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TOPOLOGY MORPHING IN LATTICE STRUCTURES THROUGH TENSILE BUCKLING

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ABSTRACT

Incorporating, by design, tensile buckling into the macroscopic response of lattice structures offers a novel approach for adaptive (meta-)material/structure development. In this study, we explore the potential of utilizing the simultaneous tensile buckling of adjacent cells to induce a transformation in lattice topology. Unit cells are passively transformed from rectangle-like to triangle/pentagon-like unit cells, with an associated change in the effective macroscopic properties. This approach provides a new route to elastically tailor the non-linear response of (meta-) materials/structures. The paper explores the behavior of such a system through finite element analysis. The results identify: i) that the initial lattice internal topology (rectangular) is dominated by membrane effects, ii) a negative region of stiffness is associated with the transformation phase, and iii) once formed, the new topology (triangular/pentagonal) exhibits positive stiffness in both compression and tension.

Keywords: Topology, Morphing, Tensile Buckling, Sliding

NOMENCLATURE

l	Length of the column [mm]
l_s	Length of the slider rail [mm]
θ_s	Rotation of the slider rail [rad]
w	Displacement in y-direction [mm]
F	Load [N]

1. INTRODUCTION

1.1 Topology Morphing

Morphing structural design harnesses adaptivity to meet multiple competing requirements. Furthermore, structures that can reconfigure have the potential to provide multifunctionality as demonstrated in examples across a range of applications: automotive, aerospace, robotics and energy domains [1–4]. Shape morphing structures, in particular, have been extensively studied by researchers due to their desirable properties such as low weight, high specific stiffness, increased energy absorption, design flexibility, adaptability and packing efficiency [5–9].

The most prevalent approach for shape-changing structures to adapt is by changing the geometry of the constituent elements i.e. the fundamental topology/connectivity and associated load paths do not change. However, by altering the load path, the structure can operate under multiple loading conditions. Examples of this include, variable topology truss structures that possess topology morphing capability by merging or splitting nodes with the help of linear actuators [10–12]. These truss structures have found applications such as stowage, locomotion and workspace manipulation in its various topologies. As a development in the theoretical guidance for topological structure design, researchers have also explored the topologies of Rubik's cube using graph theory by kinematically analysing its structural composition [13]. Similarly, kinematic studies on linkages-based metamorphic mechanisms, with topology reconfiguring properties, show their potential in deployable mechanisms with large folding ratios and adaptive properties [14, 15]. However, these structures rely on relatively rigid truss-type members which do not undergo significant elastic deformations. In order to benefit fully from topology changes, designs for (meta-)materials/structures should also exploit elastic tailoring of constituent members to achieve intended properties such as variable macroscopic stiffness, energy absorption capability and mechanical wave propagation control.

One example of the synergy between elasticity and topology changes is demonstrated by bistable helical lattices. Here the combined tailoring approach widens the deployment capacity from a linear extension to one which will trace a curvilinear path [16, 17]. Topological changes in lattice-based (meta-) materials/structures show that a bending-dominated topology can be transformed into a stretching- (or membrane-) dominated topology or vice-versa either via programming with external stimuli

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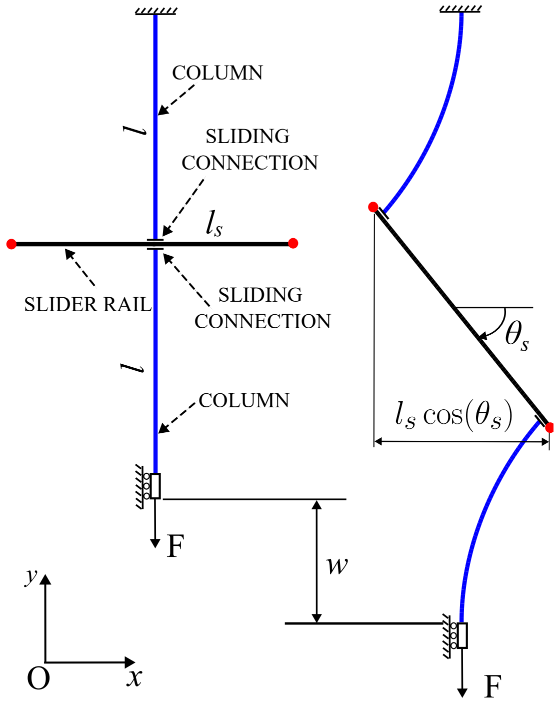


FIGURE 1: SCHEMATIC OF A UNIT CELL (LEFT) SUBJECTED TO TENSILE AXIAL LOAD IN THE INITIAL CONFIGURATION. DEFORMED SHAPE (RIGHT) OF THE UNIT CELL WHEN THE COLUMNS (BLUE LINES) REACH THE ENDS OF THE SLIDER RAIL (SOLID BLACK LINES). RED DOTS REPRESENT THE END COORDINATES OF THE SLIDER RAIL.

[18–20] or via passive morphing [21, 22] by the formation of contact connections. These lattice structures often achieve step changes in their stiffness properties, still within the elastic regime, by transitioning from a complaint topology (bending-dominated) to a stiff topology (stretching-dominated).

1.2 Tensile Buckling

The flexible meta-structures discussed in Sect. 1.1 [21, 22] demonstrate passive topology change under tension, compression and shear by forming new connections via self-contact. These contacts are typically achieved through large deformations of pre-curved members. These members initially exhibit bending-dominated behavior followed by a stiffening effect once self-contact has been established. In the present work, a novel approach is proposed to achieve topology transformation. It is shown that a lattice system with initially straight members responds in a stiff membrane-dominated manner under tensile loads. At larger load intensities the large flexural deformations of the columns cause an associated softening effect until the new lattice topology is formed. The new topology then demonstrates a step increase in its effective stiffness characteristics. To achieve this response, the tensile buckling phenomenon demonstrated by Zaccaria et al. [23] is utilized within the columns of a rectangular lattice structure. Buckling under tensile loads in continuous elastic structures has been realized in practice using sliding elements that allow the development of a hidden length within the system. The elastica of such a structure, derived using analytical expressions, are in good agreement with the experimental observations [23]. Ongoing work has paid considerable attention to utilizing sliding elements and the tensile buckling phenomenon as constituents in the design of structures that are mainly subjected to tensile loads [24–30]. Employing tensile buckling in (meta-)material/structural design paves a new way to control the elastic characteristics at large deformations where plastic or catastrophic failure of the constituent materials would normally have occurred. This approach can further be exploited to bring in new mechanisms for structural adaptivity as is the case of the topology morphing lattice presented in this paper.

This paper proceeds as follows: Section 2 discusses the lattice's unit cell and its assembly into a multi cells system together with details of the supporting Finite Element (FE) analysis. Section 3 explains the mechanical behavior of initial and transformed topologies and discusses the impact of contact/fixed connections at the topology transformation point. Finally, Section 4 summarizes the key contributions of this study.

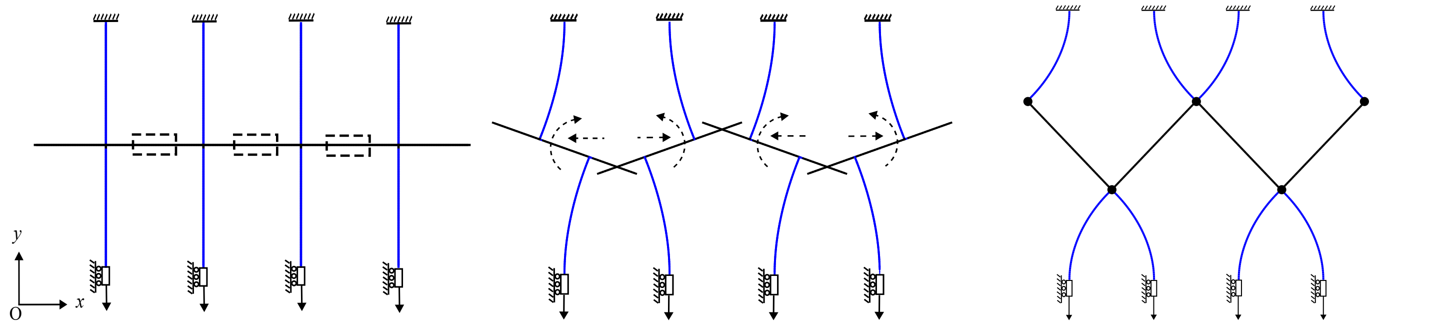
2. THE LATTICE

2.1 The Unit Cell

The structure presented by Zaccaria et al. [23] can bifurcate under both tension and compression. Herein the focus is placed on the tensile response, as only in the tensile buckling mode the sliding ends of the beams reach the ends of the slider rails to form new connections thus facilitating topology transformation. Each unit cell in the proposed structure consists of two collinear columns of length l whose ends are connected, independently of each other, to a horizontal slider rail of length l_s. Figure 1, illustrates the initial and deformed (post-buckling) configuration of the system.

2.2 Configuration With Multiple Unit Cells

To design (meta-)structures/materials with the intended topology transformation capability, several unit cells must be arranged in parallel and in series to form the lattice structure. When a unit cell deforms under tension, the horizontal distance between the ends of the slider rails reduces due to its rotation, as illustrated in Figure 1. To connect unit cells in parallel (see Figure 2a), a conventional hinge connection between slider rails is not sufficient to compensate for the contraction of slider rail end coordinates (see Figure 1). Therefore, a connection is introduced between slider rails which facilitates the connectivity of unit cells in parallel. This connection allows sliding and rotational motions between adjacent slider rails which initially overlap as illustrated in Figure 3. This sliding and rotational connection allow each unit cell to deform independently until the columns reach the ends of the corresponding slider rail, thereby forming a new connection with adjacent cells. Column length l, column flexural rigidity, and slider rail length l_s are sufficient to calculate the initial overlap between slider rails and can be estimated using finite element analysis or elastica formulas [23]. In practice, such a connection could be achieved by offsetting alternate cells in the out-of-plane direction, as shown in Figure 3. Since this offset is small relative to the lattice's overall dimensions the behavior can still be considered planar, and the FE analysis proceeds assuming this offset is zero. The lattice with initial topology (rectangle-like cells),



(a) Initial topology (b) Intermediate deformation state (c) Transformed topology

FIGURE 2: SCHEMATIC OF THE LATTICE WITH FOUR UNIT CELLS IN PARALLEL SUBJECTED TO TENSILE LOADS. (A) INITIAL TOPOLOGY WITH RECTANGLE-LIKE UNIT CELLS. BOXES IN DASHED LINES INDICATE THE INITIAL OVERLAP OF SLIDER RAILS, SEE FIGURE 3. (B) INTERMEDIATE DEFORMATION STATE DURING TOPOLOGY TRANSFORMATION. DASHED LINES AND ARCS WITH ARROWS INDICATE THE DIRECTION OF SLIDING AND ROTATIONAL MOTIONS OF SLIDER RAILS RESPECTIVELY, DURING TOPOLOGY TRANSFORMATION. (C) TRANSFORMED TOPOLOGY OF THE LATTICE WITH TRIANGLE- AND PENTAGON-LIKE UNIT CELLS. BLACK DOTS REPRESENT THE NEWLY FORMED CONNECTIONS.

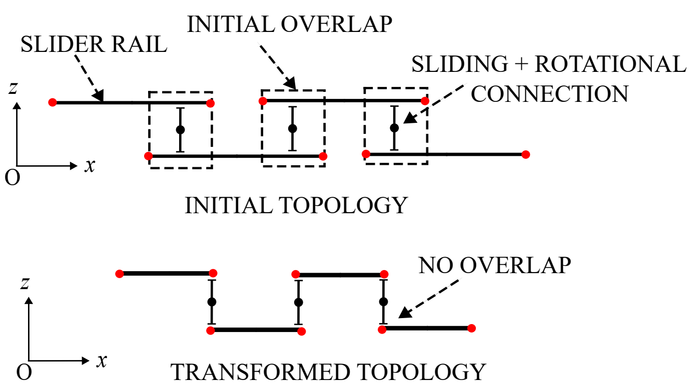


FIGURE 3: SCHEMATIC OF SLIDER RAILS FOR INITIAL AND TRANSFORMED TOPOLOGIES OF THE LATTICE. THIS VIEW IS IN THE PLANE (xz-PLANE) ORTHOGONAL TO THE AXIS (y-AXIS) OF APPLIED LOAD.

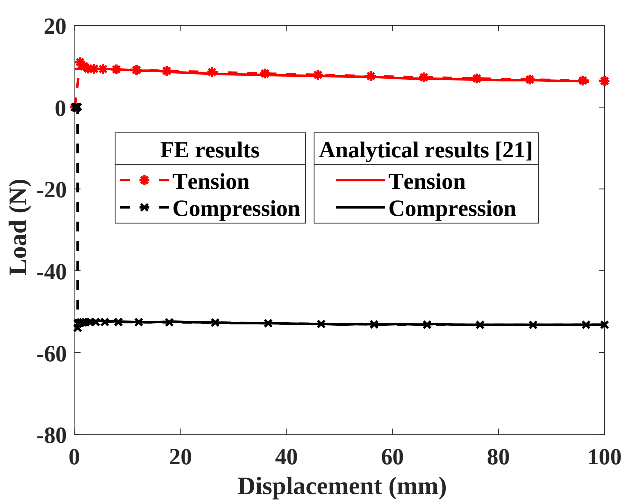


FIGURE 4: LOAD VERSUS DISPLACEMENT OF A SINGLE UNIT CELL COMPARED WITH ANALYTICAL RESULTS OBTAINED BY ZACCARIA ET AL. [23].

an intermediate deformation state with slider rails undergoing relative sliding and rotational motions between them, and transformed topology (triangle- and pentagon-like cells) are shown in Figure 2.

Repeating unit cells in series does not require a special connection type as in the case of cells in parallel. Therefore, in the current work, the analysis is restricted to a system of four parallel unit cells (see Figure 2a) which is sufficient to understand the general behavior of the lattice.

2.3 Finite Element Analysis

Static nonlinear FE analysis is utilized to study the tensile buckling of the unit cell and lattice using Abaqus/Standard 2020 [31]. Two-noded linear beam elements (B21) are used to model the columns and slider rails using a 1 mm mesh size determined via a convergence study. The columns have length l = 250 mm, width = 25 mm and depth = 1 mm, and the slider rail has length l_s = 250 mm. For the given column and slider rail lengths, the initial overlap between two adjacent unit cells is estimated to be 94 mm. A Young's modulus of 3.31 GPa is assumed, representative of 3D-printed polylactic acid [32], and the system is modeled as fully isotropic/homogeneous. The sliding between the columns and the slider rail is modeled using a SLOT + ALIGN connection [31] with a friction coefficient assumed to be 0.01. An initial rotation of 0.1 degree is applied to the slider rail to seed a symmetry-breaking imperfection, before subjecting it to the tensile load. In all analyses, the top columns are fully fixed and the bottom column is allowed to translate vertically, i.e. horizontal and rotational motion is constrained, see Figure 2a. The slider rail is modeled with two stiffness constants, i) "rigid" where the flexural rigidity is greater than 100 times that of the columns and ii) "elastic" with flexural rigidity equal to that of the columns.

A two-regime modeling approach is utilized to simplify the analysis. The initial behavior of the lattice, i.e. the transformation phase, is modeled assuming each cell acts independently i.e. no loads are transferred between adjacent cells. Modeling the cells independently accounts for negligible friction in the sliding

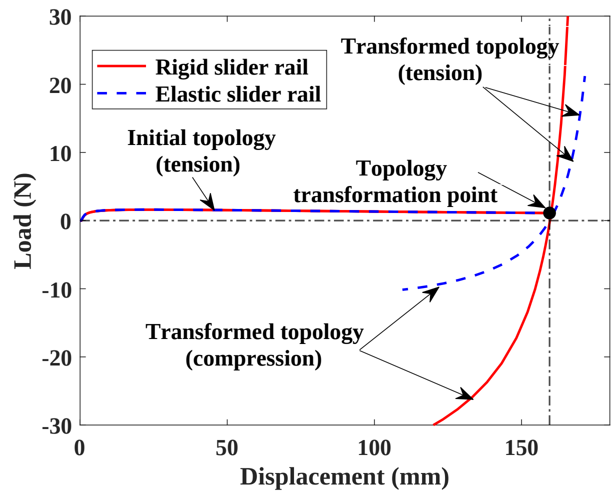


FIGURE 5: LOAD VERSUS DISPLACEMENT OF THE LATTICE OBTAINED FROM FE ANALYSIS. THE LATTICE WITH INITIAL TOPOLOGY IS SUBJECTED TO TENSION. POST-TOPOLOGY CHANGE, THE LATTICE IS SUBJECTED TO TENSION AND COMPRESSION IN SEPARATE ANALYSES AND THE CORRESPONDING LOAD-DISPLACEMENT CURVES ARE INDICATED IN THE FIGURE. THE DISPLACEMENT PLOTTED FOR TRANSFORMED TOPOLOGY (COMPRESSION) REFERS TO THE DISPLACEMENT ATTAINED BY THE BOTTOM ENDS OF THE COLUMNS FROM THE INITIAL UNDEFORMED STATE.

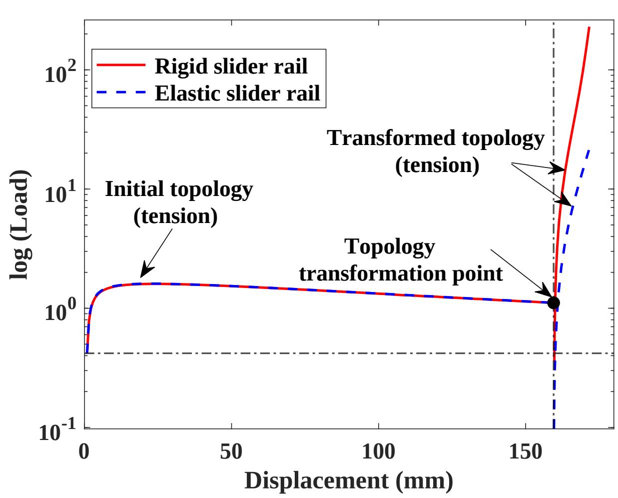


FIGURE 6: LOAD VERSUS DISPLACEMENT OF THE LATTICE UNDER TENSION. LOGARITHMIC SCALE FOR LOAD (Y- AXIS) IS PLOTTED TO VISUALIZE THE ORDER OF MAGNITUDE INCREASE IN STIFFNESS AFTER TOPOLOGY CHANGE.

+ rotational connection between unit cells. This model is used to obtain the final stress state and column/slider rail deformations at the end of the transformation phase. In the transformed topology state, i.e. beginning in the state shown in Figure 2c, the structure is analysed with fixed connections between the ends of the slider rails (achieved by merging the nodes). A pre-stress corresponding to the final state of the transformation analysis is applied before subsequent analysis of the new topology. This two-regime approach obviates the need for modeling contact between adjacent cells whilst still capturing the key underlying physics.

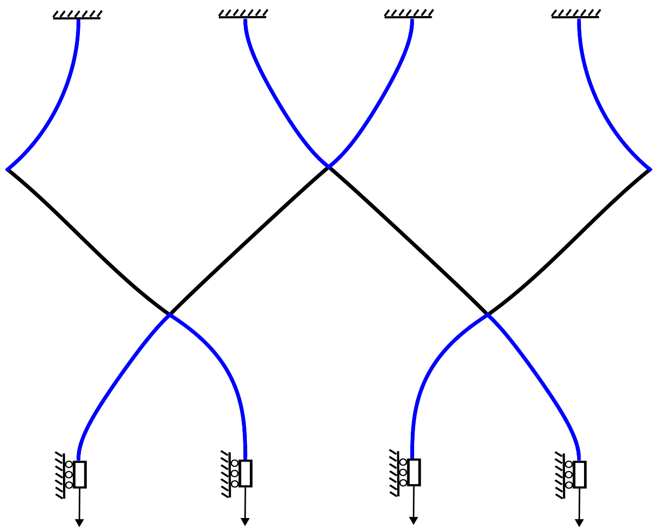
3. RESULTS AND DISCUSSION

The results of the present FE analysis for a single unit cell are compared with the analytical results presented by Zaccaria et al [23]. The results closely agree with each other as shown in Figure 4.

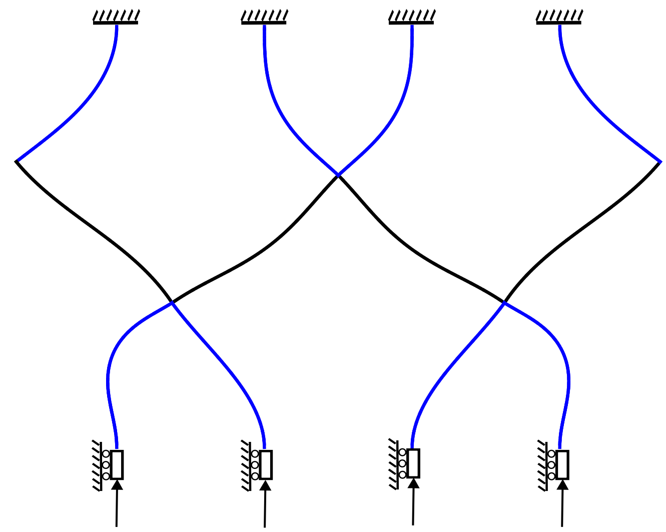
Under vertical displacement of the lower column, the unconstrained ends of both columns translate along the slider rail, which itself rotates, to accommodate the change in length of the system. An illustration of this effect is shown in Figure 2b. It can also be seen how the slider rail-to-slider rail connection (i.e., the connection between adjacent cells, see Figure 3) permits this slider rail rotation without inducing a change in the horizontal dimension of the system. This response corresponds to the initial portion (displacements smaller than the topology transformation point) of the load-displacement curve shown in Figure 5. An initially stiff phase of membrane-dominated behavior is observed before a softening to negative stiffness and is clearly shown in Figure 6 plotted with a logarithmic scale for load (vertical axis). This response is associated with the induced large rotations of the rigid rail, and transition to a flexural response of the columns. Such a softening effect is unusual when compared with traditional elastic lattice structures where stiffening is more commonly observed under large tensile deformations [33, 34].

The topology transformation point occurs when vertical displacement is sufficient to cause the slider rail to reach the end of the run. This transition occurs simultaneously with new connections between adjacent cells. The resultant topology change, from rectangle-like to triangle/pentagonal-like unit cells, is illustrated in Figure 2c. The transformed topology can be reversed i.e., the lattice with triangle/pentagon-like unit cells can transform back to a lattice with rectangle-like unit cells under compression. However, a hysteresis which could be desirable can be introduced in the system by locking the connections when the new topology is formed. This locking of connections could be realized, for example by using snap-fit joints. Therefore, in the locked configuration, the lattice can exhibit increased stiffness under both tension and compression after transforming, see Figure 5.

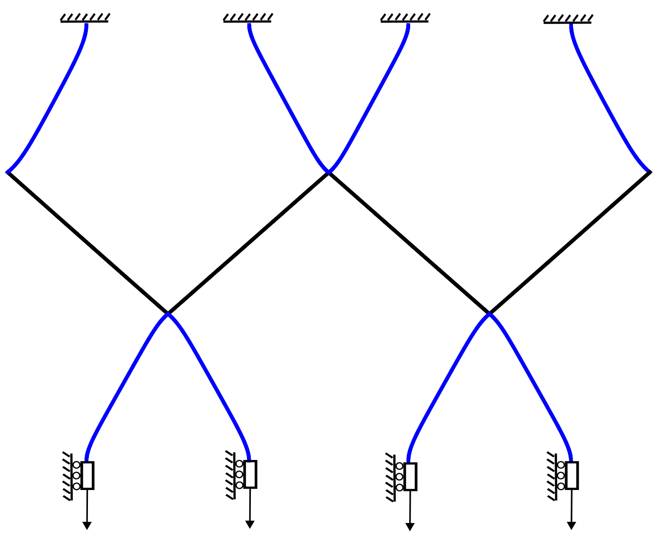
The load-displacement curves for both the "rigid" and "elastic" slider rails are nominally identical until the topology transformation point. The transition occurs at load/displacement values of 1.11 N/159.64 mm and 1.11 N/159.52 mm for rigid and elastic slider rails respectively. This result is confirmed by observing there are no stresses present in the slider rail and corroborates the analytical expressions presented by Zaccaria et al. [23] that possess no dependence on the flexural rigidity of the slider rail. In the initial topology, the slider rail acts only to ensure relative



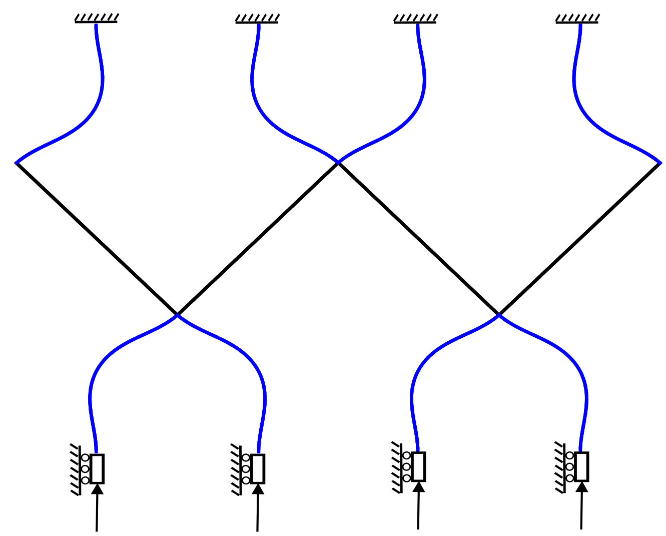
(a) Lattice with elastic slider rails subjected to tension.



(b) Lattice with elastic slider rails subjected to compression.



(c) Lattice with rigid slider rails subjected to tension.



(d) Lattice with rigid slider rails subjected to compression.

FIGURE 7: SCHEMATICS OF THE DEFORMED SHAPES OF LATTICES WITH TRANSFORMED TOPOLOGY OBTAINED FROM FE ANALYSIS.

horizontal sliding between the columns. For practical implementations, this stiffness independence is significant. Since most readily available slider rails are flexurally rigid, including those used in experiments by Zaccaria et al. [23], there is likely to be an undesirable mass penalty. Using elastic sliding concepts [26] could offer a route for light-weighting of the lattice, for example by reducing the mass of the slider rail by changing its geometry (which would likely reduce its effective flexural rigidity) but would not alter the underlying behavior.

Post-topology change, the flexural rigidity of the slider rail does affect the structural response as shown in Figures 5 and 6. The stiffness of lattice with elastic slider rails under both tension and compression is less than that of the lattice with rigid slider rails. This behavior occurs because the elastic slider rails deform significantly as they transmit axial and bending forces between the columns, due to their low flexural rigidity. This deformation is evident from the shapes of lattices with elastic slider rails, shown in Figures 7a and 7b, which is in contrast to the lattices with rigid slider rails shown in Figures 7c and 7d, in which the deformations of slider rails are not significant.

The proposed concept would be helpful in designing a structure which fails at a desired tensile load, i.e. the load at which bifurcation occurs for initial topology. However, this failure under tension can be harvested to induce a topology change which can significantly increase the stiffness of the structure under tension/compression. As this concept is independent of length scale, it can be applied in the design of structures at any length scale. This lattice structure is tunable by defining the initial length and flexural rigidity of columns and slider rails.

4. CONCLUSION

A new lattice design concept that utilizes tensile buckling to reconfigure its unit cells is presented. A topology change within the system is observed that harnesses the elastic behavior of the constituent members. In this concept, no external stimuli or actuation is required other than the applied load on the structure to induce the topology change. Finite element analysis reveals that initial topology, with rectangle-like unit cells, exhibits an initial membrane response under tension while breaking the connections and softens under flexure of the members, leading to a negative stiffness phase during transformation. At the topology transformation point, a step change in stiffness is observed, associated with the formation of new load paths. The behavior is reversible but with the inclusion of a topology fixing connection (e.g. snap fitting) upon transformation, the system exhibits high-stiffness hysteretic behavior under subsequent compression. This study is an initial exploration of the behavior of such a system and indicates a rich route for further exploration of synergistic elastic/topological designs. The concept presented in this paper has the potential to aid in the development of new (meta-)material/structures and provides a benchmark for continued investigation, both numerically and experimentally.

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