

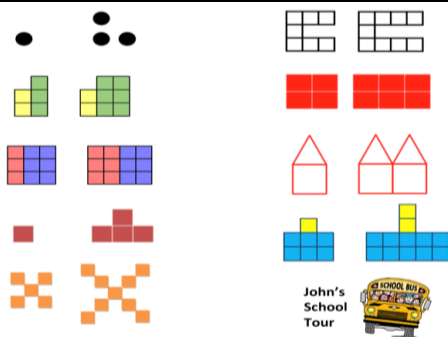
Literacy & Numeracy for Algebra Teaching in Ireland

Workshop 4

www.projectmaths.ie

SEC sample 2012

JC HL 2009



www.projectmaths.ie



- (i) Factorise $25x^2 - 36y^2$.
- (ii) Factorise $11x^2 + 75x - 14$.
- (iii) ✍ Simplify $(3 - 4x)^2 - (3 - 5x)^2$.

Bernie O'Donoghue

Learning Intentions

- Understand aspects of literacy and numeracy in the Algebra syllabus
- Introduce 'Explanations' and 'Multiple procedures and solution methods'
- Appreciate the way we teach algebra can enhance problem solving skills

Mapping Numeracy to Mathematics

Stand 3 Number

**Think and communicate
quantitatively**

Make sense of data

Strand 2 Geometry and Trigonometry

Have spatial awareness

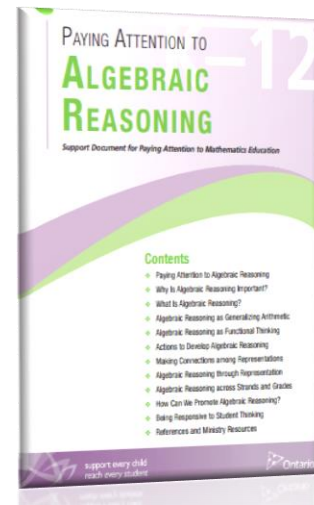
Strand 4&5 Algebra & Functions

**Understand patterns and
sequences**

Solve problems

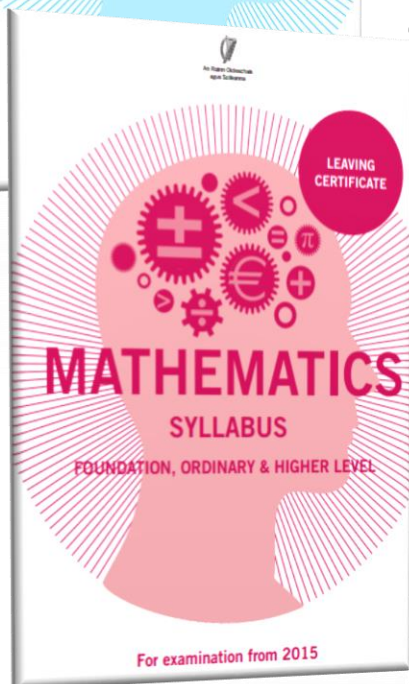
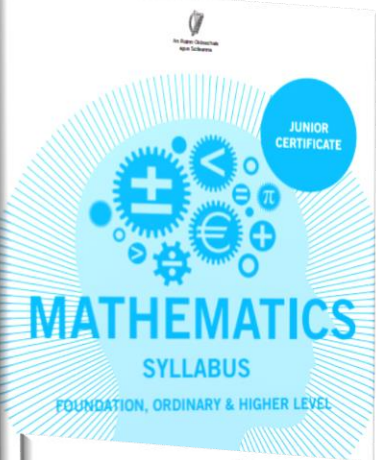
Numeracy: Algebraic Reasoning

- Everyone has the capacity to think algebraically because algebraic reasoning is essentially the way humans interact with the world. We look for patterns, pay attention to aspects of the pattern that are important, and then generalize from familiar to unfamiliar situations (p.3).



Algebra Syllabus

- algebra as generalised arithmetic
 - algebra as syntactically guided transformations of symbols
- collecting like terms, factoring, expanding, substituting, solving equations, and simplifying expressions.*



JCOL 2015: Algebra Skills

Paper	Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking (Examination)	Main Topic
1	1	20.3 / 25	81	6	3.1, 3.2 Number
1	2	13.7 / 20	69	14	3.5 Number
1	3	21.0 / 25	84	3	3.3, 3.6 Number
1	4	16.4 / 25	66	15	3.3, 3.4 Number
1	5	9.4 / 10	94	1	3.5 Number
1	6	18.7 / 20	94	2	3.3, 3.6 Number
1	7	8.3 / 20	42	23	5.2 Functions
1	8	14.3 / 20	72	11	4.5 Algebra 3.4 Number
1	9	12.7 / 40	32	24	4.6 Algebra
1	10	24.8 / 35	71	12	5.2, 5.3 Function 4.2 Algebra
1	11	17.0 / 40	43	22	4.7 Algebra
1	12	16.6 / 20	83	4	3.1, 3.6 Number 4.6 Algebra

JCHL 2015: Algebra Skills

Paper	Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking (Examination)	Main Topic ⁴
1	1	12·0 / 15	80	5	3.5 Number
1	2	10·9 / 15	73	11	3.3 Number
1	3	11·8 / 25	47	27	3.3 Number
1	4	6·0 / 10	60	20	5.1 Functions
1	5	12·0 / 15	80	4	4.6, 4.7 Algebra
1	6	22·9 / 30	76	9	5.2, 5.3 Functions
1	7	15·3 / 20	77	8	4.6 Algebra

Literacy: Expression and Equation

It is recommended that teachers and candidates give due attention to distinguishing between equations and expressions, and understanding why some procedures may validly be applied to one and not the other (p.21)

Chief Examiners Report JC 2015

Algebra Literacy: Quiz

Expression

Equation

Evaluate

Simplify

Solve

Algebra Literacy: Quiz

Expression

A group of terms separated by + and - signs

Equation

**An equation is a mathematical statement
in which two expressions are equal.**

Evaluate

Find the value of..... an expression

Simplify

Express the expression in simpler terms

Solve

Find a value(s) for an unknown quantity

Textbooks supporting Literacy and Numeracy

Mathematical Expressions

$$17 + 3$$

The sum of 17 and 3

$$12 - 8$$

The difference of 12 and 8

These are called **mathematical expressions**, or **expressions** for short.

$$17 + 3 = 20, 12 - 8 = 4.$$

20 is the **value** of the first expression.

4 is the **value** of the second expression.

Read the following expressions and find their values:

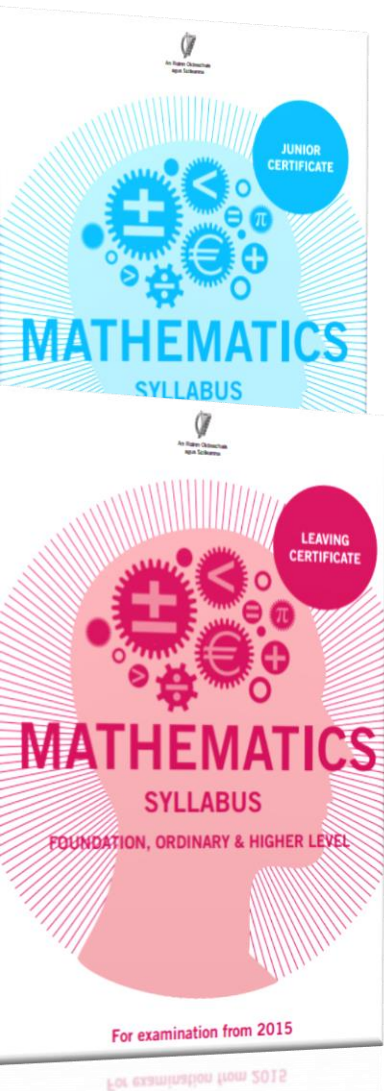
$$90 - 4 \quad 38 + 20 \quad 61 - 50 \quad 8 + 42 \quad 19 - 7$$

JCFL 2015: Literacy and Numeracy

Write each term into the table below to match it with the correct description.

Description	Term
The square root of the number.	\sqrt{x}
The number is divided by 2.	
The number is multiplied by 2.	
The number is added to 2.	
The number is squared .	

LC&JC Algebra: Objectives



- make use of letter symbols for numeric quantities
- emphasise relationship-based algebra
- connect graphical and symbolic representations of algebraic concepts
- use real life problems as vehicles to motivate the use of algebra and algebraic thinking
- use appropriate graphing technologies (calculators, computer software) throughout the strand activities

Question 2. The first three stages of a pattern are shown below. Each stage is made up of a certain number of white tiles and a certain number of shaded tiles.



Assuming the pattern continues:

Write down the relationship (using words or symbols) between the number of white tiles and the number of shaded tiles in any stage of the pattern. State clearly the meaning of any symbols where used.

Question 20. In a hospital, there are 5 times as many nurses as doctors. If we let n equal the number of nurses and d equal the number of doctors, which one of the following is correct?

- (a) $5n = d$ (b) $\frac{n}{d} = 5$ (c) $\frac{d}{n} = \frac{5}{1}$ (d) $n = 5 + d$

Question 5. Apples cost a cents each. Bananas cost b cents each.

If I buy 3 apples and 2 bananas, what does $3a + 2b$ represent?

- (a) 3 apples and 2 bananas (b) The total amount of fruit I buy (c) The total cost of 3 apples and 2 bananas

- (c) Niamh is in a clothes shop and has a voucher which she **must** use. The voucher gives a €10 reduction when more than €35 is spent. She also has €50 cash. Write down an inequality in x to show the range of money she could spend in the shop.

$$\boxed{} \leq x \leq \boxed{}$$

Write down an inequality in y to show the price range of articles she could buy.

$$\boxed{} \leq y \leq \boxed{}$$

Junior Certificate 2012 – sample paper

- make use of letter symbols for numeric quantities
- emphasise relationship-based algebra
- connect graphical and symbolic representations of algebraic concepts
- use real life problems as vehicles to motivate the use of algebra and algebraic thinking
- use appropriate graphing technologies (calculators, computer software) throughout the strand activities

NCCA, JC Syllabus, p.26

NCCA, LC Syllabus, p.35

A red banner with a dark grey outline, shaped like a wide arrow pointing to the right. It has a small notch on the left side.

Mathematical Quality of Instruction

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language
- Mathematical Sense Making
- Explanations
- Multiple Procedures or Solution Methods
- Remediation of Student Errors and Difficulties
- Teacher uses student mathematical contributions

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language
- Mathematical Sense Making
- Explanations
- Multiple Procedures and Solution Methods

Multiple Procedures and Solution Methods

$$3a + 2b$$

Multiple Procedures and Solution Methods/Explanations

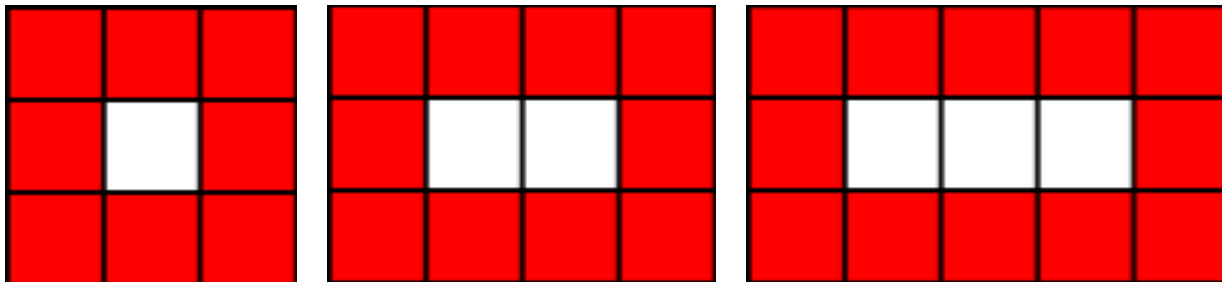
$$s = 6 + 2w$$

$$s = 8 + 2(w - 1)$$

$$s = 3(w + 2) - w$$

$$s = 2(w + 2) + 2$$

Where s = number of shaded tiles and
 w = number of white tiles.



Explanations: Scenario #1

*A toy train has a 100 cars. The 1st car is red, the 2nd is blue, the 3rd is yellow, the 4th is green, the 5th is red and 6th is blue, and so on.
What is the colour of the 39th car?*

- *Student: Yellow*
- *Teacher: Correct*

Explanations: Scenario #2

*A toy train has a 100 cars. The 1st car is red, the 2nd is blue, the 3rd is yellow, the 4th is green, the 5th is red and 6th is blue, and so on.
What is the colour of the 39th car?*

- *Student: Yellow*
- *Teacher: Explain how you got your answer*

Explanations: Scenario #2

*A toy train has a 100 cars. The 1st car is red, the 2nd is blue, the 3rd is yellow, the 4th is green, the 5th is red and 6th is blue, and so on.
What is the colour of the 39th car?*

- *Student: Yellow*
- *Teacher: Explain how you got your answer*
- *Student: 39 is a multiple of 3*

Question?

- How did you teach the rules of arithmetic for integers?

Arithmetic: Look for a Pattern

Table 1		
3 Times	Result	
3×4	$= 12$	Positive Answer
3×3	$= 9$	Positive Answer
3×2	$= 6$	+
3×1	$= 3$	+

- ☐ Make a Conjecture about
- ☐ Try out other Examples
- ☐ Generalise in words
- ☐ Make a Rule

Arithmetic: Look for a Pattern

Table 1

3 Times	Result	
3×4	$= 12$	Positive Answer
3×3	$= 9$	Positive Answer
3×2	$= 6$	+
3×1	$= 3$	+
3×0	$= 0$	0
$3 \times (-1)$	$= -3$	-
$3 \times (-2)$	$= -6$	-
$3 \times (-3)$	$= -9$	-
$3 \times (-4)$	$= -12$	-
$3 \times (-5)$	$= -15$	-

- ❑ Make a Conjecture about $n \in \mathbb{Z}$
- ❑ Try out other Examples
- ❑ Generalise in words
- ❑ Make a Rule

Collect Data/Look for Patterns

Table 1

3 Times	Result	
3×4	$= 12$	Positive Answer
3×3	$= 9$	Positive Answer
3×2	$=$	
3×1	$=$	
3×0	$= 0$	0
$3 \times (-1)$	$=$	
$3 \times (-2)$	$= -6$	
$3 \times (-3)$	$=$	
$3 \times (-4)$	$=$	
$3 \times (-5)$	$=$	

Table 2

5 Times	Result	
5×4	$= 20$	Positive Answer
5×3	$= 15$	Positive Answer
5×2	$=$	
5×1	$=$	
5×0	$= 0$	0
$5 \times (-1)$	$=$	
$5 \times (-2)$	$= -10$	
$5 \times (-3)$	$=$	
$5 \times (-4)$	$=$	
$5 \times (-5)$	$=$	

Literacy and Numeracy

Table 3		
-2 Times	Result	
-2×4	$= -8$	Negative Answer
-2×3	$= -6$	Negative Answer
-2×2	$= -4$	
-2×1	$=$	
-2×0	$= 0$	0
$-2 \times (-1)$	$=$	
$-2 \times (-2)$	$=$	
$-2 \times (-3)$	$=$	
$-2 \times (-4)$	$=$	
$-2 \times (-5)$	$=$	

What can we say about -2 times an integer?

What can we say about $-2 \times$ integer?

What can we say about $-2 \times n$?

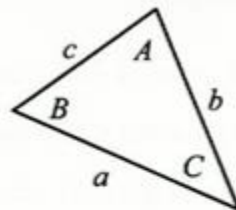
What can we say about $-2n, n \in \mathbb{Z}$?

Algebra: substitution

P16 Formulae and Tables

Triantánacht an triantáin

Trigonometry of the triangle



achar

area

riail an tsín

the rule

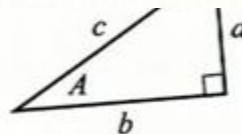
riail an cho

the rule

Triantán d

angle

$$a^2 = b^2 + c^2 - 2bc \cos A$$



$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

teoirim Phíotagaráis

$$c^2 = a^2 + b^2$$

Pythagoras' theorem

Algebra: substitution

P20 Formulae and Tables

Ailgéabar

Algebra

fréamhacha na cothromóide cearnaí
 $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

roots of the quadratic equation
 $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x+y)^n = \sum_{r=0}^n \binom{n}{r} x^r y^{n-r} = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{r} x^r y^{n-r} + \cdots + \binom{n}{n} x^0 y^n$$

comhéifeachtaí déthéarmacha

$$\binom{n}{r} = {}^nC_r = C(n, r) = \frac{n!}{r!(n-r)!}$$

binomial coefficients

Mapping Problem Solving to Literacy and Numeracy

communicate mathematics verbally and in written form	Literacy
explain findings	
justify conclusions	
explore patterns and formulate conjectures	Numeracy
apply their knowledge and skills to solve problems in familiar and unfamiliar contexts	
analyse information presented verbally and translate it into mathematical form	Literacy and Numeracy
devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.	

Evaluation