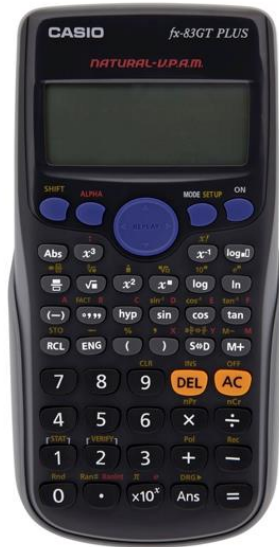
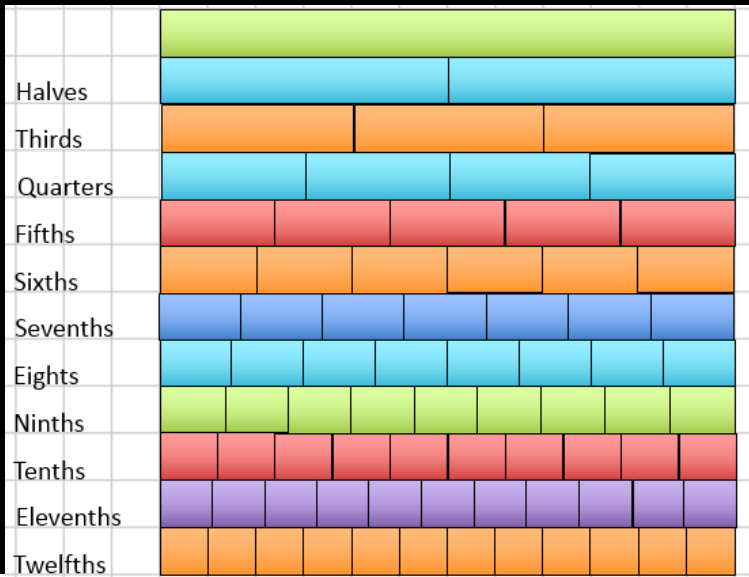
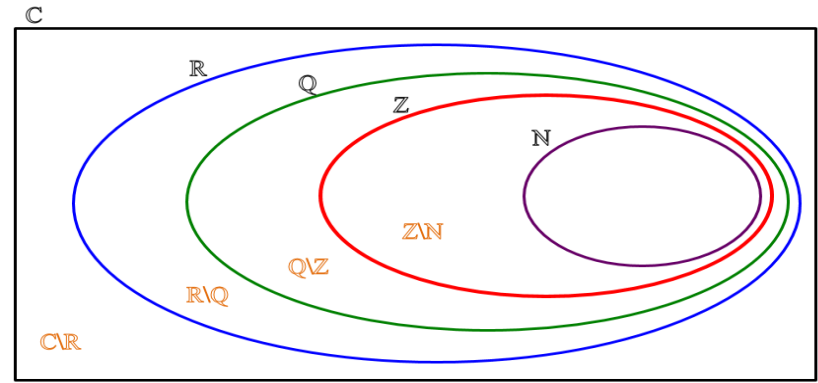


# Literacy & Numeracy for Number Teaching in Ireland

## Workshop 3



Natural numbers:	$\mathbf{N} = \{1, 2, 3, 4, \dots\}$
Integers:	$\mathbf{Z} = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$
Rational numbers:	$\mathbf{Q} = \left\{ \frac{p}{q} \mid p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0 \right\}$
Real numbers:	$\mathbf{R}$
Complex numbers:	$\mathbf{C} = \{a + bi \mid a \in \mathbf{R}, b \in \mathbf{R}, i^2 = -1\}$



Bernie O'Donoghue

# Learning Intentions

- Recap on Geometry knowledge
- Introduce 'Mathematical sense making'
- Appreciate Ratio and Proportion as a key learning priority in number
- Explore Calculator use for L&N in maths class

# Mapping Numeracy to Mathematics

**Stand 3 Number**

**Think and communicate  
quantitatively**

**Make sense of data**

**Strand 2 Geometry and Trigonometry**

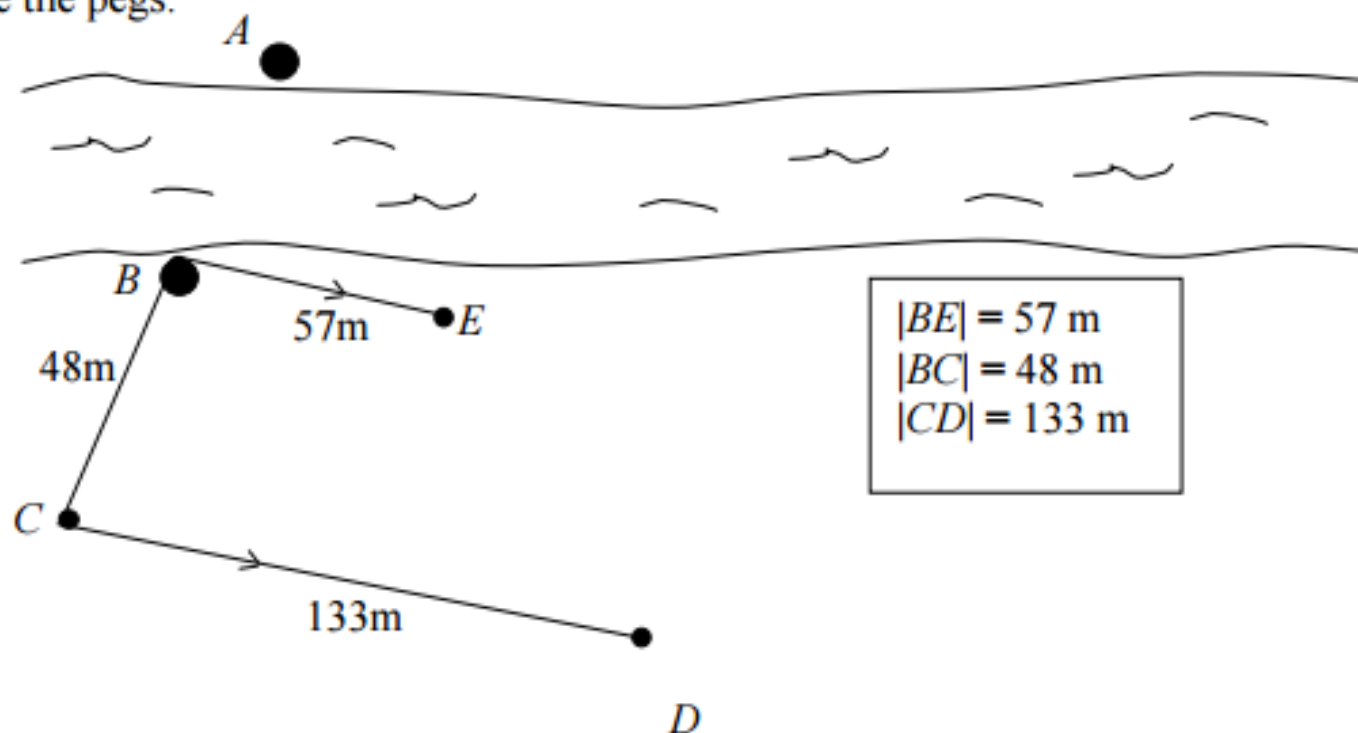
**Have spatial awareness**

**Understand patterns and  
sequences**

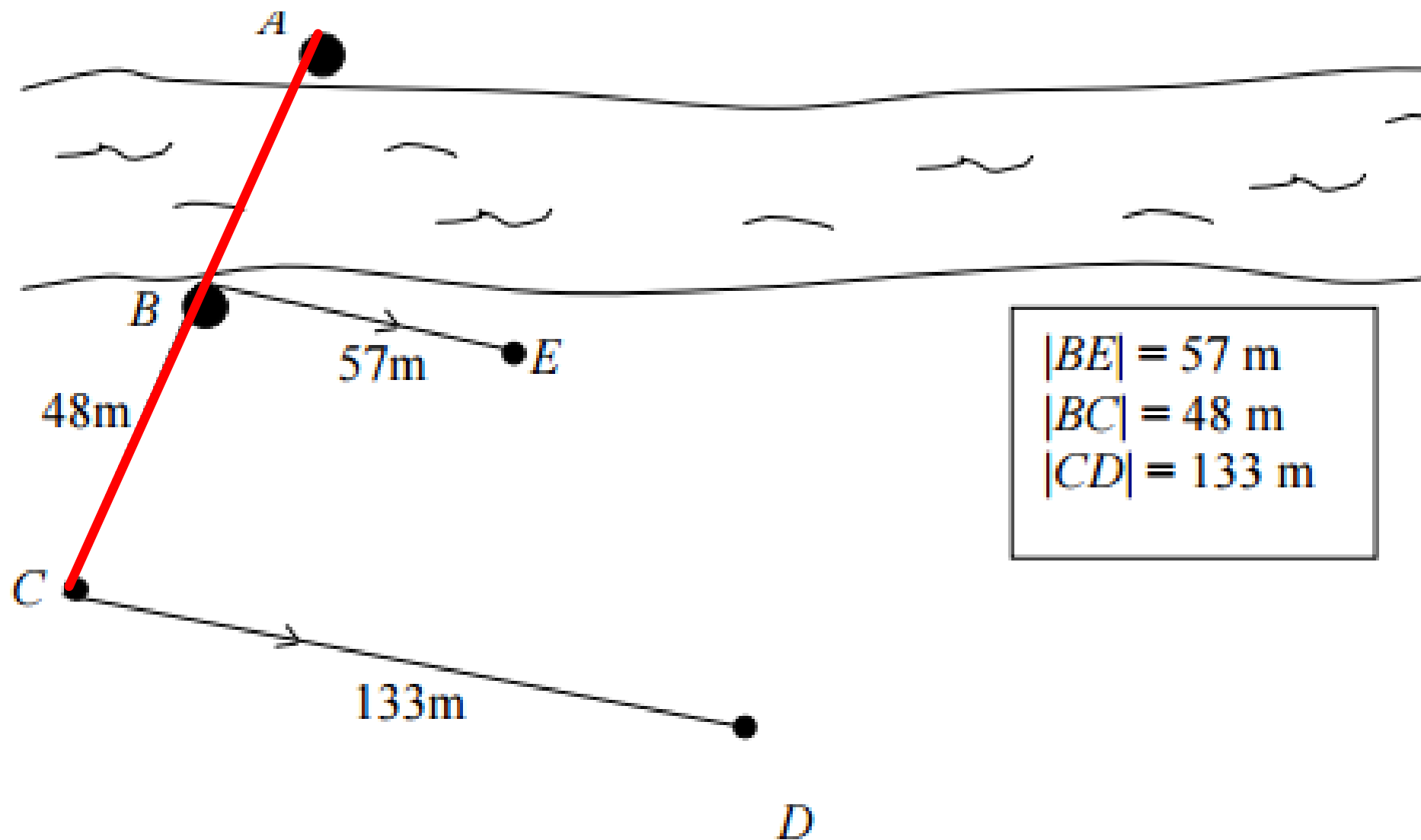
**Solve problems**

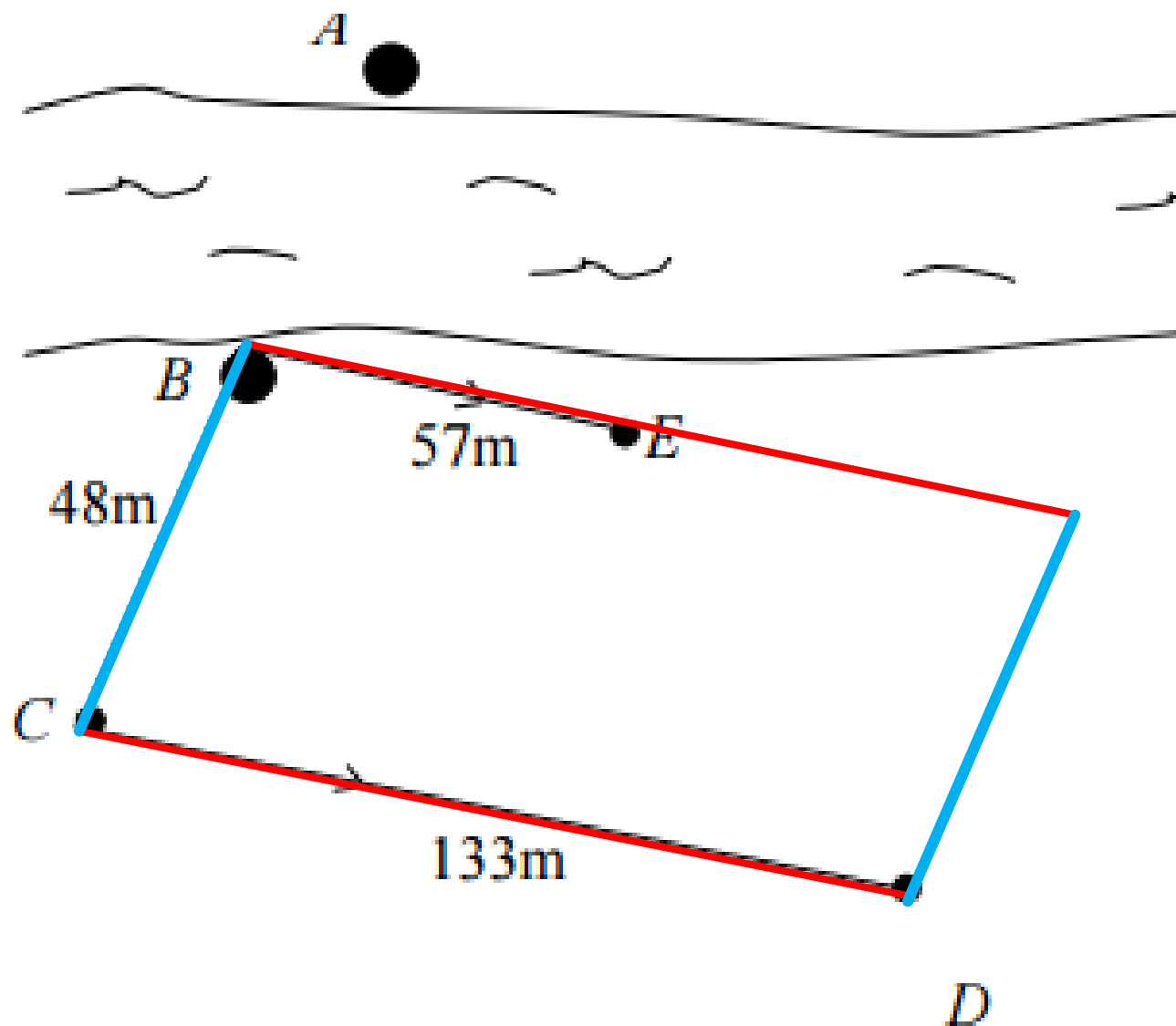
# Strategic Competence

A group of students were trying to find the distance between two trees on opposite sides of a river using pegs, a measuring tape and a large amount of string. They align the pegs in a particular way, take several measurements and sketch this diagram. On the diagram,  $A$  and  $B$  are the trees and  $C$ ,  $D$  and  $E$  are the pegs.



- (a) In what way must the pegs and the trees be aligned if the students are to use these measurements to calculate  $|AB|$ .

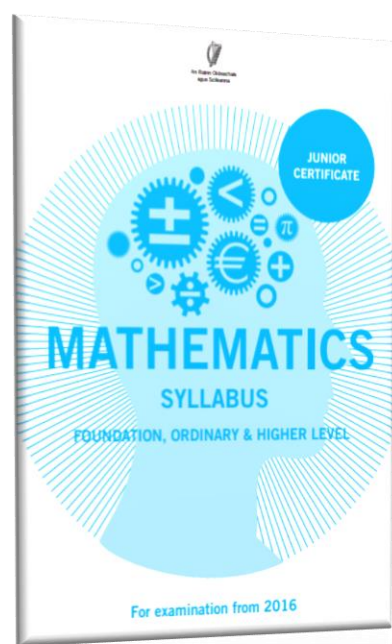




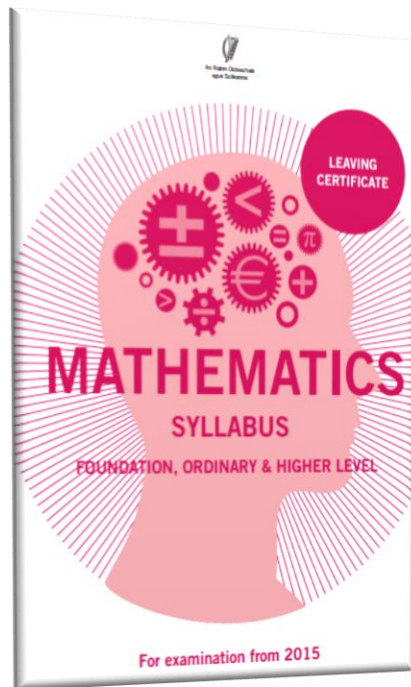
- (d) Suggest how the group of students might have ensured that  $[BE]$  was parallel to  $[CD]$ .

# JC Number: Objectives

- revisit previous learning on number and number operations
- develop a meaningful understanding of different number types, their use and properties
- engage in applications of numeracy to solve real life problems
- apply set theory as a strategy for solving problems in arithmetic



# LC Number: Objectives (excerpt)



Learners continue to make meaning of the operations of addition, subtraction, multiplication and division of whole and rational numbers and extend this sense-making to complex numbers.

They utilise a number of tools: a sophisticated understanding of proportionality....



A red banner with a dark grey outline, shaped like a wide arrow pointing to the right. It has a small notch on the left side.

## Mathematical Quality of Instruction

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language
- Mathematical Sense Making
- Explanations
- Multiple Procedures or Solution Methods
- Remediation of Student Errors and Difficulties
- Teacher uses student mathematical contributions

- Linking between Representations
- Patterns and Generalisations
- Mathematical Language
- Mathematical Sense Making

# Teaching & Learning Plans

## Ratio and Proportion

Junior Certificate Syllabus

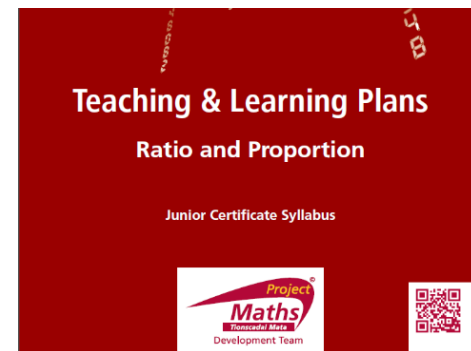




# Mathematical Language

## Mathematical Sense-Making

- When we compare €60 to €20, using the idea of how many “times” €60 is “more” than €20 instead of the difference between €60 and €20, we call this the ratio 60:20.
- How does 60 compare to 20?
- Write as a ratio.
- 60:20
- The current standard pupil-teacher ratio is 1:19. What is wrong with this statement?



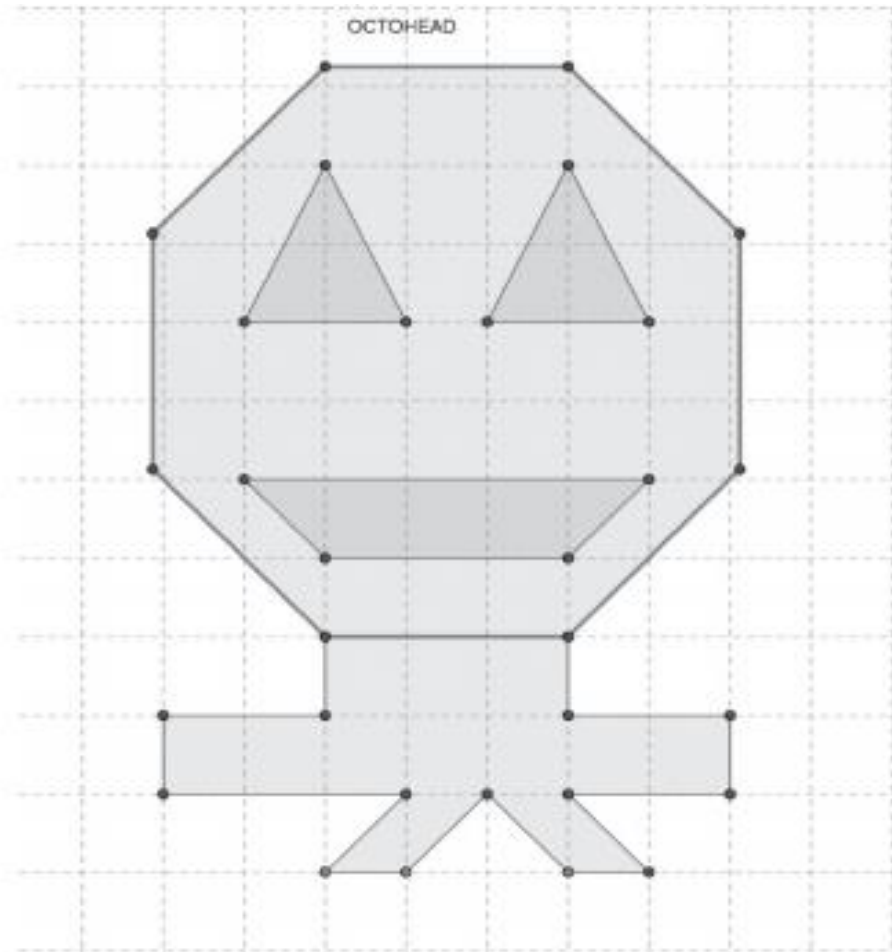
# Patterns and Generalisations

- Write down other numbers in the same ratio as 60:20.
- What is the simplest form of the ratio 60:20?
- How did you generate ratios equivalent to 60:20?

# Mathematical Language

- When we have a statement of equivalent ratios, we have a proportion.
- Set up a proportion for 6:4.

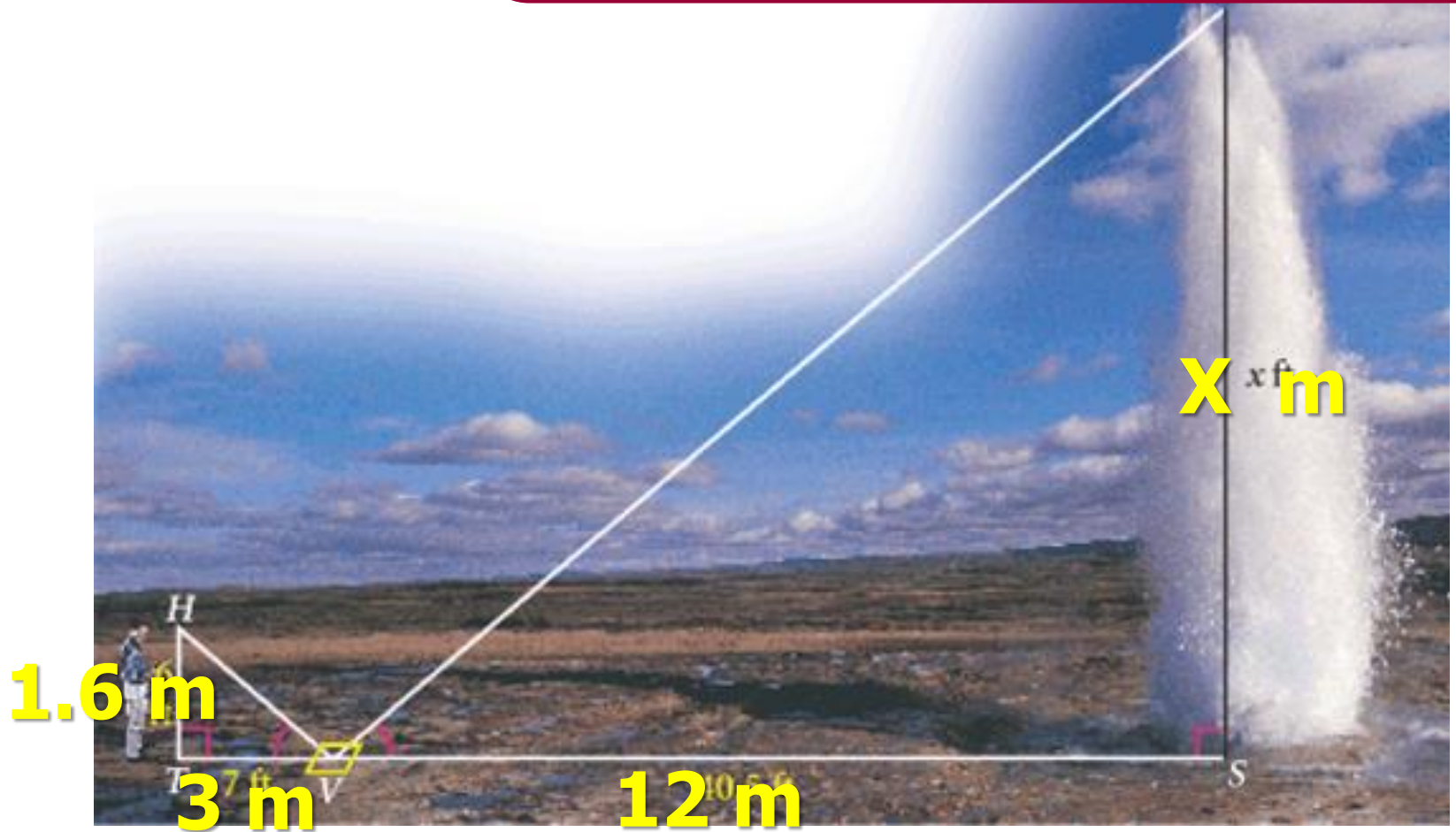
# Proportional Reasoning: 'twice as big'



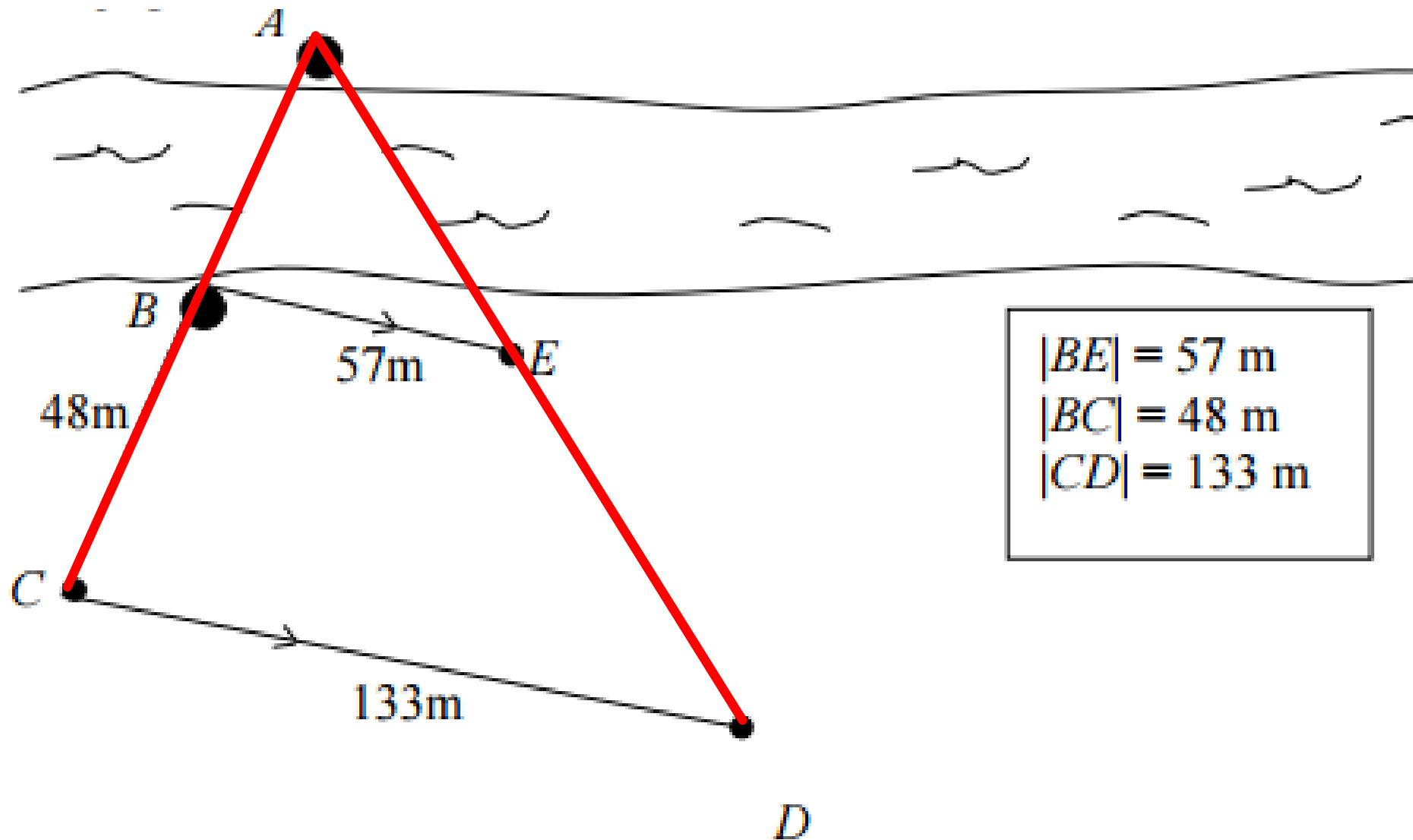


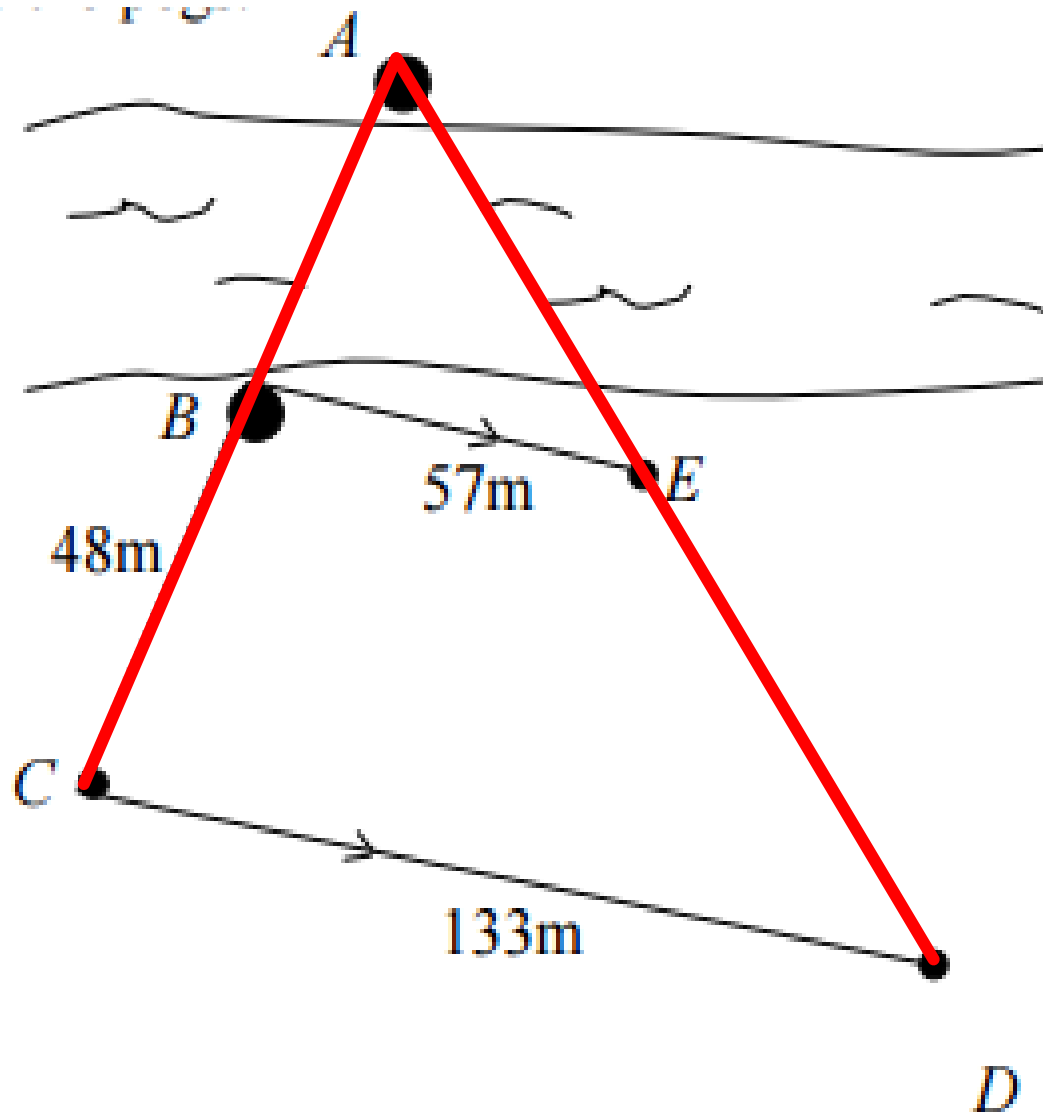
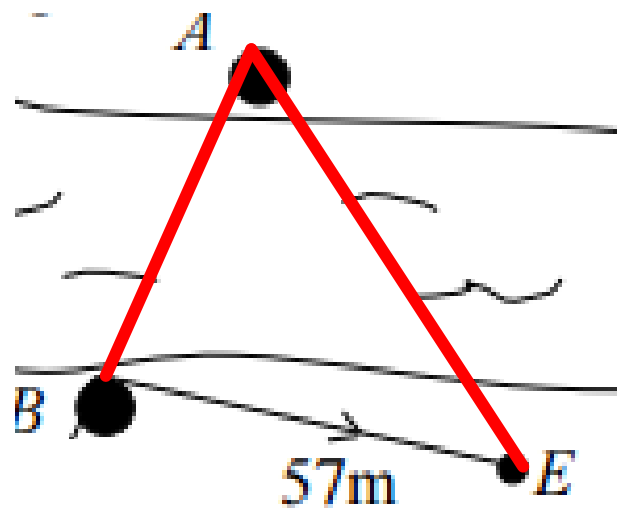
## Height of the Geyser

*If two triangles are similar, then their sides are proportional in order*



(b) Calculate the distance between the trees.



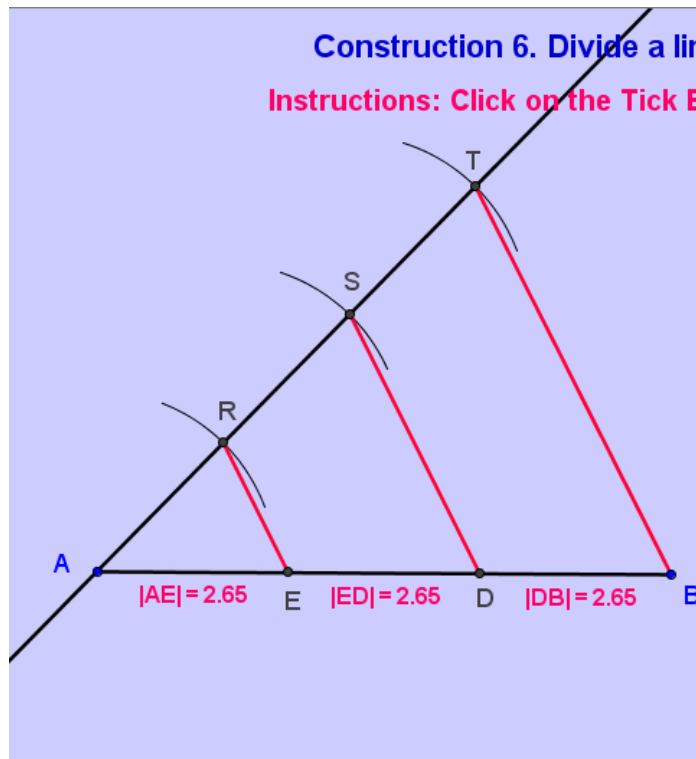


# Linking Representations

Construction 6. Divide a line segment [AB] into three equal parts.

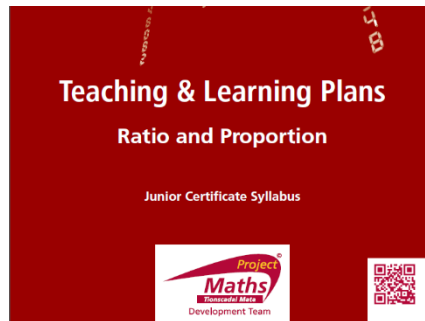
Reset

Instructions: Click on the Tick Boxes in sequence to show construction steps



## Tick Boxes

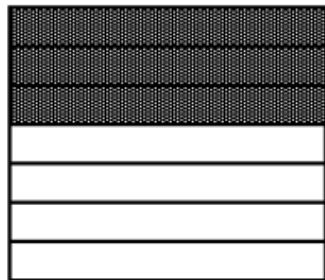
- ☒ A Draw a line segment [AB].
- ☒ B Through A draw a line at an acute angle to [AB].
- ☒ C On this line use circle arcs of the same radius to mark off three line segments of equal length [AR], [RS] and [ST].
- ☒ D Join T to B.
- ☒ E Through S and R draw line segments parallel to [TB] to meet [AB] at D and E. Use set square to do this.
- ☒ F Now  $|AE| = |ED| = |DB|$ .



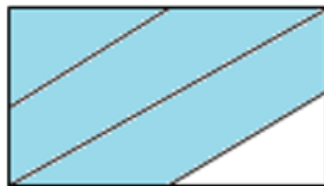
Sylvia is sharing a box of chocolates with her brother Dan. She says “2 for you and 3 for me” as she divides them out in the ratio 2:3. She continues until all the chocolates have been divided up. When she has finished she says to Dan “OK, you got  $\frac{2}{3}$  of the sweets.” because I divided them in the ratio 2:3. Why is Dan frowning? Is there a difference between 2:3 and  $\frac{2}{3}$ ? Discuss.

1. Which of these rectangles has  $\frac{3}{4}$  shaded in? Is it more than one rectangle?

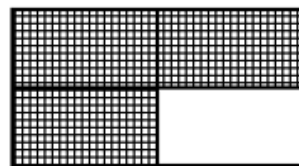
(i)



(ii)



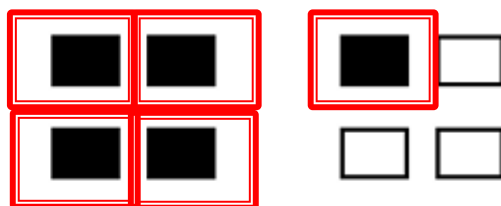
(iii)



(iv)



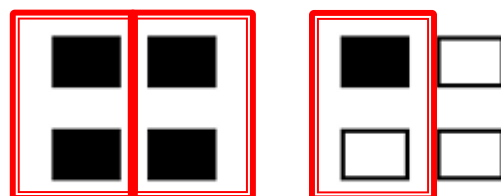
2.



The shaded part of this diagram could represent the numbers

- A:** 5    **B:**  $2\frac{1}{2}$     **C:**  $\frac{5}{8}$     **D:**  $1\frac{1}{4}$

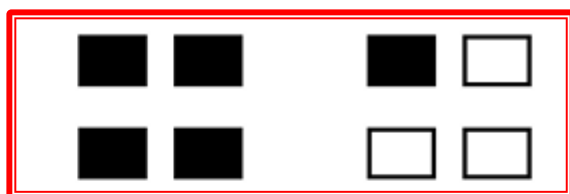
2.



The shaded part of this diagram could represent the numbers

- A:** 5    **B:**  $2\frac{1}{2}$     **C:**  $\frac{5}{8}$     **D:**  $1\frac{1}{4}$

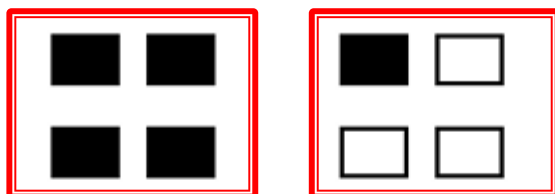
2.



The shaded part of this diagram could represent the numbers

- A:** 5    **B:**  $2\frac{1}{2}$     **C:**  $\frac{5}{8}$     **D:**  $1\frac{1}{4}$

2.

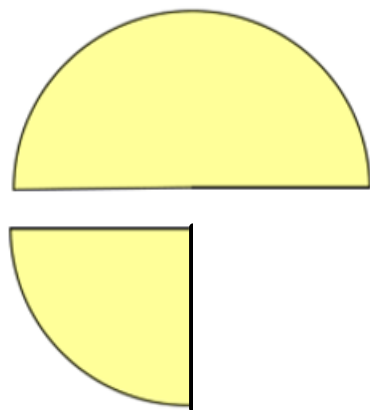


The shaded part of this diagram could represent the numbers

- A:** 5    **B:**  $2\frac{1}{2}$     **C:**  $\frac{5}{8}$     **D:**  $1\frac{1}{4}$



3. The shaded area below represents  $\frac{2}{3}$  of a patio area. What shape would represent the whole patio?



# JCOL: Linking Representations

Candidates struggled to show procedural fluency in a number of areas that have caused difficulty for students at this level in the past. In Paper 1, while candidates were generally able to find 10% of a given price (Question 3(a), mentioned above), they struggled to write 60 as a percentage of 80 (Question 4(c)). They also had great difficulty splitting 4000 in the ratio 3:5, with most candidates dividing 4000 by 3 and by 5 (Question 4(d)).

Paper	Q	Mean Mark / Total Mark	Mean Mark (%)	Mark Ranking (Examination)	Main Topic <sup>4</sup>
1	1	12·0 / 15	80	5	3.5 Number
1	2	10·9 / 15	73	11	3.3 Number
1	3	11·8 / 25	47	27	3.3 Number
1	4	6·0 / 10	60	20	5.1 Functions
1	5	12·0 / 15	80	4	4.6, 4.7 Algebra
1	6	22·9 / 30	76	9	5.2, 5.3 Functions
1	7	15·3 / 20	77	8	4.6 Algebra

### Question 3

**(Suggested maximum time: 10 minutes)**

Eleanor has a **gross** income of €38 500 for the year.

She has an annual tax credit of €3300.

The standard rate cut-off point is €33 800.

The standard rate of income tax is 20% and the higher rate is 40%.

- (a) Find Eleanor's **net** income for the year (i.e. after tax is paid).

Eleanor receives a pay rise. As a result, her **net** income for the year is €34 780.

- (b) Find Eleanor's new **gross** income for the year.

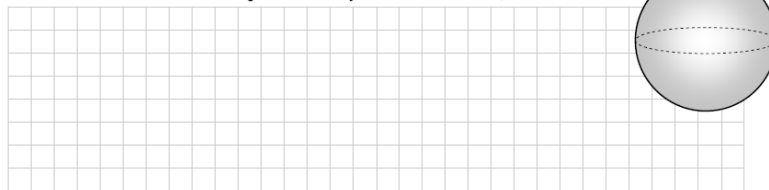
2	6	11·9 / 20	60	21	2.3 Geom. & Trig.
2	7	7·8 / 15	52	25	2.1 Geom. & Trig.
2	8	7·7 / 15	51	26	2.1, 2.4 Geom. & Trig.
2	9	19·6 / 30	65	15	1.3, 1.6, 1.8 Stats & Prob.
2	10	10·6 / 15	71	13	1.6, 1.8 Stats & Prob.
2	11	11·4 / 20	57	22	2.1 Geom. & Trig.
2	12	15·4 / 20	77	7	2.1 Geom. & Trig.
2	13	18·3 / 35	52	24	2.4 Geom. & Trig. 3.4 Applied measure
2	14	11·1 / 20	56	23	3.2 Number 3.4 Applied measure

Question 14

(Suggested maximum time: 5 minutes)

A small sphere has a radius of  $1.5$  cm.

- (a) Find the **volume** of the small sphere. Give your answer in  $\text{cm}^3$ , in terms of  $\pi$ .



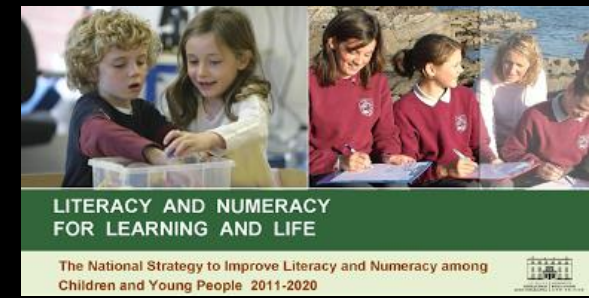
The volume of a large sphere is three times the volume of the small sphere.

- (b) Find the **radius** of the large sphere.

Give your answer in cm, in the form  $\frac{a\sqrt[3]{a}}{b}$ , where  $a, b \in \mathbb{N}$ .

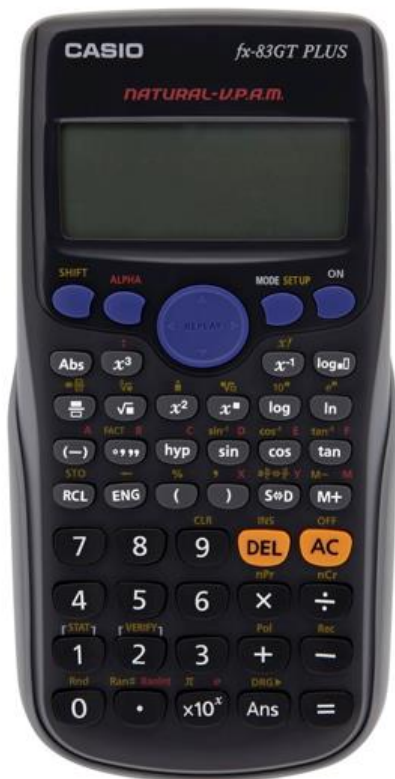
Candidates also struggled to find the radius of a sphere when told that its volume was three times a particular value (Paper 2, Question 14(b)). Many candidates struggled to successfully set up the required equation, and those who did set it up successfully had difficulty expressing the radius in the required (surd) form.

# Literacy Definition



Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

# Numeracy Model: Mathematical Sense Making



Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings.
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position.



# Evaluation