regarding recent reforms in mathematics and literacy and numeracy skills. This indicates a satisfactory outcome for the intervention.

7.3.1.3 Recommendations

Improvements and revisions to the intervention were identified by answers to Question 5:

Would you recommend this module to future PME students of mathematics teaching?

This summative evaluation question addressed the benefits or otherwise of continuing the programme. All of the participants considered the programme worthwhile for future PME students. The reasons offered aligned with three of the four specific learning targets for the module outlined in Section 6.2. Table 7.3 presents the description of the LNMTI learning target and a sample of quotations from the survey data for Question 5.

Table 7.3 LNMTI learning targets and sample quotations from survey data

<i>LNMTI Learning Target</i>	<i>Sample Quotations from Question 5</i>
To understand the interpretation of numeracy and mathematical proficiency in the mathematics syllabuses and mathematics assessment items	<i>P11: Yes. Some interesting topics covered and relating to syllabus was good. I feel it is assumed maths teachers are automatically good at numeracy and we might not be on all aspects.</i>
To understand elements of Mathematical Quality of Instruction and how it can be operationalised in a classroom setting	<i>P10: I would recommend this module as it provided a great insight into various pedagogy styles and more knowledge about maths numeracy and literacy.</i>
To appreciate problem solving as a central element of literacy and numeracy	<i>P8: Yes, problem solving and relating words to maths concepts is an increasingly important aspect of maths teaching and should be prioritised more.</i>
To expand the repertoire of communication formats such as digital media and broadcast media to explore mathematics	No data available from this survey Data is available from survey on workshop 6 and post interventions focus group interview

Mandeville and Liu (1997) discuss how teachers with higher mathematics content knowledge were better prepared to make connections and involve the students in more

complex learning and reasoning tasks, part of the reformist agenda in post-primary mathematics education in Ireland. However, Prendergast *et al.* (2013) report on Irish mathematics teaching graduates having ‘often little relational understanding’ (p.635) of the subject. The notion of relational understanding in mathematics for the current reforms in Junior Cycle has been scaled up to include supporting students learning across all aspects of the curriculum (National Council for Curriculum and Assessment (NCCA) 2017). The responses above demonstrate the LNMTI module was effective in contributing to the reformist approach in preparing the pre-service teachers to implement change. Although the comment made by P11 revealed an unexpected outcome:

P11: I feel it is assumed maths teachers are automatically good at numeracy and we might not be on all aspects.

This raises questions about reform education policy documents that position mathematics as a subject and mathematics teachers develop numeracy across the curriculum (National Council for Curriculum and Assessment (NCCA) 2014a; Ireland, Department of Education and Skills 2011). More research in this area is needed to answer a vital question: are Irish mathematics teachers numerate?

Moreover, it must be noted a more expanded repertoire of literacy formats was not acknowledged by the respondents, with no available data to assign to this category suggesting this objective did not make an immediate impact. On the other hand, data from the focus group interview and the expert appraisal on calculator use as an example of digital literacy, was compelling. Secondly, recommendations were sought to improve or revise the LNMTI module. This was an example of a formative evaluation where respondents were invited to critique the structure, content and presentation of the workshops (Worthen 1990). 4 of the respondents did not think the programme required any modifications, 1 made no response and 7 made recommendations summarised in Table 7.4.

Table 7.4 Participants' suggested recommendations for LNMTI intervention improvement

<i>Dimension</i>	<i>#Responses</i>	<i>Recommendations</i>
Structure	2	Active participation in practical activities Schedule the module to take place in PME 1
Content	3	More examples from state examinations assessments More real life examples More worksheets
Presentation	2	Model more teaching strategies More working through the mathematics of questions presented

A preference for scheduling the module in year 1 of the PME studies, engaging participants in more practical activities as well as equipping them with more ‘hand-outs’ they could keep and reuse for future teaching scenarios was expressed. Providing more examples from state examinations assessments and real life was suggested. Finally, one participant, P9, recommended working through the mathematics in more detail during the workshops. In the final comments section, the same participant expressed a similar view:

***P9:** While this was an interesting module, it would have been more useful to teach us what is on the course (JC and LC) and go through hard questions and how to do them.*

The mathematics content explored in the module did not go beyond Junior Certificate /middle school standard and all the participants met the Irish Teaching Council (2013) subject registration standards having ‘acquired sufficient knowledge, skills and understanding to teach the Mathematics syllabus to the highest level in post-primary education’ (p.37). However, the LNMTI survey revealed participants had difficulties with areas of mathematical content, most notably in probability, statistics and geometry (Sections 5.5.3 and 5.5.4). Improvements in two of these content areas were acknowledged in the final comment box by P7 who obtained the poorest result in the LNMTI survey:

***P7:** I feel I really learned a lot, particularly in the areas of geometry and statistics.*

Similarly, P9 also scored poorly in the LNMTI survey with 50% of the responses in the ‘Not Present-Low’ category however the comment by P9 above illustrates the content component did not go deep enough for all participants in addressing the issue of having a sound mathematical content base before engaging in their professional role (Lowrie and Jorgensen 2016). This is another serious issue that needs to be challenged going forward in Irish PME mathematics programmes if the desired teacher quality as mandated by the Teaching Council (2013) is to be fully achieved.

7.3.1.4 Expert Appraisal

As part of the EDR process, it is recommended to plan for implementation and spread of the design artefact (McKenney and Reeves 2012). For this study, this was achieved by receiving feedback from a fully qualified mathematics teacher with over five years’ experience who is also involved in numeracy research at third level (Krajcik *et al.* 2008). Before observing the workshops, the expert identified two key problems for literacy and numeracy implementation in the mathematics classroom:

- The lack of support for teachers to promote understanding of what literacy and numeracy for mathematics teaching entails.
- Challenging teacher beliefs about literacy: *‘it isn’t part of a maths teacher’s job’* (Appendix O) and numeracy: *‘it is already been taught as part of the mathematics curriculum’* (*ibid*).

The expert attended the last three workshops out of six and evaluated the effectiveness of the intervention by answering the question: has your understanding of ‘Literacy and Numeracy for mathematics teaching in Ireland’ changed as a result of observing these workshops?’ The response revealed three main learning outcomes:

- Change in understanding of LNMTI,
- The importance of developing literacy in the mathematics classroom,
- Numeracy strategies.

The expert acknowledged the workshop content and activities had enabled a broader view of how literacy and numeracy in the Irish mathematics classroom could be

enacted. The balanced emphasis on literacy and numeracy in the teaching and learning interaction was identified as a major strength of the intervention. The final comment related to the expert's enhanced numeracy knowledge, '*which I didn't think was possible*', (Appendix P) to promote numeracy across all subjects. Particular reference was made to the multiple uses of the random number generator in the calculator, an example of digital literacy in the mathematics classroom (*ibid*).

In summary, expert input on the effectiveness and utility of the workshops helped not only to validate the feedback from the participants' survey but it identified a need from an Irish professional teaching perspective to articulate, model and enact literacy and numeracy skills for the mathematics classroom.

This section reported on the evaluation of the LNMTI intervention from two points of view: the participants and an expert. Both perspectives endorsed the value of the intervention for its clarity of purpose and valid contribution to literacy and numeracy for mathematics teaching in Ireland. The section that follows will present findings from the focus group interview and 1-1 interviews on evaluating the strengths and weaknesses of authentic learning principles that guided the development and construction of the LNMTI intervention.

7.3.2 LNMTI Intervention Alignment with Design Principles

The post intervention interviews were used as an evaluative tool for the intervention to ascertain to what extent the LNMTI intervention addressed the principles underpinning the design. The purpose of this analysis was to assess the impact of the intervention from a design perspective that would enable an improved learning experience for future pre-service teachers. The nine design principles are listed in Table 7.5 and Herrington and Oliver's (2000) Instructional Design Framework where these principles were derived was introduced to readers in Section 3.5.3.

Table 7.5 Design principles from Herrington and Oliver's (2000) instructional design framework

1	authentic contexts that reflect the way the knowledge will be used in real life
2	authentic activities
3	access to expert performances and the modelling of processes
4	multiple roles and perspectives
5	collaborative construction of knowledge
6	reflection to enable abstractions to be formed
7	articulation to enable tacit knowledge to be made explicit
8	coaching and scaffolding by the teacher at critical times
9	authentic assessment of learning within the tasks.

The interview transcripts were analysed using pre-determined nodes from eight of the nine design principles. As previously mentioned, design principle 9 ‘authentic assessment of learning within the tasks’ was deployed by assessing participants’ enactment of LNMTI in the classroom environment as discussed in Section 7.3.4. In the following analysis, the author will reference both the focus group interview responses from the five participants as well as the telephone interviews given by P1 and P7. As outlined in Section 3.7.5, the first eight design principles were grouped thematically by the author on the basis of a shared attribute followed by the design principle number from Table 7.5 above. For example, design principle 1, *authentic contexts that reflect the way the knowledge will be used in real life* and design principle 2, *authentic activities* are paired under the heading ‘Authentic (1,2)’:

- Authentic (1,2),
- Experts, Coaching and Collaboration (3,5,8),
- Perspectives and Articulation (4,7),
- Reflection (6).

Quotations from the respondents will be provided as evidence of the design principle in action to support the learning of the pre-service teachers (Prendergast *et al.* 2013).

7.3.2.1 Authentic (1, 2)

1: Authentic contexts that reflects the way the knowledge will be used in real life.

2: Authentic activities.

All seven of the participants spoke of the relevance and validity of the knowledge they acquired in the workshops, captured by P1's response:

P1: I learned so much. Even some of the things you were going through I was actually teaching at the time and it was so, so helpful.

Secondly, all participants agreed they would have preferred to have experienced the workshop in the first year of their initial teacher education. They also spoke about the broader understanding of numeracy in mathematics they gained from the maths specific LNMTI intervention because when it was addressed in the generic literacy and numeracy module provided by the school of education, P5 commented:

P5: We were just left, we were told to pick the other subject when we got into groups.

These findings reflect those of Todorova *et al.* (2017) who in their study of pre-service teachers' professional vision of instructional support for teaching of science found that a content specific focus was necessary.

The respondents also valued the activities as authentic and novel:

P6: Cool examples, to do a series that way was amazing, you would just never think of it.

However, Herrington *et al.* (2007) argue for more complex and sustaining examples of authentic activities instead of providing good illustrations from real life contexts. There is evidence from the respondents that this was achieved by P2's comment:

P2: I think as well, just how to structure it with examples of what questions you ask the students and how to go through it from start to finish. Because I find I'd confuse them so it was really useful getting those kinds of specifics as well.

These quotes demonstrate the authentic component of the design principles for authentic learning has been achieved in both generating authentic activities and the contexts the knowledge will be used in Irish classroom settings.

7.3.2.2 Experts, Coaching and Collaboration (3, 5, 8)

3: Access to expert performances and the modelling of processes.

5: Coaching and scaffolding.

8: Collaborative Construction of Knowledge.

Ethell and McMeniman (2000) highlight that pre-service teachers observing an expert teacher is not sufficient to improve their practice. They contend that the ‘how’ of teaching should be aligned with the ‘why’ for authentic learning to occur. This is substantiated by Kopcha and Alger (2014) who promote technology-assisted cognitive apprenticeships through online platforms to support the situated learning of the pre-service teacher. Concepts of legitimate peripheral participation and communities of practice underpin the theory of situated learning (Lave and Wenger 1991) which is a constituent part of this study’s theoretical framework. The author was positioned as the expert in the community of practice to deliver the LNMTI intervention and the respondents regarded having access to an ‘expert model’ as beneficial. These viewpoints are reflected by the following comments:

***P1:** I really liked it in terms of the fact you are a teacher and you’re in the classroom and it was really nice to have it coming from someone who is teaching day in day out. I think what you went through was practical,*

***P7:** Even your use of resources, you were very visual as well. It inspired me to be more visual within a classroom.*

Furthermore, Vygotsky’s (1986) research on learning focused on the interactional nexus between learner and more experienced other which is manifested in the coaching and facilitation role of the author as teacher of the LNMTI module. However, there was limited evidence of coaching and scaffolding having an impact from the

respondents which points to a weakness in the enactment of this principle. Only one implied reference came from **P1** in the 1-1 interviews:

***P1:** Both yourself and (maths methods module supervisor) have shown us the need for problem solving and definitely you got it across how to go about it.*

In addition, the collaborative construction of knowledge was referenced once however in this case, the respondent's comment demonstrates a collaborative learning atmosphere was facilitated in the workshops and valuable learning ensued:

***P7:** Your activities were focused on problem solving and especially the group work. Your methods of how to tackle the different problems and how to teach in that way as well as seeing all the different methods that students could come up with, that was another good way, as well.*

To conclude, these data indicate a mixed evaluation on the adherence to the principles of modelling, coaching and collaboration. While there is strong evidence to suggest the modelling of expert performances and processes was successful, consideration of more explicit coaching and collaboration elements need to be addressed. These responses suggest the culture of coaching for collaborative learning that is fostered by the initial teacher education programme in this university was not fully upheld, illustrated by a comment from P1 in the pre-intervention focus group on a group project on literacy:

***P1:** we were put into groups.....we learned a load from actually taking the pillars and applying it to our subject.*

Consequently, as time allocation for the LNMTI module was limited, consideration of digital technology mediated platforms could be explored in the future to scaffold a shared learning environment as a social activity (Rhetulla and Xiu 2005).

7.3.2.3 Perspectives and Articulation (4, 7)

4: Multiple Roles and Perspectives.

7: Articulation to make tacit knowledge explicit.

Participants were exposed to multiple manifestations of numeracy in the syllabus, state examination questions, workshop activities and Goos *et al.*'s (2012) model in response to a need, expressed at the pre-workshop focus group interview, to bridge their understanding of numeracy from a theoretical perspective to a practical one. All seven respondents spoke about an improved understanding in numeracy and its relationship to mathematics which was enabled by showcasing the varied educational materials and academic models that included numeracy as a foundational domain as evidenced from the following comments:

P1: It opened up my eyes to a few different areas that we hadn't been introduced to before. It was great linking the whole course up: linking numeracy with the curriculum and all the different strands, the aims of the syllabus,

P3: I am actually thinking about all the concepts now from Goos model because of the workshops.

However, respondents also spoke exclusively about the advantage of identifying numeracy moments in the mathematics syllabus:

P2: I think I can relate the numeracy to the syllabus more than I used to before. Even just the language of what numeracy is, to put it into words, for our lesson plans. I feel I am more confident in it now that I was before.

P5: It makes it easier to do numeracy now it is in the syllabus.

What is clear is the awareness of the respondents of the importance of the official syllabus documentation in lesson planning. This emphasis underpins the thinking of Long *et al.* (2014) who call for a 'common language' (p.1) across the educational processes of educational objectives, classroom activities and assessment. The data also show participants developing a professional agency (Lynch *et al.* 2017) in the domain of numeracy and mathematics which didn't exist prior to the workshops (See Section 5.4.2).

Furthermore, five out of the seven participants had also spoken about their lack of confidence in choosing to complete a numeracy assignment in mathematics and had opted to do another teaching subject instead. Following the workshops, all of the

participants had gained confidence in the domain and its relationship with mathematics which is echoed in the following comment:

P4: I would have found it [numeracy assignment] interesting to do it in maths.

However, despite the positive comments on the impact the LNMTI workshops had on making the tacit knowledge of numeracy more explicit, two respondents, P1 and P7 said they hadn't fully connected with this domain:

P1: In terms of numeracy, I'm still a bit unsure of what numeracy is.

P7: I still wouldn't be 100% sure, I would be more 80% and where I was at was 50%.

They both refer to the temporal issues around the assimilation and embedding of numeracy knowledge preferring instead to have access to an extended module on LNMTI over a two year period. This was substantiated by all of the respondents who believed accessing the LNMTI module in year 1 of their education would have been beneficial.

The following conclusions can be drawn from the above data: having multiple perspectives and roles for embedding knowledge of LNMTI was successfully implemented in the workshops that enabled a richer understanding of numeracy and its relationship to mathematics. Secondly, translating the tacit knowledge of the curriculum documents into workable and practical knowledge for the classroom was worthwhile. On the other hand, the closing of the knowledge gap for numeracy was not fully achieved for all participants.

7.3.2.4 Reflection (6)

6: Reflection to enable abstractions to be formed

The reflection element as espoused by Herrington and Oliver (2002) refers to the availability of platforms such as reflective journals and online discussions forums to ascertain the growth in participants learning through recorded thoughts and observations for an authentic learning experience. These channels were not made

available to the participants in the LNMTI workshops because of pre-service teacher workload. Instead participants voluntarily completed a pen and paper survey at the end of each workshop. The data from this practice has been discussed in Sections 6.6.3 and 7.2.1. However, the focus group as a method of social reflection (Herrington, n.d.) allowed the participants to describe in a group setting the status of their learning as a result of the LNMTI workshops. The following comments reveal, that respondents advanced in their learning by communicating reflections that ‘enabled abstractions’ (Herrington and Oliver 2000, p.27). One respondent, P4, generalised the domain of numeracy *as a set of skills to be practised instead of numbers and symbols on a page* as well as *being able to communicate data and spatial awareness*.

In addition, P3’s comment on numeracy knowledge illustrates a connection between the content from the generic module on numeracy provided by the university and the LNMTI module.

***P3:** It’s going beyond, let’s say we are just subtracting fractions or something like that. I am actually thinking about all the concepts now from Goos model because of the workshops.*

Gentener *et al.* (2003) found that in a lot of cases, novice adult learners rely on concrete features to represent a phenomenon and fail to abstract or transfer the learning to a new situation. They reported that learners were enabled to form abstractions when they were involved with comparing various examples of the phenomenon of interest. This method was employed in workshop 1 to launch the definition of literacy and numeracy for mathematics teaching in Ireland when examples from the syllabus, state examinations assessments and Goos *et al.*’s (2012) model of numeracy were aligned.

The following comment also illustrates an improved relational understanding in all the domains addressed at the LNMTI workshops: literacy, numeracy and mathematics:

***P6:** Well, I think my approach to teaching knowledge that I have of maths, we’ll say, similar triangles, I’d say: ‘look, big over small’, that’s it. I would never reference ratios or proportions or anything. But now I can nearly see it in every class which I would never have before. For trigonometry, I would never have really used it. I know they are trigonometric ratios but I’d never*

have said the word 'ratio', ever. Whereas now I am looking out for stuff like that.

It is clear in this example the participant has linked her understanding of Geometry and Trigonometry to Number and making connections is an element in the 'unifying strand' contained in the Junior Cycle mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017). This reflection has also demonstrated a generalised application of one of the key mathematical ideas in post-primary mathematics: ratio and proportion (Morgan 2012).

As previously mentioned in Section 7.3.3.2, digitally supported collaborative learning platforms could have supported learners to deepen participants' relational understanding and generate abstractions similar to P6 above. However, even in the absence of this support, the participants' thoughts and observations conveyed in the focus group interview establish that relational abstractions were formed. Therefore, the LNMTI workshops achieved a positive outcome for this design principle.

In summary, the findings show principles underpinning authenticity (1, 2) and reflection to enable abstractions (6) were achieved. On the other hand, experts, coaching and collaboration (3, 5, 8) and perspectives and articulation (4, 7) were partially achieved. This analysis has served to identify and clarify the strengths and the weaknesses of the module that will be used to improve its overall quality.

The previous section provided an evaluation of the first eight principles of authentic learning in the LNMTI workshops. The next section focuses on the 9th principle which addresses the assessment of authentic learning.

7.3.3 LNMTI Classroom Observation Instrument: Design and Development

To assess the learning of the LNMTI module authentically, observing the participants enacting the skills in their school placements was the most appropriate form. This form of evaluation is endorsed by Prendergast *et al.* (2013) however the difficulties associated with this practice are outlined (p.10). As LNMTI was a new construct or educational artefact/product for this study, the need for an instrument to collect data in

a classroom setting arose. In the analysis and exploration phase for this product design, the author reviewed the university materials used by school placement tutors in their assessment of pre-service teachers in school placements. Although literacy and numeracy are regarded as key skills underpinning all teaching and learning interactions (Ireland, Department of Education and Skills 2015a), the key principle for this design was in defining the role for this instrument for school placement tutors. It was important to acknowledge the established university evaluation protocols that school placement tutors used therefore this instrument would provide a separate but specific literacy and numeracy lens to observe pre-service teachers in their teaching practice. Consequently ‘ease of use’ was a priority issue.

Moreover, a major influence on the design came from an examination of materials from research currently being carried out by Schoenfeld and the Teaching for Robust Understanding Project (TRU) (2016a) which set to clarify and characterise successful teaching and learning interactions in classrooms. To date various methods have been developed and introduced to measure teacher effectiveness in the classroom (Schoenfeld 2013) and one such instrument for video based observations, Mathematical Quality of Instruction (MQI), is an element of this study’s theoretical framework. On the other hand, the TRU approach concentrates on actual classroom visits, relevant to this phase of the study, where ‘each observation is one of a series of classroom visits contributing to teacher growth’ (Schoenfeld and Teaching for Robust Understanding (TRU) Project 2016b, p.2). Therefore, to answer the research question:

How do we measure pre-service teachers’ LNMTI skills in the mathematics classroom?

the author developed a classroom observation sheet for the classroom visits of seven participants, P1, P2, P3, P4, P5, P6 and P7 who volunteered to be observed and took part in both pre and post intervention Focus Group interviews or 1-1 interviews. The classroom visits took place post intervention and post interviews in March/April 2017. One school placement tutor, one of the author’s PhD supervisors, volunteered to use the instrument. As previously discussed in Section 3.8.2, this could be seen as

compromising the research findings. However, as the results will reveal, this conflict of interest had no discernable impact on the integrity of the outcomes.

The next section will give an overview of the structural and theoretical influences of the TRU project observation guide on the LNMTI classroom observation instrument.

7.3.3.1 TRU Observation Guide

Firstly, the TRU observation guide comprises of five dimensions:

1. *The Content* defined as the richness of the mathematics being taught and opportunities for students to make sense of the mathematics,
2. *Cognitive Demand* refers to how the students are facilitated to engage with challenging mathematical experiences,
3. *Equitable Access to Content* describes whole class' meaningful involvement in the learning,
4. *Agency, Ownership, and Identity* is used to describe the development of student positive, productive disposition towards the content,
5. *Formative Assessment* refers to the use of structured activities engaging the student in the learning process where understanding can be deepened and misconceptions can be addressed.

Similarly, Sections A, B, C, D in the LNMTI classroom observation instrument reflected the four fields in the LNMTI framework:

- A. *Numeracy in Mathematics Content Domain* refers to the mathematical content as represented by the Irish mathematics syllabuses at Junior/Leaving Certificate.
- B. *Literacy and Numeracy Cognitive processes* is the extent to which students have opportunities to grapple with and make sense of the mathematics being taught.
- C. *Mathematical Quality of Instruction for LNMTI* describes the enabling of literacy and numeracy skills development by the enactment of rich instruction while working with students and mathematics.

D. *Literacy Forms* are the various forms of communication in the classroom interaction to enable Literacy and Numeracy Skills development.

Secondly, like the TRU observation sheet, each section was visually displayed in a matrix. However, the TRU instrument made a point of not itemising elements. The teacher and observer for the TRU context are required to discuss in advance of the lesson the features to be addressed in the live lesson and evidence of student and teacher practice are recorded. In contrast, individual elements from the LNMTI framework were included in the LNMTI observation sheet as they defined LNMTI skills for an Irish classroom context, previously discussed in Section 4.4.2. The next section will describe specific features of the main components of the LNMTI observation instrument.

7.3.3.2 Section A: Numeracy in Mathematics Content Domain

Figure 7.3 shows Section A, the Numeracy in Mathematics Content domain and Section B, ‘Literacy and Numeracy Cognitive Processes’ of the observation instrument. Section A identified the five strands in the current Irish mathematics syllabuses for Junior and Leaving Certificate that are also included in Ireland’s national strategy’s numeracy definition. The school placement tutor was required to place a tick against the featured strand in the lesson. In addition, a cell was assigned to whether the lesson referenced a ‘real life context’ to further reflect the numeracy definition (Boaler 1994, Ngcobo and Julie 2012). The detail, length or relevance of this episode could be captured in the adjacent comment box. Unlike the TRU content definition, defined as ‘the richness of the mathematics being taught and opportunities for students to make sense of the mathematics’ (Schoenfeld and Teaching for Robust Understanding (TRU) Project 2016a, p.1), this section refers only to the category of content to be taught and its relevance to real life.

LNMTI Observation Sheet for the Planning and Enactment of Mathematics Lessons

Student: _____ **Class Group:** _____ **Date & Time:** _____

Section A: Numeracy in Mathematics Content Domain: the mathematical content as represented by the Irish mathematics syllabuses at Junior/Leaving Certificate

Strand 1 Probability and Statistics <input type="checkbox"/>	Strand 2 Geometry and Trigonometry <input type="checkbox"/>	Strand 3 Number <input type="checkbox"/>	Strand 4 Algebra <input type="checkbox"/>	Strand 5 Functions <input type="checkbox"/>
Reference to real life context	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	Comment		

Section B: Literacy and Numeracy Cognitive Processes: the extent to which students have opportunities to grapple with and make sense of the mathematics being taught

Understand	Use	Critically Appreciate
Acquisition of new mathematical Knowledge <input type="checkbox"/>	Application of new mathematical knowledge <input type="checkbox"/>	Recognise where mathematical reasoning can be used to solve problems <input type="checkbox"/>
Comment:		

Figure 7.3 Section A and Section B from the LNMTI classroom observation instrument

7.3.3.3 Section B: Literacy and Numeracy Cognitive Processes

As illustrated in Figure 7.3, Section B describes the ‘Literacy and Numeracy Cognitive Processes’, characterised in TRU as ‘cognitive demand’, comprising three levels: understand, use and critically appreciate. These reflect Maguire’s (2003) concept of numeracy, the literacy and numeracy definitions (Ireland, Department of Education and Skills 2011) and Bloom’s Taxonomy (Samo 2017). To clarify the cognitive demand, the learning objectives from the syllabus were referenced. For example, acquisition of new mathematical knowledge referred to the conceptual understanding objective in both the Leaving and Junior Certificate syllabuses (NCCA 2013a, 2013b). These objectives have been duplicated in the Junior Cycle mathematics specification (NCCA 2017). More than one box could be ticked here to reflect the lesson intention or cognitive demand of the lesson for the students. This section also contained a comment box to allow the school placement tutor to give a reason for choosing one category over another. For example, commenting on P2, the school placement tutor writes:

The primary focus of the lesson was on the acquisition of new knowledge relating to theoretical and experimental probability. This was undertaken using a coin tossing activity.

7.3.3.4 Section C: *MQI for LNMTI*

The next section, Section C, the *MQI for LNMTI* (Harvard Graduate School of Education 2018a) category from the LNMTI framework was aligned with the richness of mathematics element from the TRU content definition. Figure 7.4 lists the eight elements from this domain which is comprised of two selected dimensions from Mathematical Quality of Instruction (MQI) (*ibid*) ‘richness of mathematics’ and ‘working with students and mathematics’. See Section 4.4.2, for a detailed discussion on the relevance of the following features to LNMTI.

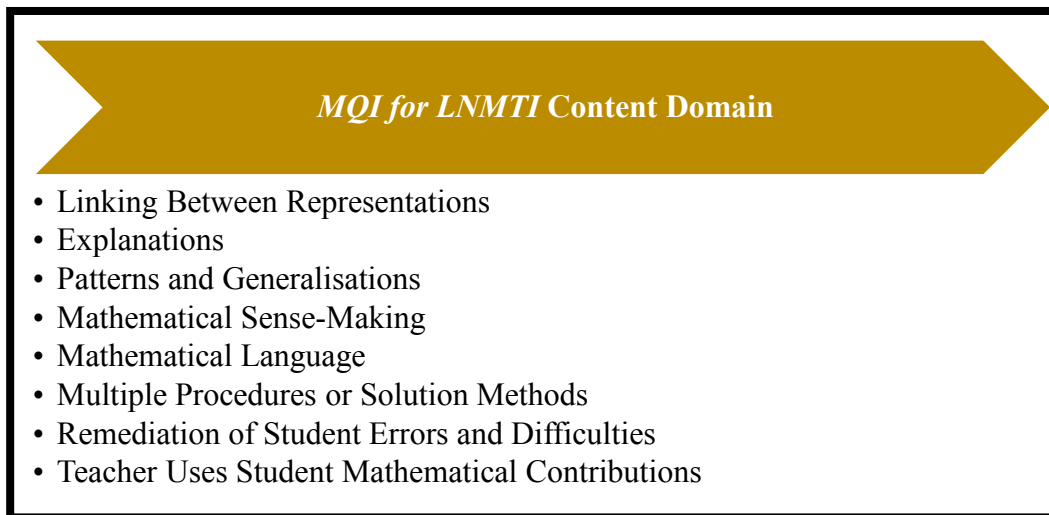


Figure 7.4 Eight elements from MQI for LNMTI (Harvard University 2018b)

‘Mathematical Sense Making’ features as a stand-alone element in the LNMTI framework whereas the TRU guide encompasses it as a feature under ‘cognitive demand’. Also, the category ‘Explanations’ listed above is identified in the ‘Agency, Ownership and Identity’ domain of TRU.

To facilitate the ‘easy to use’ principle for the LNMTI classroom observation sheet, six elements were assigned to the *MQI for LNMTI* (Harvard Graduate School of Education 2018b) category as shown in Figure 7.5 but the ‘Mathematical Language’ and ‘Teacher Uses Student Mathematical Contributions’ features were listed under the *Literacy Form* domain, Section D, for reasons outlined in the next section.

MQI for LNMTI	Explanation		Comment
<i>Linking between representations</i>	Captures an explicit link made by the teacher between representations of mathematical ideas. This can be visual (using graphs or physical models), numerical, algebraic, verbal	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
<i>Explanations</i>	Describes the way in which the teacher (a) answers a question of clarification from a student or (b) explains why a mathematics procedure, solution etc. works	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
<i>Mathematical Sense-Making</i>	Focuses on the importance of number sense, reasonableness of a solution, mathematical definitions in the teaching and learning interaction	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
<i>Multiple Procedures or solution methods</i>	The presence of different mathematical approaches to solving a problem from teacher or student	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
<i>Patterns and generalisations</i>	Describes the examination of an example and its development into a generalisation from teacher or student	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
<i>Remediation of student errors and difficulties</i>	Captures the way in which a teacher deals with a student misconception and difficulty with an area of mathematics	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	

Figure 7.5 Section C from LNMTI classroom observation instrument

7.3.3.5 Section D: Literacy Forms

Finally, the fourth field, Section D, comprised of dimensions from the literacy form domain defined as the various forms of communication in the classroom interaction to enable literacy and numeracy skills development. As set out in Figure 7.6, spoken language was the principle element in the domain and was sub-divided into:

- a) *Mathematical Language* that focused on the fluency of the teacher and the support given by the teacher to develop mathematical language use in the students.
- b) *Teacher Uses Student Mathematical Contributions* that described how the teacher managed student answers/responses/work to advance the mathematics under instruction.

Three other literacy forms were also included:

- *Printed Text* defined as the use of textbook, hand-out etc. non-digital representational systems for mathematics such as hand drawn graphs, diagrams, tables etc.
- *Digital Media* characterised as the use of PowerPoint, digital representational systems for mathematics (such as graphs/diagrams generated for example in GeoGebra), calculator and internet.
- *Broadcast Media* focused on the use of video, film, newspapers, television, and radio.

Highlighting the variety of platforms that exist to communicate ideas is reflected in the TRU dimension of ‘Equitable Access to Mathematics’. Ireland’s national literacy and numeracy strategy was built on values of equity and access to education for all citizens of Ireland (Ireland, Department of Education and Skills 2011). A variety of literacy forms such as digital and broadcast media facilitate engagement with mathematics for all as demonstrated by the work of Meyer (2009), previously discussed in Section 6.18.

Section D: Literacy Forms: the various forms of communication in the classroom interaction to enable Literacy and Numeracy Skills development

Literacy Form Domain	Explanation		Comment
Spoken Language (a) Mathematical language	Focuses on the fluency of the teacher and the support given by the teacher to develop mathematical language use in the students.	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
Spoken Language (b) Teacher uses student mathematical contributions	Describes how the teacher manages student answers/responses/work to advance the mathematics under instruction.	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
Printed Text	Focuses on the use of textbook, handout etc. non digital representational systems for mathematics such as hand drawn graphs, diagrams, tables etc	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
Digital Media	Use of PowerPoint, digital representational systems for mathematics (such as graphs/diagrams generated for example in GeoGebra) Calculator and internet	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	
Broadcast Media	Use of video, film, newspapers, radio, television	Present <input type="checkbox"/> Not Present <input type="checkbox"/>	

Figure 7.6 Section D from LNMTI classroom observation instrument

For the purposes of analysis of the enactment of LNMTI in a live classroom setting, the eight elements that were originally in the *MQI for LNMTI* from the LNMTI framework were selected and coded. To recap, these eight elements comprised of the six features from Section C and the two parts from the Spoken Language element in Section D from the LNMTI classroom observation instrument. An LNMTI classroom score was then calculated using frequencies and percentages. The other data that was collected from Section A, B and D was used to explore how content, cognitive demand and literacy forms supported the enactment of the LNMTI skills captured by the *MQI for LNMTI* elements.

In summation, the LNMTI classroom instrument was designed and constructed using the TRU observation guide as a theoretical and practical framework. It comprised of four sections derived from the LNMTI framework:

- A. Numeracy in Mathematics Content Domain,
- B. Literacy and Numeracy Cognitive Processes,
- C. Mathematical Quality of Instruction elements for LNMTI,
- D. Literacy Forms such as spoken language, printed text, digital and broadcast media.

The next section will describe the coding procedure for the LNMTI classroom observation.

7.3.4 Development of the Coding Procedure

Coding and categorisation of actions witnessed in the classroom was guided by the binary coding procedure, ‘Present’ or ‘Not Present’, used by the Learning Mathematics for Teaching (LMT) (2011) project. This simplified the assessment of the interaction for the user of the instrument to comply with the principle of an easy to use supplementary evaluation resource for school placement tutors. More sophisticated systems of classroom observation have been developed, pioneered by the work of Flanders (1962) whereby verbal interactions are classified into mutually exclusive categories every three seconds and the proportion of time allotted to each category is

measured. To collect evidence of the enactment of LNMTI in this preliminary stage of the classroom observation prototype, it was sufficient to acknowledge any aspect of the feature as 'Present', even if it was cursory or fleeting. The Learning for Mathematics Teaching (LMT) project (2011) observation protocol also included whether a specified feature of instruction was appropriate or not appropriate in the context. The author did not include this element in the final draft of the observation sheet, instead, like the structure of the TRU observation sheet (Schoenfeld and Teaching for Robust Understanding (TRU) Project 2016b, p.3) space for comment was provided which enabled the tutor to explain the reason for the occurrence or omission of a feature. For example, digital media is a literacy form specifically referenced in the literacy definition and consequently was a feature in the literacy form domain. However, for P3 the use of digital media was 'Not Present' because as the school placement tutor explained, the classroom the pre-service teacher was operating out of did not have digital media facilities. Similarly, this participant scored 'Not Present' on the 'patterns and generalisations' element which forms part of the numeracy definition and is a characteristic of Mathematical Quality of Instruction (MQI). But in this case, the school placement tutor noted the lesson would have benefitted from the inclusion of this component.

The author developed a second layer of coding if evidence of the LNMTI element was appropriate for the lesson and was coded 'Present'. Similar to Bakker and Van Eerde's (2015) data analysis tools, the author identified four categories from the 'Present' coding procedure. These categories were labelled 'Present', 'P+', 'P++' and 'P+++', indicating various levels of engagement with a LNMTI element. 'Present' indicated that the element was observed to a limited extent; P+ that it was observed to some extent; P++ showed the element was observed extensively and P+++ that it was observed to a high degree (Coben *et al.* 2007). The following example using the 'linking between representations' feature will illustrate this coding process. Linking between representations is an element identified in the Junior Cycle mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017) and it was defined in the LNMTI observation sheet as an explicit link made by the teacher

between representations of mathematical ideas. This can be visual (using graphs or physical models), numerical, algebraic or verbal.

The school placement tutor determined that all seven participant lessons would have benefited from showcasing the ‘linking between representations’ element. All of the strands from the syllabus except Number, Strand 3, featured in the participants LNMTI classroom assessment. P2 and P7 facilitated lessons on probability (strand 1), P1 and P5 taught content from geometry (strand 2) and P3, P4 and P6 used algebra and functions (strands 4 and 5) for their lesson content. Out of the seven participants only one was assigned a ‘Not Present’. Out of the six who demonstrated ‘linking between representations’ in the lesson, the comment made by the school placement tutor determined whether the participant remained in the basic category, ‘Present’, or whether there was sufficient evidence to secure a higher grade: P+, P++ or P+++.

P1, P5 and P7 were coded ‘Present’ because there was an over-reliance on visual/verbal representations and given the nature of the topic under instruction, there was ‘greater scope to incorporate others’ (Observation notes from school placement tutor, Appendix II). P2, P6 and P4 obtained P+, P++ and P+++ respectively. The comments from the school placement tutor to support this grading allocation were as follows:

P2 utilised a table/visual representation to capture the data being collated by the students.

P6 Very good - used images of a pattern, converted info to a table and then plotted graph.

P4 incorporated multiple representations moving from tables to graphing and incorporating algebraic representations.

As illustrated, each of these three participants included a representation other than visual and verbal. P2 included a table representation, P6 had the previous two and added a graphical representation and finally P4 explored multiple representations of the topic in detail.

Therefore, to code for the enactment of LNMTI, a five point scale was applied: ‘Not Present’, Present, P+, P++, P+++. The next section will describe the results from the

eight featured elements from Section C and part of Section D that evaluates pre-service teacher enactment of LNMTI skills in the classroom.

7.3.5 LNMTI Classroom Intervention: Eight Elements

Figure 7.7 presents individual participant scores on a five point scale for the following eight elements:

- (1) Teacher uses Student Mathematical Contributions,
- (2) Mathematical Language,
- (3) Remediation of Student Error and Difficulties,
- (4) Patterns and Generalisations,
- (5) Multiple Procedures and Solution Methods,
- (6) Mathematical Sense Making,
- (7) Explanations,
- (8) Linking Between Representations.

The following example demonstrates how each bar in the bar graph was constructed for each participant. The percentages in the accompanying data table reference the frequency a participant demonstrated a *MQI for LNMTI* element as a percentage of the total. Table 7.6 presents the breakdown of percentage relative frequencies awarded to P1 as an example:

Table 7.6 Breakdown of % relative frequencies awarded to P1

<i>Five Point Scale</i>	Not Present	Present	P+	P++	P+++
<i>% Relative Frequency</i>	12.5%	50%	0%	37.5%	0%

As the bar graph with accompanying data table illustrates (Figure 7.7), six participants were assigned a ‘Not Present’ category for one or two elements with P3 acquiring a ‘Not Present’ on four. All participants obtained at least a ‘Present’ for four or more features. Three of the participants achieved a P+ grade for at least one and at most three elements whereas five of the participants achieved a P++ grade for two, three and five elements. P4 earned the highest award, P+++ for four elements.

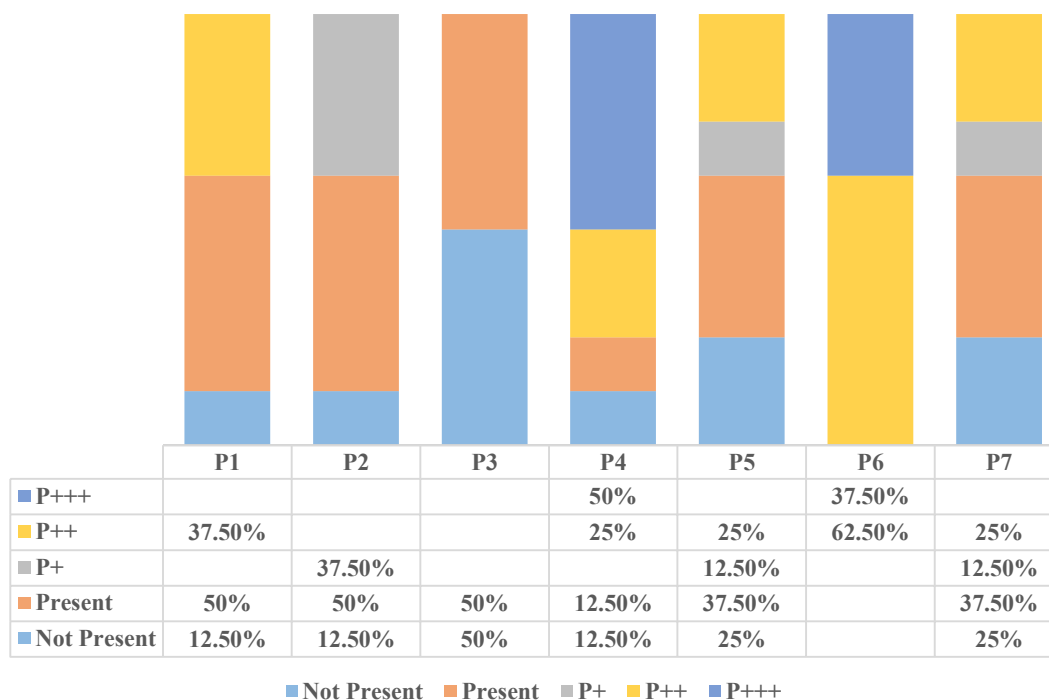


Figure 7.7 Participants' % acquisition of the eight elements from MQI for LNMTI

P6 was awarded either P++ or P+++ grades for all eight elements conveying a very high level of engagement with the LNMTI strategies. In contrast, only one participant, P3, did not obtain a grade higher than 'Present' and combined with 50% 'Not Present' grades, this performance overall demonstrates a poor enactment of LNMTI skills in the classroom setting. The school placement tutor remarked on P3's poor performance on *Mathematical Language*:

Poor use of mathematical language throughout the lesson and inaccuracies evident e.g. Teacher using 'equation' instead of 'expression'.

This signals the actual learning trajectory from LNMTI workshop 4 that specifically addressed the difference between expression and equation for this participant was not achieved. Evidence from the post workshop evaluation showed participants were confused about the correct use of basic mathematical language for algebra (Section 6.4.2.1). However, despite having exposure to expert models (the author and maths

methods tutor) P3 did not make sufficient progress in this observed lesson. This is a worrying outcome and points to a need for increased attention in the area of basic mathematical language to support pre-service teachers in this endeavour. Consequently, for this participant, Renkl *et al.*'s (1996) research on knowledge compartmentalisation is relevant. They cite a similar example in a university course where students positioned the knowledge of a concept next to a flawed conception of that knowledge and similar to this example, the 'incorrect' knowledge was not replaced. They also note when the knowledge is to be applied the 'old' deficient knowledge is deployed (p.117). In addition, evidence from the post workshop focus interview convey this participant found the workshops very valuable culminating in the comment:

P3: I would have actually have preferred to have all of this (the LNMTI workshops) last year.

This comment indicates that more time, reflection and practice was required by this participant to support her learning (Perez-Sanagustín *et al.* 2015). This invites future research into assessment procedures for the LNMTI module.

7.3.5.1 Frequency of 'Present' Awards on Eight Elements

To understand the strengths and weaknesses in the enactment of the skills, an analysis of the 'Present' grades, meaning the demonstration of the element irrespective of standard shown, across the eight elements was conducted. Figure 7.8 illustrates the distribution of 'Present' awards across the eight elements.

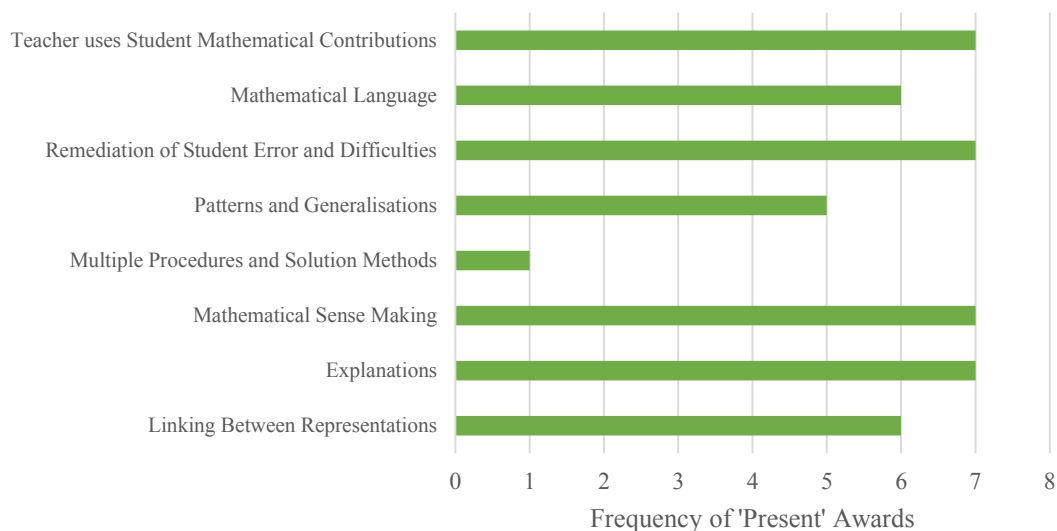


Figure 7.8 Frequency of 'Present' awards across the eight elements from MQI for LNMTI

These results show that all of the participants enacted a variety of skills associated with LNMTI. The element participants did not exhibit, except for P6, was 'multiple procedures and solution methods' defined as the presence of different mathematical approaches to solving a problem from teacher or student. The school placement tutor remarked that opportunities did exist for participants to employ this element but they were not exploited. In the case of P6, the tutor noted:

Students were encouraged to develop their own pattern and build on this. Very skilful use of different patterns by the teacher.

To enhance an understanding of the quality of enactment of the elements, an analysis of the distribution of hierarchal 'Present' categories, P+, P++, P+++ was applied to the participant performances. This produced interesting results presented in Figure 7.9.

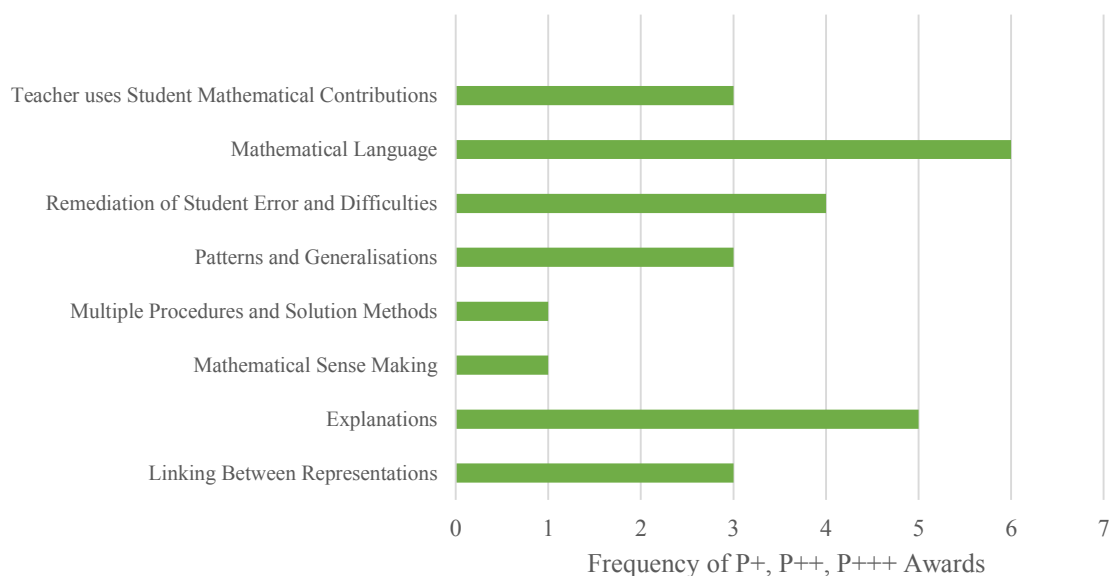


Figure 7.9 Frequency of P+, P++, P+++ awards across the eight elements from MQI for LNMTI

Apart from P3, participants scored well in *Mathematical language* and *Explanations*. These two elements are associated with the literacy domain and the coding in each of these two domains generated a nearly perfect correlation, $r = 0.89$, illustrating that although these dimensions are treated separately, they have an internal coherence (Schoenfeld 2013). The high scoring in these dimensions also validate the emphasis placed by the university on the enactment of literacy skills in the classroom. The scoring for *Teacher Uses Mathematical Contributions*, *Remediation of Student Error and Difficulties*, *Patterns and Generalisations* and *Linking Between Representations* was lower, which ultimately reflected how successful or otherwise the participant was in executing the lesson overall. For example, P4 and P6 both attained P++ or P+++ in each of these four categories, whereas P1, P5 and P7 managed to demonstrate good proficiency in only one of these elements: *Patterns and Generalisations* for P1; *Remediation of Student Error and Difficulties* for both P5 and P7. However, the main weakness that is revealed in these results is in the enactment of the numeracy associated elements:

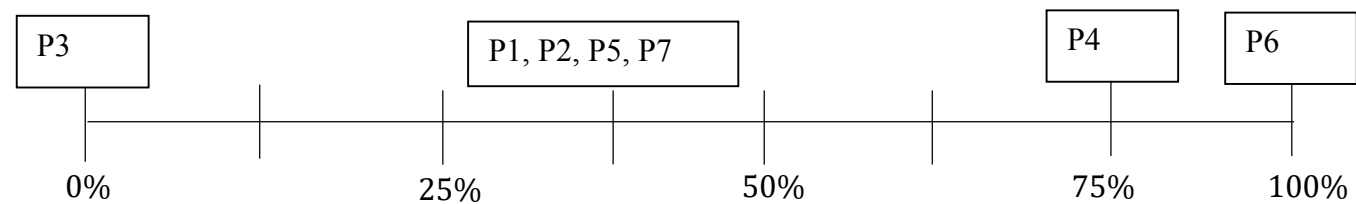
- *patterns and generalisations* which describes the examination of an example and its development into a generalisation from teacher or student,
- *mathematical sense making* which focuses on the importance of number sense, reasonableness of a solution, mathematical definitions in the teaching and learning interaction.

Three participants, P1, P4, and P6 displayed skills in facilitating patterns and generalisations, a specified element in the unifying strand of the Junior Cycle mathematics specification (National Council for Curriculum and Assessment (NCCA) 2017). However, it is reported that teachers find the skill of synthesising and generalising in a mathematics lesson very challenging (Swan, n.d.). The *mathematical sense making* definition has similar features to the ‘conceptual understanding’ learning objective in Irish post-primary mathematics syllabuses that refers to ‘comprehension of mathematics concepts, operations and relations’ (National Council for Curriculum and Assessment (NCCA) 2013a, 2013b). Only one participant, P6 attained an award greater than ‘Present’ in *mathematical sense making*. This is a surprising and important result indicating that this element needs specific attention and focus when planning mathematics pedagogy learning outcomes. In summary, these findings suggest by using the lens of numeracy for evaluating pre-service teachers, an interesting perspective on the strengths and weaknesses in mathematics teaching has been enabled.

7.3.6 Overall Score

Similar to the analysis of grade allocations for the LNMTI survey where the percentage of awards in ‘Mid’ and ‘High’ categories were calculated to generate a LNMTI Adequate score, a participant LNMTI classroom observation score was computed by finding the percentage of awards in the P+, P++ and P+++ categories. As shown in Figure 7.10, results were compared with LNMTI survey scores to investigate if any interesting patterns could be uncovered. Data on P5 is missing from the LNMTI survey results as this participant did not complete the survey.

Participant LNMTI Classroom Observation score: percentage of awards in P+, P++, P+++



Participant knowledge of LNMTI Adequate score from the LNMTI survey

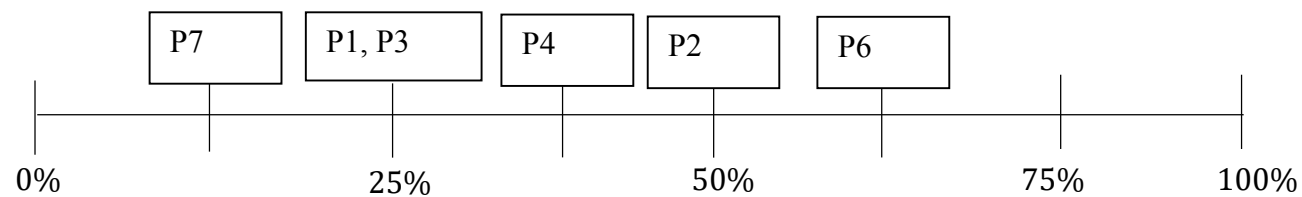


Figure 7.10 Participants' LNMTI survey and LNMTI classroom observation scores

By firstly examining the extreme cases of P3 and P6 who obtained 0% and 100% respectively in the LNMTI Classroom Observation, it is clear the highest LNMTI survey score with 62.5% was also achieved by P6 and P3 attained one of the lowest survey scores. This is consistent with research in the correspondence between Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) (Blömeke *et al.* 2011). However, no other discernable pattern existed amongst the other participants. Instead, the other data collected from the LNMTI observation sheet was examined to investigate if it was possible to offer other explanations for the LNMTI classroom observation results. To explore other links and connections, the next section will analyse data from:

- Section A: Numeracy in Mathematics Content Domain,
- Section B: Literacy and Numeracy Cognitive Processes,
- Section D: Literacy Form Domain expect for the Spoken Language element which has already been discussed under the *MQI for LNMTI* heading.

7.3.6.1 Section A: Numeracy in Mathematics Content Domain

There was a variety of mathematics content taught in the seven lessons. P2 and P7 explored content from strand 1, Statistics and Probability. P2 facilitated a lesson on how experimental probabilities approach the theoretical probability as the number of trials increase while P7's lesson involved the use of set theory in probability. P1 explored the geometry of angles from strand 2. From the LNMTI survey results, P2 and P7 scored a 'Not Present' in the probability domain and P1 scored a 'Low' on ***Geometry Definitions***, which exposed weaknesses in all of the participants' basic content knowledge of geometry. P5 who didn't take the survey also facilitated a lesson on geometry. P1, P2, P5 and P7 scored 37.5% in the LNMTI classroom observation. These results suggest a link exists between an average performance in the classroom when facilitating lessons in probability and geometry. By contrast, the abstract algebra question in the LNMTI survey generated the highest scores overall while the results on the pattern based approach to algebra were weaker (Section 5.3.3). P6 scored a

‘Low’ on this aspect of algebra but successfully facilitated an investigation of linear patterns in the classroom (P6=100%). P4 scored a ‘Mid’ in this LNMTI survey question and also successfully facilitated a class on the exploration of exponential functions (P4=75%). It was noted by the school placement tutor that both these participants’ lessons involved the students constructing their own learning. This result raises more questions than it answers. Does certain mathematical content align better with more student centred pedagogies? In contrast, P3 focused on simplifying algebraic expressions attaining the lowest classroom score (P3=0%) but the highest score in the LNMTI survey on a similar item. The school placement tutor reported that the focus on P3’s class centred on completing procedural algebraic questions. This example demonstrates that more emphasis on facilitating lessons where the learning outcome is skills based should be addressed. The next section will examine the cognitive processes element in the LNMTI observation sheet.

7.3.6.2 Section B: Literacy and Numeracy Cognitive processes

In the LNMTI classroom observation sheet, the three cognitive processes, to understand, to use and to apply were listed as learning goals. Out of the seven participants who were observed only one participant, P6, who attained the highest LNMTI classroom score, fulfilled all three. The learning goal prepared by the following four participants, P1, P2, P4, P5 was for students to ‘understand’ and finally, P3 and P7, facilitated lessons whereby students had to ‘use’ prior knowledge in a new context. The least successful lessons (P3=0% and P7=37%) had students use prior knowledge in a new context. Oyinloye and Popoola’s (2013) study reports on the necessity to access students’ prior knowledge in mathematics before adapting it to a new situation, however, P7’s lesson relied too heavily on the prior knowledge aspect of the lesson which was very teacher led. Contrastingly, P3’s lesson provided ‘very little scaffolding of student learning’ (school placement tutor observation notes) suggesting the prior knowledge was not accessed sufficiently for the students. These findings suggest the management of prior knowledge in the dynamic environment of the classroom requires further research to fully understand its impact on the enactment of LNMTI skills in the classroom environment. However, what is certain is that

teachers who differentiate learning and access all cognitive processes for students, as was the case for P6, create ‘powerful classroom environments’ (Schoenfeld 2013, p.618).

7.3.6.3 Section D: Literacy Form Domain

The literacy form domain is comprised of four elements: Spoken Language, Printed Text, Digital Media and Broadcast Media. As previously mentioned, the Spoken Language dimension has been analysed as part of the eight elements in the *MQI for LNMTI* domain. Therefore, for this analysis, the focus will be on the other three. Figure 7.11 presents an overview of these literacy forms that were utilised in the lessons.

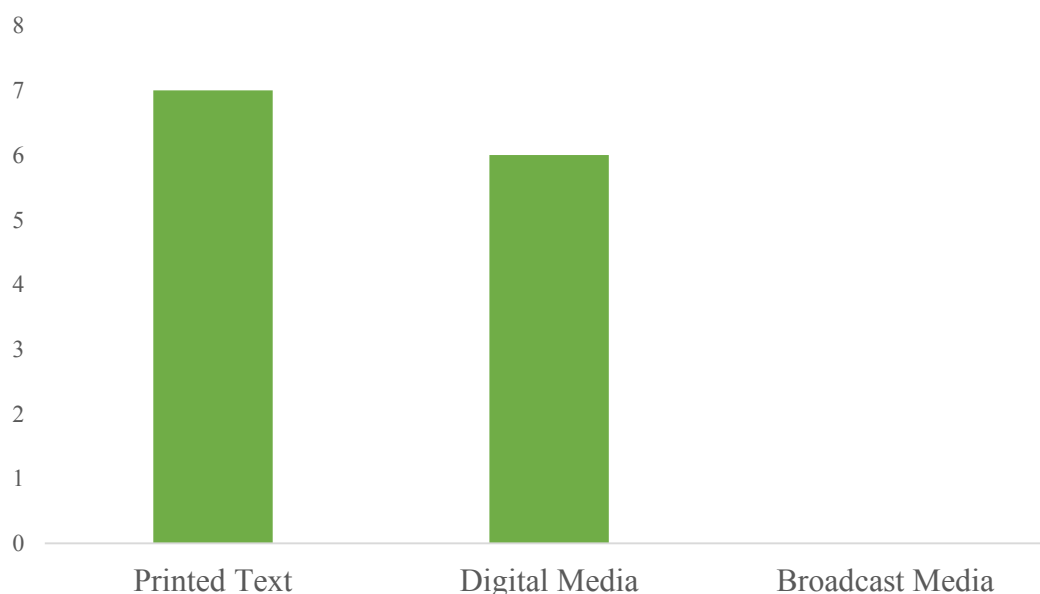


Figure 7.11 Literacy forms used by participants during LNMTI classroom observation

It is not surprising printed text forms dominate the post-primary mathematics classroom in Ireland (Maths Development Team, n.d.a). However, a notable difference observed in these lessons was all but one of the participants had a pre-prepared hand-out with the aim to support the student learning. In the most successful lesson, P6, students were building their learning on a previous hand-out that had been completed. In the least successful lesson, P3, students had to complete a hand-out comprised of a set of procedural questions. Similarly, all participants, except P3, used

digital media by incorporating PowerPoint slides to guide the lesson. The classroom P3 was assigned had no ICT infrastructure. Inoue-Smith (2016) reports that using PowerPoint does not automatically support learning, however learners tend to prefer PowerPoint over blackboard/whiteboard presentations which could improve productive disposition, one of the learning objectives from the mathematics syllabuses/specifications (National Council for Curriculum and Assessment (NCCA) 2013a, 2013b).

As can be seen from Figure 7.11, none of the participants used broadcast media resources defined as the use of video, film, newspapers, radio and television. This was the literacy form focus of workshop 6, the last workshop of the LNMTI module and the classroom visits took place the following week. However, in the workshop evaluation, the participants made positive comments about the usability and the appeal of this literacy form to engage students in learning by relating the mathematics to real life contexts (Section 6.4.1.2). In addition, it is interesting to note that apart from one participant, P4, who applied the lesson content to real-life contexts, for the other participants this feature was considered limited or lacking by the school placement tutor. Although this is beyond the scope of this study, further research into the use of broadcast media as a conduit to link mathematics to real life contexts for Irish students would be recommended.

7.3.7 Limitations of the instrument

The study examined participants' enactment of LNMTI in Irish post-primary classroom settings, therefore its generalisability to other contexts, cultures and settings has its limitations. Secondly, the school placement tutor who coded the observations and made observation notes on the lesson attended all of the LNMTI workshops delivered by the author and was familiar with the concepts, content and approach. In addition, a non-random sample of seven Irish female teachers is not generalisable to a larger population. However, the results demonstrate that despite the instruments' limitations, it is a foundation on which a more thorough and detailed classroom observation instrument for the Irish educational context could be developed.

7.4 Conclusion

This chapter set out to describe the evaluative and reflective cycle of the EDR process for the LNMTI module and answer the four research questions that related to LNMTI effectiveness, LNMTI alignment with principles of design, measuring LNMTI in the classroom environment and the enactment of LNMTI skills. The intervention was highly valued for improving knowledge of literacy and numeracy for mathematics teaching, which was clearly supported by the findings from the survey and observation notes from the expert appraisal. The results of this evaluation also demonstrated numeracy is still an issue for participants, although engaging with syllabus documents was a key factor in supporting learning in this domain. The second major finding emerged from the focus group and interview data that showed authentic learning principles were mostly achieved. Areas for improvement reside in developing a more structured collaborative practice. Also, the LNMTI classroom observation instrument appeared to be effective as a diagnostic tool in identifying problem areas in mathematics teaching, however further research is necessary to validate this assertion. Finally, the LNMTI classroom observation instrument showed variations existed in the enactment of these skills in the classroom setting. Although content knowledge was a factor in some cases, specific features that appeared to support successful LNMTI implementation were: *Teacher Uses Mathematical Contributions, Remediation of Student Error and Difficulties, Patterns and Generalisations and Linking between Representations*.

Chapter 8: Conclusion, Contribution and Future Work

8.1 Introduction

The key objective of this study, using Bernstein's (1990) Social Construction of Pedagogic Discourse as the orientating framework, was to understand the message of educational reform being transmitted and to investigate the readiness of pre-service teachers of mathematics for an educational environment in Ireland with increased demands, challenges and diversification. Other Irish context research has explored supporting pre-service teachers in this climate of reform such as Guerin (2017) who researched developing pre-service teachers' problem solving strategies, Walsh (2015) who focused on pre-service teachers' knowledge of Trigonometry and Prendergast *et al.* (2013) investigated the mathematics content knowledge of pre-service teachers for teaching. What is novel about the author's Irish based research for mathematics teaching is the use of literacy and numeracy as a lens to explore mathematics content knowledge and pedagogical knowledge of pre-service teachers in preparation for these reforms.

The National Strategy for Literacy and Numeracy expresses the values of the Irish education system where sophisticated literacy and numeracy skills are the desired outcome for all children and young people to thrive in an equal and just society (Ireland, Department of Education and Skills 2011). Philosophers such as Dewey espoused the idea of the democracy of education in 1916 as did Irish educator Pádraic Pearse who fought for Irish independence from British rule (Dewey 1916; Pearse 1916; Cronin 2016). However, O'Donoghue *et al.*'s study (2016) describes Irish Free State education provision in Ireland where ideals of democratic learning and learner agency did not feature because teachers lacked basic qualifications in subject content knowledge notwithstanding pedagogical knowledge. Nowadays, ideals and objectives are being translated into action plans with targets (Galvin 2018). Ireland's Action Plan for Education 2018 promotes the purpose of education as targeting excellence in the form of standards and performance indicators (Ireland, Department of Education and Skills 2018).

Figure 8.1 illustrates the current policy documents that mandate systemic reforms for pre-service teachers of mathematics:

- The current syllabuses for Junior Certificate and Leaving Certificate (2010),
- The national strategy for literacy and numeracy (2011),
- Teaching council curricular registration requirements (2013),
- Junior Cycle Framework (2015),
- Junior Cycle Mathematics specification (2017).

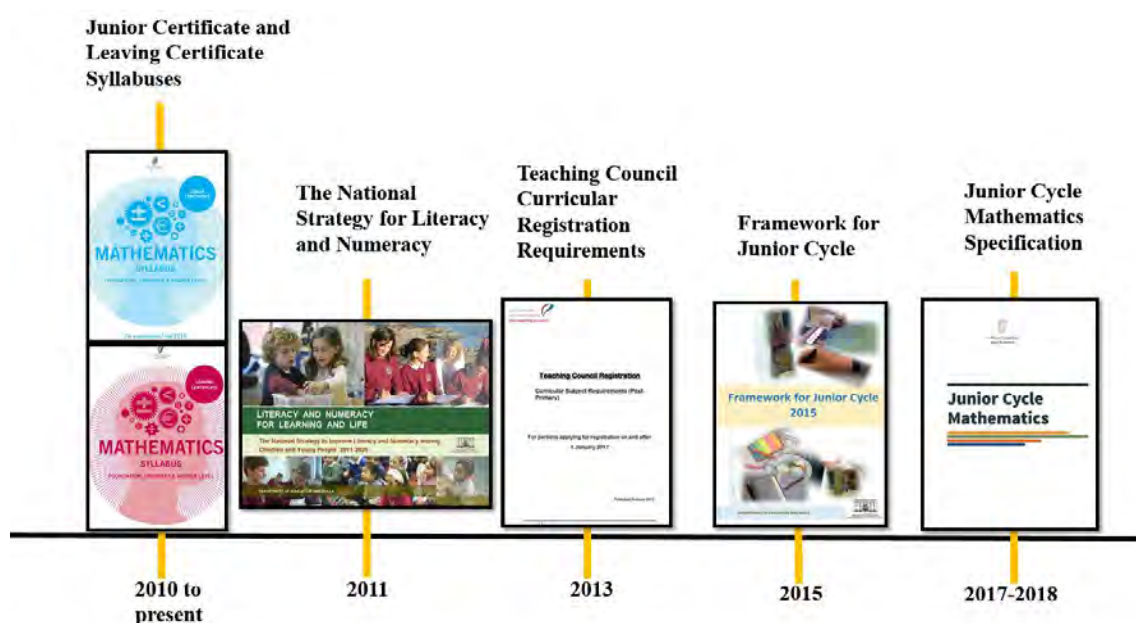


Figure 8.1 Educational reform documents from 2010 to present

Aligned with these mandated reforms is a change in the description of the mathematics curriculum document from ‘syllabus’ to ‘specification’ which highlights a process oriented approach that underpins the development of mathematical competences such as making connections, linking representations etc. (Rabbin 2002). In addition, school self-evaluation, introduced in 2012 by the Department of Education and Skills is a process whereby schools are expected to engage in internal reviews to promote school improvements using a quality framework (Ireland, Department of Education and Skills 2016b). As part of current developments, ‘Teachers’ individual practice’ (*ibid*, p.17) is listed as an element in the quality framework with the standards focusing on teacher

subject content knowledge and pedagogical knowledge. While teaching as a profession in Ireland has traditionally attracted high calibre candidates, the issue of teacher supply and ‘out of field’ teachers is now a contemporary issue (Ní Ríordáin and Hannigan 2011; Gleeson 2012; Teaching Council 2015).

In the next section, an overview of the study is outlined and the research questions that guided the study are revisited.

8.2 Study Overview and Research Questions Revisited

As previously mentioned, the research is located in Ireland during a period of unprecedented educational reform where government policy and initiatives aim to position the nation’s education system as the best in Europe by 2026 (Ireland, Department of Education and Skills 2017d). Despite the local context, the author believes the study has implications for policy makers in education internationally in communicating reform to practitioners as well as supporting agents to translate the reform into practical actions.

To address a key element of the reform agenda, the author set out to determine the meaning and application of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI). This required providing grounded and practical solutions by producing ‘tools’ or ‘artefacts’ (Akkerman *et al.* 2011). Therefore, Educational Design Research was chosen as the research methodology whose dual purpose is to produce educational artefacts and make a contribution to theory (McKenney and Reeves 2012). For the remainder of the chapter, the acronym LNMTI will be used in place of ‘Literacy and Numeracy for Mathematics Teaching in Ireland’.

The study was comprised of four main phases. Phase 1 analysed and explored literacy and numeracy for mathematics teaching where a definition and framework for this domain was constructed. This work has made a contribution to research in the area of the relationship between literacy and numeracy as well as their relationship to the discipline of mathematics and mathematics pedagogy. This was followed by the design of an instrument for assessing pre-service teachers of mathematics knowledge

and skill level for this domain (Chapter 4). Phase 2 focused on investigating the readiness of pre-service teachers of mathematics who were nearing completion of a two year Professional Master of Education (PME) programme to teach in Ireland to facilitate LNMTI (Chapter 5). Phase 3 arose as significant knowledge gaps were identified in participants' knowledge of LNMTI therefore an intervention which comprised of six workshops was designed and constructed (Chapter 6). Evaluating the impact of the intervention from a static (learning in workshops) and dynamic (enacting in classrooms) perspective was the focus of Phase 4 which encompassed the design and construction of the LNMTI classroom observation sheet (Chapter 7). In Sections 8.2.1 to 8.2.4 the four research questions are listed and the main findings of these phases are summarised. In the final Sections, 8.3 to 8.7, the contribution to theory building in relation to LNMTI, contributions, recommendations and future work are addressed.

8.2.1. How is LNMTI Defined?

Assessments for literacy and numeracy are now part of Australia and UK entry requirements and/or teacher accreditation (Australia, Department of Education and Training 2016; United Kingdom, Department for Education 2018). This study explored the potential and value of formally assessing pre-service teachers of mathematics in literacy and numeracy skills for teaching. The key challenge at this phase of the research was in defining what was to be assessed. Consequently, the Research Question for phase 1 of the study was:

***RQ1:** How is Literacy and Numeracy for Mathematics Teaching in Ireland defined and characterised as a knowledge domain for assessing pre-service teachers of mathematics in order to monitor how well prepared they are to meet the challenges of current educational reforms in Ireland?*

In addressing this research question, the author began by exploring the domain of literacy and numeracy from an Irish context. Certain literature was identified such as O'Breacháin and O'Toole's (2013) criticism of the increased emphasis on literacy and numeracy at the expense of other curricular areas in the Irish primary curriculum and Shiel and Gilleece's (2015) comparison of Irish primary, post-primary and adult

performances in literacy and numeracy with Northern Ireland, a province of the United Kingdom that shares a border with Ireland. In addition, Flanagan (2016) explored the implementation of numeracy strategies in the post-primary classroom to improve student disposition towards mathematics. However, no research was found that focused on supporting pre-service teachers to facilitate literacy and numeracy skills in the post-primary mathematics classroom. Consequently, a gap in the literature was exposed and the construct was then decomposed into three main dimensions: numeracy, literacy and mathematics teaching. Relevant literature from these dimensions that aligned with the Irish context were then read and analysed.

With the various definitions of numeracy that exist and the lack of agreement on numeracy as a concept (Coben 2002), an acceptance of this status quo to allow for adaptations based on context and human capital was proposed by Withnall (1994). Moreover, O'Donoghue (2002) and Maguire (2003) discussed the issue of numeracy as a shortcoming following a mathematics education in Ireland and internationally. Despite conceptions of numeracy possessing different attributes, the author, like Kaye (2009), considered two types of definitions: firstly, numeracy as a cognitive process with literacy and mathematical elements featured in international definitions by Penny (1984), Grawe *et al.* (2010) and OECD (2013b), secondly, numeracy as a construct distinct from mathematics presented by Johnston (1994, cited in Cummings 1996) and Steen (2001). Collectively, the definitions highlighted cognitive, mathematical and literacy elements. The synthesis of literacy and numeracy components in the numeracy definitions was an important finding leading the author to conclude that contrary to the National Council for Curriculum and Assessment's (2015) bid to separate these domains, the recoupling of literacy and numeracy resulted in a more coherent definition and framework for LNMTI. This synthesis is also implied from Australians Goos *et al.*'s (2012) 'context' component from their model for numeracy in the 21st century. Influenced by Steen's (2001) conceptualisation, a person's ability to make informed decisions, interpret data and/or think in a logical way across many domains and disciplines aligns with the cognitive processes from the literacy definition of understand, use and critically appreciate. In addition, the model contains the

element ‘tools’ which can be representational, physical and digital. This idea is manifested in the Irish literacy definition as ‘forms of communications’ whereas in the 21st century model numeracy model, it is described as ‘mediators of mathematical thinking and action’ (Goos *et al.* 2012, p.4).

In order to understand literacy and numeracy for mathematics, the definitions of literacy and numeracy from the national strategy were mapped, using domain and taxonomic coding (Saldana 2009), to the post-primary mathematics syllabuses to identify correspondences or deviations. The findings clearly showed that the content of the texts were perfectly aligned and the Project Maths syllabus was an appropriate vehicle to disseminate literacy and numeracy outcomes. From the work on content mapping and the literature review, the author noticed that the learning objectives outlined in Irish mathematics syllabuses were sourced from American research documents written by Kilpatrick *et al.* (2001). This directed the research to American curriculum documents to investigate similarities and differences to the Irish context. Once more there was a strong connection between the U.S. Common Core standards for mathematical practice and Ireland’s problem solving learning outcomes (Common Core State Standards Initiative 2016).

However, references to pedagogical mechanisms in official definitions of numeracy listed previously were absent. This study sought to remedy this problem by analysing the literature that focused on good mathematics teaching. Several studies highlighted mathematical content knowledge as a vital element to the enterprise of good mathematics teaching (Nodding 1990; Ball, Hill and Bass 2005; Ball, Thames and Phelps 2008; O’Meara 2011; Learning Mathematics for Teaching (LMT) 2011). Literacy components such as mathematical language and explanations also emerged (Kitchen *et al.* 2007; Pollak 2007; Schleppegrell 2007; Ní Ríordáin 2011; Usiskin 2012). A broader perspective was adapted by the Harvard University Center for Education Policy Research group (2018a) who identified specific dimensions in the dynamic classroom environment that encompassed quality mathematics instruction referred to as the Mathematical Quality of Instruction (MQI) framework (Hill 2011; Learning Mathematics for Teaching (LMT) 2011;). This was developed to observe

teachers teaching the U.S. Common Core standards of mathematics which, as previously mentioned, aligned with curriculum objectives in Irish syllabuses. The dimensions, *Richness of Mathematics* and *Working with Students and Mathematics* from the Mathematical Quality of Instruction (MQI) framework provided performance indicators for pre-service teachers in the enactment of literacy and numeracy skills in the mathematics classroom. Therefore, this study can be considered as an extension of the work of the Learning for Mathematics Teaching (LMT) project (2011) in such a way that the following four classification mechanisms for quality mathematics teaching at post-primary level in Ireland were identified in terms of literacy and numeracy:

- Literacy cognitive processes,
- Numeracy in mathematics content,
- Mathematical Quality of Instruction for literacy and numeracy,
- Literacy forms.

Moreover, these four categories formed the domains of the LNMTI framework that guided the development of a new survey used to assess pre-service teachers' knowledge of LNMTI, with combined mathematics content knowledge and pedagogy elements. This subscribes to similar work done by international researchers such as Ball, Thames and Phelps (2008), Hill *et al.* (2008), Learning Mathematics for Teaching (LMT) project (2011); Prescott *et al.* (2013) and Adler (2017) who have developed explicit criteria to evaluate content knowledge and pedagogy dimensions. However, the author's study makes an original contribution to research on the work of mathematics teaching by exploring the potential of criteria specific literacy and numeracy elements as evaluative tools in this domain.

8.2.2. Are Pre-Service Teachers Prepared?

For phase 2 of the study, the Research Question asked was:

RQ2: *Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms?*

This is the first study to challenge accepted beliefs that mathematics teachers are automatically numerate. To collect data on pre-service teachers' knowledge of LNMTI, a pen and paper survey, devised by the author, was conducted. Eleven pre-service teachers of mathematics who attended one out of seven universities in Ireland offering a Professional Master of Education (PME) in 2015-2017 answered the survey. All of the universities were contacted regarding number of Professional Master of Education students and literacy and numeracy provision. There was a university response rate of over 85%. The number of PME course applicants ranged from 4 to 16 and literacy and numeracy provision was embedded in the mathematics module in all of the universities. However, what is interesting about the participants recruited for this study is that all of the participants as part of their Initial Teacher Education programme had to attend an additional 12 x 2 hour lecture module on Literacy and Numeracy Development in the Post-Primary Classroom. They also had to submit two reflective research papers on each of the domains of literacy and numeracy. Therefore, the participants from this university appeared to have greater access to a literacy and numeracy education suggesting the findings from this study should make an important contribution to the field of mathematics education in Ireland.

The participants comprised of two mutually exclusive groups: individuals who completed their post-primary studies of mathematics pre-2012 or the year of the first Project Maths national assessment in 2012. Comparing the results of these two cohorts did not show any one group was advantaged in any way. Therefore the study did not detect any evidence that for these participants experiencing the reformed mathematics assessment in 2012 made them better prepared to teach the reformed mathematics syllabus (Barnes 2000). Other relevant findings showed participants overestimated their basic knowledge of mathematics for teaching particularly in the area of geometry and probability and statistics. For example, from the four point survey assessment rubric that categorised responses as 'not present', 'low', 'mid' and 'high', 82% of responses were graded in the 'Not Present-Low' category for a question that asked participants to identify and define geometric shapes from a photograph of a tiled floor. The 'Not Present-Low' category was described in the study as LNTMI Inadequate,

indicating the participants did not have sufficient knowledge to implement the reform agenda. Although all of the participants rated this question as ‘Easy’, only 36% of the participants defined a square correctly. In contrast, abstract mathematics knowledge for teaching questions in the survey were better answered in comparison to questions situated in a real life context. Also, difficulties in developing the formulation of generalisations in a mathematics teaching context was evident coupled with poor use of mathematical language.

A focus group interview was conducted following the pen and paper survey and the most surprising aspect of this data was participants’ lack of understanding about numeracy and the relationship between numeracy and post-primary mathematics. In contrast, participants displayed more confidence in the domain of literacy. The data suggested there was an assumption made by university tutors that pre-service teacher of mathematics were implementing numeracy because mathematics was their specialist subject, consequently in the numeracy module the emphasis fell on finding numeracy moments in other subject areas. This assumption is also endorsed in the Junior Cycle Framework where the critical role of mathematics in numeracy development is outlined (Ireland, Department of Education and Skills 2015a). Facilitating problem-solving lessons was also highlighted as a priority issue.

8.2.3. Teaching and Learning Module

Increasing pre-service mathematics teachers’ competency levels has been a primary focus of mathematics education research (Shulman 1986; Ball *et. al* 2008). New models and methods, grounded in practice, have been developed to contribute to better Continuous Professional Development opportunities for improved student outcomes (Hill 2011; Adler and Ronda 2015). Consequently, this study is contributing to literature in this area by generating data to answer the third Research Question:

RQ3: What content and characteristics should a teaching and learning module have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?

This phase explored ways to address the knowledge gaps that were identified from the LNMTI survey and focus group interview through the design, construction and implementation of the LNMTI module. Using the author's LNMTI framework, design principles by Herrington and Oliver (2000) and the construct of the hypothetical learning trajectory (HLT) (Simon 1995; Dierdorp 2013; Bakker and Van Eerde 2015) to map the learning goals, sequences and processes, a six workshop module was developed and delivered by the author to support participants' knowledge growth in this domain. Each workshop targeted a priority issue identified in the survey and the focus group interview as summarised in the previous section. Four of the workshops focused on a strand content area: Geometry and Trigonometry, Number, Algebra and Functions and Statistics and Probability. The first workshop examined the concept of LNMTI and while problem solving strategies networked all of the workshops, a specific problem solving workshop was designed to present different literacy formats to introduce a problem. A matrix was constructed to chart the development of the six workshops in adhering to the instructional design principles of authentic learning in the design phase (Section 6.4.3). Following each workshop, evaluative survey data was gathered from participants.

The data from individual workshop evaluations indicated that the teaching and learning module was authentic and valid for participants' daily practice in the classroom but challenges for future degree programmes in mathematics as well as pre-service teacher of mathematics education programmes were revealed. All of the participants found the individual workshops either good, very good or excellent. Out of a possible twelve participants, the minimum number of evaluations received for each workshop was 9 and the maximum was 12. By focusing on the 'excellent' response frequency, the most successful workshops were Algebra (72%), Statistics and Probability (82%) and Problem Solving (89%). The most striking result from the Algebra workshop was 37% of participants found the 'letters as numbers' concept as the most useful part of the workshop whereas the Problem Solving workshop was valued for addressing student disposition through the use of digital and broadcast media to introduce a problem. The excellent response rate for the Number workshop,

in comparison, was low at 46%. The understanding of the ratio concept in number was the central focus of this workshop but making connections between ratio and similarity in geometry was also presented. The geometry workshop with a 50% excellent response rate was valued by the participants for making connections within the subject of mathematics as well as linking geometry to real life contexts. A challenge for participants, however, was in describing ratios in words and notation in the effort to address common student errors. For example, one participant commented:

W3.9: Although I got what you meant with describing the difference between $\frac{2}{3}$ of the sweets and $2:3$ of her sweets, I'd be afraid highlighting [it as] that could confuse them.

As the understanding of part-part and part-whole relationships is fundamental in developing mathematical competency (Nagar *et al.* 2015), an amendment was made to this task. This was achieved by introducing Chinn and Ashcroft's (2007) work on special learning difficulties in mathematics who advise against 'dumbing down' of mathematics for students, irrespective of their ability, and focus instead on maintaining mathematical integrity at all times.

In addition, workshop 1 that dealt with conceptualising LNMTI had relatively low excellent response rates (37%), yet the exploration of official documents such as mathematics syllabus, national examination assessment questions as instruments to convey literacy and numeracy for mathematics teaching was highly valued by the participants indicating the instructional design principles for authentic contexts were fulfilled. This finding also demonstrates that official curriculum and assessment materials are a rich and valuable source for pre-service teachers to guide lesson planning and lesson enactment.

In developing the Statistics and Probability workshop for LNMTI, the author deviated from the LNMTI framework and used Adler's (2017) Mathematical Discourse for Instruction (MDI) framework to guide the development of the workshop. The MDI framework was developed to assess teaching strategies that would be classified as basic and procedural in more sophisticated frameworks such as Mathematical Quality of Instruction (Harvard University Center for Education Policy Research 2018a; Adler

2015). Considering the results from the LNMTI survey for the statistics and probability questions, where 70% of the participants misread a bar chart and 91% of them failed to correct a student misconception in a basic probability problem, it was important to address fundamental content knowledge and pedagogical strategies in these domains. The data showed that going back to basics such as defining probability and understanding the concept of mean in statistics as a measure of central tendency was hugely beneficial for the participants. These results indicate that basic knowledge of probability and statistics even amongst individuals who have a third level degree in mathematics and mathematics related modules with a probability and statistics component, as was the case for these participants, should not be assumed. In addition, considering the positive response by participants to using the calculator as a digital media tool, more time should be given to this aspect in future iterations of this module.

8.2.4. Evaluation of Module

There is general agreement amongst researchers that pre-service teachers of mathematics need to understand mathematics well to teach mathematics well (Hill *et al.* 2008; Ma 2010; Rowland and Ruthven 2011). Morris *et al.* (2009) argue that to improve the teaching of mathematics, pre-service teachers need to focus on learning skills to ‘unpack mathematical learning goals’ (p.518). This study has re-cast the problem of preparation of teachers in a different way in terms of literacy and numeracy knowledge for teaching mathematics. Therefore, the final phase of the study is contributing to literature on curriculum content for Initial Teacher Education programmes (Griffin 1990; Li and Castro Superfine 2018) guided by the fourth Research Question:

***RQ4:** What is the potential of a teaching and learning module based on authentic design principles to support pre-service teachers of mathematics develop an understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that implementation of the domain in the dynamic classroom environment is enabled?*

Data was collected to measure the potential of the teaching and learning module for supporting pre-service teachers’ knowledge of LNMTI by an evaluative pen and paper

survey taken by participants following the conclusion of the workshops. All of the participants, $n = 12$, reported a gain in understanding of the LNMTI construct with 92% claiming their literacy and numeracy outcomes in their lesson planning were better and the same percentage said the workshops had influenced their teaching. The module seemed effective in improving participants' understanding of numeracy and its relationship with mathematics, which was supported by presenting mathematical concepts in abstract as well as real life contexts and making connections between mathematical ideas from different strands. However, one participant, P11, did acknowledge a surprising but interesting outcome about numeracy and mathematics that demands more attention in future research:

P11: I feel it is assumed maths teachers are automatically good at numeracy and we might not be on all aspects.

Secondly, data from the post intervention focus group and two 1-1 interviews showed that principles for authentic learning were achieved or partially achieved. The findings show principles underpinning authentic contexts and tasks and reflections that enabled knowledge transfer and abstract thought were achieved. On the other hand, opportunities for more reflection to monitor and deepen the learner experience were not sufficient from a coaching and collaborative construction of knowledge perspective (Herrington and Oliver 2000). In future iterations of this module adaptations will be necessary to accommodate this aspect.

For an authentic assessment of participants' learning, a classroom observation instrument was designed and constructed by the author (Herrington and Oliver 2000). The design was guided by Schoenfeld and the Teaching for Robust Understanding Project (TRU) (2016), a framework that characterises effective classroom teaching and learning. The participants were involved in one observation. The data indicated that the facilitation of a lesson on material from the reformed syllabus, the functions based approach to algebra, catered for a diverse learning experience for the students because the teacher used a range of teaching and learning interactions, however, the question that explored student misconceptions on this topic was poorly answered in the LNMTI survey. In contrast, the participant who facilitated a lesson on algebraic skills was less

successful even though a question on explaining an algebraic procedure in the LNMTI survey generated the best responses from the participants. Consequently, there is a need to review the alignment between the LNMTI survey questions and the expectations in a teaching performance from the LNMTI classroom observation sheet. In addition, one observation of participants' classroom performance is insufficient to draw conclusions about the enactment of LNMTI skills, however, it might seem reasonable to conclude more research is needed to identify specific LNMTI standards in the teaching of procedural skills versus problem solving.

Moreover, despite these limitations, what was interesting in the data was numeracy based processes in the LNMTI observation sheet were poorly executed. For instance, *mathematical sense making* which focuses on the teachers' ability to highlight number sense, mathematical definitions and the reasonableness of a solution was underdeveloped in all of the lessons except one participant, P6. This finding suggests by evaluating pre-service teachers' performance in the classroom from a numeracy perspective, areas of strengths and weaknesses in mathematics teaching can be determined.

The next section will describe the study's contribution to theory in the form of design principles.

8.3 Policy to Practice Design Principles

A conclusion to Educational Design Research methodology is a contribution to theory (McKenney and Reeves 2012) and for this study, the author developed a set of design principles for the purpose of making the tacit knowledge or messages in policy documents explicit for the dynamic classroom environment. These design principles are not specific to the constructs of literacy and numeracy and would be useful to national and international organisations set with the task of designing Initial Teacher Education modules or delivering Continuous Professional Development or as a starting point to decode and transmit messages from educational policy to classroom practice. To put a structure on these general rules, the author returned to Bernstein's

(1990) three tiered hierarchical organisational structure for Social Construction of Pedagogic Discourse from the study's theoretical framework: production, recontextualisation and secondary (Kirk and Macdonald 2001). In brief, production is the source of the message conceived by government agencies, recontextualisation is the translation of the message for distribution and secondary is the transmission of the message. The author identified seven design principles listed in Table 8.1 and explanations of each principle are given below:

Table 8.1 Policy to practice design principles

1	Decode and classify the message	Primary (Production)
2	Create mapping relations for clarity and coherence	
3	Define the construct	
4	Develop a construct framework	
5	Produce tools/artefacts to assess knowledge and fill knowledge gaps of construct	Recontextualisation
6	Delivery of artefacts by practising classroom experts	
7	Assess performance in classroom setting	Secondary (Reproduction)

8.3.1 Primary (Production)

(1) Decode and classify the message

The first step in bridging policy and practice is to decode the message from policy documents using domain and taxonomic coding processes (Saldana 2009). This coding procedure was specifically developed to investigate tacit knowledge embedded in cultural expressions of human behaviour and experiences (*ibid*). For this study the decoding process involved taking the definitions of literacy and numeracy from Ireland's national strategy to improve literacy and numeracy levels in children and young people in Ireland and apply the coding process. Literacy and numeracy were treated as two different domains, but as elements from the definitions were categorised, it soon became apparent that the cognitive processes classifications from both definitions were similar. In addition, the content from the numeracy domain aligned

with the mathematics syllabus strand classifications. Moreover, as numeracy was regarded as an instrument of communication (Penny 1984), the text formats from the literacy taxonomies (spoken language, printed text, digital and broadcast media) provided frames to communicate numeracy moments.

(2) Creating mapping relations for clarity and coherence

Once the central message is decoded and classified, similar messages in curriculum texts should be networked to the central message for clarity and coherence. For this study, mapping the literacy and numeracy definitions to the mathematics syllabuses illustrated these documents were the appropriate vehicle for disseminating messages about literacy and numeracy for mathematics. Messages embedded in the mathematics syllabuses were mapped with U.S. Common Core standards (Common Core State Standards Initiative 2016) because they shared identical learning objectives. A perspective on teaching surfaced because of the U.S. connection. This led to identifying the Mathematical Quality of Instruction (Harvard University Center for Education Policy Research 2018a) framework as a schema suitable for the Irish context.

(3) Develop a construct

A statement that provides the exact meaning of the construct to enable a common understanding is necessary to provide clarity and direction for any actions that are to be taken. For this study, the definition of Literacy and Numeracy for Mathematics Teaching in Ireland (LNMTI) was as follows:

Literacy and numeracy for mathematics teaching in Ireland (LNMTI) encompasses the ability to understand, use and critically appreciate number, algebra and functions, geometry and trigonometry, statistics and probability in various forms including spoken language, print, broadcast media and digital media as well as the enactment of rich instruction while working with students and mathematics.

This definition aligned with literacy and numeracy definitions for the Irish context but it also aligned with international conceptualisations of numeracy that included cognitive mathematical and communication elements.

(4) Develop a construct framework

A definition of a construct encompasses basic domains (Landauer and Rowlands 2001). However, a domain requires content collections of descriptive elements for further clarity and coherence. The construct framework presents what features of the construct are significant and what skills can be measured for future assessments for learning tasks. For instance, in this study, the domains included: cognition, content, pedagogy and text form. For the cognitive dimension, the classified elements were ‘understand’, ‘use’ and ‘critically appreciate’ which reflected Maguire’s (2003) concept of the numeracy continuum and the literacy and numeracy definitions in the study’s theoretical framework (Ireland, Department of Education and Skills 2011).

8.3.2 Recontextualisation

(5) Produce tools/artefacts to assess knowledge and fill knowledge gaps of construct

Assessing how past learning can be applied to a new construct enables the evaluation of the message being transmitted and whether future learning capabilities are required. For this study, the LNMTI survey served as a diagnostic tool for assessing participants’ LNMTI knowledge. Serious issues related to content knowledge of mathematics surfaced particularly in relation to statistics and probability (Section 5.5.4). The results of the survey directly informed the development of the LNMTI workshops, a teaching and learning module, developed to expand the learning capabilities of participants for LNMTI.

(6) Delivery of knowledge gap artefact by practising classroom experts

Using an expert from the classroom to deliver the educational artefact designed to fill knowledge gaps is based on the principle that learning is a process of assimilation from

doing and apprenticeship from experienced others (Lave and Wenger 1991). The goal of educational reform is to produce change in practice and improve learning outputs but if practising teachers don't see the educational value in a reform agenda, it will not be implemented (Akkerman *et al.* 2011). In contrast, Kirk and MacDonald (2001) argue that by empowering the teacher voice in the message transmission, this discordant relationship between the intended curriculum and the implemented curriculum is challenged. The author of this study is a practising teacher who facilitated all of the workshops in the LNMTI module and one participant remarked on the value of this principle:

P1: I really liked it in terms of the fact you are a teacher and you're in the classroom and it was really nice to have it coming from someone who is teaching day in day out. I think what you went through was practical.

8.3.3 Secondary

(7) Assess performance in classroom setting

What counts as the message to be transmitted requires an assessment of whether what was intended to be communicated was successful or not. For this study, this assessment was recorded in the LNMTI observation sheet that could potentially offer feedback to support pre-service teachers and guide instruction for better learner outcomes. This approach is supported by situated learning theory (Lave and Wenger 1991), where pre-service teachers are in the role of learner with legitimate peripheral participation. This means a learner begins on the periphery of the knowledge community he/she wishes to become part of and full membership is enabled by fluently engaging in and practising activities endorsed by the community (Morrell 2003).

These seven principles offer a roadmap to navigate the process of disseminating messages of educational reform to pre-service teachers who are preparing for professional practice. The road map has three major signposts provided by Bernstein's (1990) Social Construction of Pedagogic Discourse: primary, recontextualisation and secondary. In the primary phase the message is prepared for transmission by employing design principles of decoding, mapping and defining. The

recontextualisation phase encompasses design principles that require educational artefacts to assess prior learning and prepare for future learning. The provision of new learning should be provided by a classroom expert who has embraced the reform agenda to improve the chances of knowledge transfer. The final design principle on assessment should be applied in the secondary phase to evaluate if the intended message has been transmitted effectively by the pre-service teacher.

8.4 Contributions

From an international perspective, the relationship between literacy and numeracy has been addressed. Numeracy has been defined as a subset of literacy (Cummings 1996), as a reflection of literacy (Central Advisory Council for Education (England) 1959, as a domain independent of literacy (National Council for Curriculum and Assessment (NCCA) (2015a). In contrast, this study has identified the symbiotic relationship between literacy and numeracy where one domain cannot exist without the other in the work of mathematics teaching. The LNMTI definition and framework were developed to document this relationship. The LNMTI survey, module and classroom observation sheet made the relationship between literacy, numeracy and mathematics teaching visible and explicit.

The contribution of this study locally is extensive. Ireland is embracing major reforms in teaching and learning, curriculum and assessment, STEM education and digital learning. The author's study has contributed to all of the elements on the education reform agenda in Ireland as well as the field of mathematics education internationally. Specifically, the study's findings suggest the educational artefacts, illustrated in Figure 8.2, are practical and useable for the Irish context in promoting excellent teaching standards. In addition, improving teacher evaluation is an international issue (Cohen and Goldhaber 2016). The LNMTI classroom observation instrument for pre-service teachers was designed to produce authentic feedback to reflect the reform agenda in Ireland. The next section will outline the study's contributions to various themes published in Irish education documents: The Professional Development Service for

Teachers Strategic Plan, STEM education: implementation plan 2017-2019, TIMSS 2015 in Ireland and the Digital Strategy for Schools 2015-2020.

2015-2016

- Understand
- Use
- Critically Appreciate
- Number
- Algebra and Functions
- Geometry and Trigonometry
- Probability and Statistics
- Synthesis and Problem Solving
- Linking between representations
- Explanations
- Patterns and Generalisations
- Mathematical 'Sense-Making'
- Mathematical Language
- Multiple Representations of Solution Methods
- Remediation for Student Entry and Difficulties
- Teacher-Learner Studies
- Mathematical Contributions
- Spoken Language
- Printed Text
- Digital Media
- Broadcast Media

LNMtI Definition

Literacy and numeracy for mathematics teaching in Ireland (LNMtI) encompasses the ability to understand, use and critically appreciate number, algebra and functions, geometry & trigonometry, probability and statistics in various forms including spoken language, print, broadcast media and digital media as well as the enactment of rich instruction while working with students and mathematics.

LNMtI survey and Assessment Rubric

University of Limerick
UNIVERSITY OF LIMERICK
DEPARTMENT OF MATHEMATICS & STATISTICS
Literacy and Numeracy for Mathematics Teaching in Ireland Survey
Eimer, J. (2016)

LNMtI Intervention

Literacy & Numeracy for Mathematics Teaching in Ireland Workshop 1
Bernie O'Donoghue

LNMtI Classroom Observation Sheet

LNMtI Classroom Observation Sheet for the Training and Enactment of Mathematics Lecturers
Module: _____ Date: _____ Date & Time: _____
Section 1: Planning for Mathematics Learning (20%)
Section 2: Enactment of Mathematics Learning (80%)
Section 3: Reflection and Evaluation (20%)

2017-2018

293

The first key strategy in The Professional Development Service for Teachers Strategic Plan 2015-2020 (PDST 2017) focuses on providing high quality continuous professional development for teachers, underpinned by current research, which affords maximum impact in Irish classrooms. The author's work contributes directly to this aim. Secondly the STEM Education Implementation Plan 2017-2019 (Ireland, Department of Education and Skills, n.d.) identifies shortcomings in student learning of science, technology, engineering and mathematics. The findings from the study showed participation in the LNMTI module and the design and construction of educational artefacts produced by the author contributed to improving mathematics teaching and learning. Also, the mathematics curriculum at primary level is currently under review with a final draft specification for 4 – 8 year olds and the commencement of a draft for 8 -12 year olds due in the third quarter of 2018. The author's work would be relevant to the senior primary group as a bridging reference to demonstrate how the strand structure and pedagogical components at Junior Cycle is reflected in the proposed reform in the primary school mathematics curriculum.

TIMSS 2015 in Ireland assessed mathematics and science competencies in fourth class, aged 9-10 years at primary level and second year aged 13-14 years at post-primary level where weaknesses in algebra and geometry were evident. The findings from this study corroborate weaknesses in the teaching and learning of geometry as a result of poor content knowledge in this area. However, this study also demonstrated pre-service teachers' presented weaknesses in content knowledge of statistics and probability which differs from the results in TIMSS 2015. Irish second year students displayed strengths in this area (Clerkin *et al.* 2016) similar to results from state examinations in 2015 when Irish 15 year olds performed well on that topic (Ireland, State Examinations Commission 2016a). Notwithstanding, while the findings of this study are a commentary on the participants' major difficulties in basic statistics and probability understanding in this study, they mirror those of previous studies: Murphy *et al.* (2011) and Fitzmaurice *et al.* (2014). Consequently, the author advises universities to establish protocols to support pre-service teachers' content knowledge for Statistics and Probability.

The Digital Strategy for Schools 2015-2020 to enhance teaching, learning and assessment is also supported by the educational artefacts designed and constructed for this study. Digital media is a component part of the LNMTI definition and framework. The study found the most successful strategy was using the calculator as a digital tool in the classroom. It was referenced in over 33% of the study participant responses as a useful and worthwhile digital resource.

8.5 Recommendations

Recommendations based on the findings from the four main Research Questions are outlined in this section:

RQ1: *How is Literacy and Numeracy for Mathematics Teaching in Ireland defined and characterised as a knowledge domain for assessing pre-service teachers of mathematics in order to monitor how well prepared they are to meet the challenges of current educational reforms in Ireland?*

By regularly referencing the mathematics syllabus in the LNMTI module, participants of this study developed a greater understanding of literacy and numeracy in mathematics teaching. Therefore, a recommendation for Initial Teacher Education programmes would be mandatory study of specific subject syllabus/specification documents to identify pedagogical messages from source. In addition, curriculum materials should make visible and explicit connections between mathematical content and literacy and numeracy skills which could help address the difficulties that exist about a mathematics teacher's role in delivering literacy and numeracy skills in the classroom. For instance, recall the expert appraisal's comment on mathematics teachers' perceptions about delivering literacy and numeracy in a mathematics classroom whereby literacy '*isn't part of a maths teacher's job*' (Appendix O) and numeracy '*is already been taught as part of the mathematics curriculum*' (*ibid.*).

RQ2: *Do pre-service teachers have the required mathematical content knowledge and pedagogical practice to implement Literacy and Numeracy for Mathematics Teaching in Ireland in their classrooms?*

Rather than assuming pre-service teachers of mathematics are implementing numeracy outcomes, the author recommends university Initial Teacher Education programmes review literacy and numeracy provision in the context of mathematics teaching. Part of this review could include the use of the LNMTI survey which aimed to assess participant strengths and difficulties in this domain. It is important for the future of mathematics teaching that prospective mathematics teachers can identify numeracy moments in their teaching but the first step in achieving this is for them to understand what numeracy means in the context of the mathematics classroom and for them to feel confident in their own numeracy ability to implement the reform agenda.

RQ3: What content and characteristics should a teaching and learning module have that supports pre-service teachers of mathematics develop a deeper understanding of Literacy and Numeracy for Mathematics Teaching in Ireland?

The authentic learning experiences as a primary characteristic of the LNMTI module was valued by the participants. A key aspect of the authentic experience was the facilitation of the module by a practising classroom-based teacher. Therefore, a recommendation would be to involve more classroom-based practitioners in the curriculum planning of Initial Teacher programmes to facilitate in a more shared understanding of reform from multiple perspectives. Furthermore, a dedicated module on supporting pre-service teachers' problem solving abilities similar to the work developed by Guerin (2017) should be included in teacher education programmes and/or undergraduate mathematics degree programme.

RQ4: What is the potential of a teaching and learning module based on authentic design principles to support pre-service teachers of mathematics develop an understanding of Literacy and Numeracy for Mathematics Teaching in Ireland in such a way that implementation of the domain in the dynamic classroom environment is enabled?

The LNMTI classroom observation sheet could be adapted and used by school placement tutors to support the assessment of pre-service teachers. It provides a set of specific indicators that contribute to effective classroom practice underpinned by theory and empirically tested. With the advent of Junior Cycle mathematics in Irish post-primary schools in 2018, universities need to strongly support pre-service

teachers prepare for common level classroom based assessments in mathematics. Universities should consider, as part of the school placement assessment protocols, observing pre-service teachers facilitate and assess classroom based assessment practices.

8.6 Future Work

There is much scope to continue explorations into the research topic of interest: LNMTI. The research participants were chosen because they completed a generic education module in literacy and numeracy unlike the other universities who offer similar Professional Masters of Education programmes. Testing pre-service teachers from other universities whose literacy and numeracy education is embedded in the mathematics education modules would provide richer information on the effectiveness of engaging pre-service teachers in a mathematics specific module for literacy and numeracy. In addition, there is an opportunity to extend this research into a longitudinal study to explore whether the participants' repeated classroom practice teaching improved their literacy and numeracy skills for the mathematics classroom.

Secondly, Leaving Certificate candidates who completed a five year post-primary cycle in 2017 and those who did six years in 2018 were the first cohort to experience the fully implemented Project Maths syllabus at both Junior Certificate and Leaving Certificate Level. Therefore, further research should be done with individuals from this cohort who opt for teaching mathematics as a career to investigate if the new assessment and proposed teaching methodologies will have an impact on their teaching methodologies.

There is also much scope to develop the classroom observation sheet for mathematics supervisors' tutors in universities. Although the instrument was developed for identifying literacy and numeracy moments in mathematics classroom, its utility as a generic tool for evaluating mathematics teaching could be explored. One such approach would be to share the contents of the classroom observation sheet with pre-service teachers and observe them in a lesson developing a skill. This approach aligns

with the TRU (2016) classroom observation structure where pre-determined elements to be enacted are discussed with the observer and the observed before the lesson.

Finally, based on the findings of this study where numeracy knowledge was assumed for mathematics graduates, this study could be extended to include investigating the numeracy knowledge of graduates of science, technology and engineering.

8.7 Final Comment

The author hopes this research will foster a new way of thinking about integrating literacy and numeracy practices in classrooms for all subject areas. It is important to remember in the effort to improve education for all, we must clarify the messages for improvement. This idea and process of the author's study can be summarised by Dewey's (1916) view on knowledge construction from his influential work on *Democracy and Education*:

No thought, no idea, can possibly be conveyed as an idea from one person to another only by wrestling with the conditions of the problem first hand, seeking and finding his[her] own way out, does he [she] think..... (p.166)

To convey an outcome of a long process of wrestling of ideas and thinking by the author about literacy and numeracy for mathematics teaching as a synthesised collection, lines from the W.B. Yeats' poem, 'Among School Children' were used: *how can we know the dancer from the dance?* (Yeats 1969). Consequently, as a conclusion to the thesis, the author saw it fitting to return to the poem where Yeats in his role as a school inspector in the new Irish Free State described a visit to what was then described as a progressive school in Ireland in 1926:

*I walk through the long schoolroom questioning;
A kind old nun in a white hood replies;
The children learn to cipher and to sing,
To study reading-books and history,
To cut and sew, be neat in everything
In the best modern way—*

What will Irish classrooms look like in 2026? Will we meet the targets and achieve the performances set out in action plan documents and national strategies? The answer is yet to come but to enable the ‘best modern way’ for the best education, pre-service teachers must be explicitly supported in practical ways to implement with integrity the intended outcomes for all children and young people in the promotion of a just and equal society.

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