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Abstract

The surface electromyogram (sEMG) conveys information about the physiological properties of muscles. Unlike the power spectrum, the bispectrum can suppress noise when characterizing non-Gaussian random signals. In this paper we esablish a bispectrum based method to estimate a motor unit action potential from a simulated sEMG signal, improving on an earlier approach which combined bispectrum and power spectrum.

1. INTRODUCTION

Muscle performance diagnosis is based on the electromyogram (EMG) signal which may be obtained by needle electrode or surface electrode based electromyogram (sEMG). The detected signal consists of a train of action potentials, generated when the muscle fibres of a single neuromuscular unit (NMU) are activated by the motor neuron. Needle electrode electromyography can identify individual motor unit action potential trains (MUAPT) [1] but it is an invasive procedure and thus painful for the subject.

The sEMG is the superposition of many such MUAPTs caused by a number of NMU activities observed at the skin surface. However, as the sEMG is not invasive, it would be useful to be able to recover estimates of the motor unit action potentials (MUAP) from it. It was shown in [2] that assuming that the sEMG can be represented as a filtered impulse process, then the MUAP can be recovered and the intensity (lambda) of the impulse process can be obtained. However, due to the combination of power spectrum and bispectrum used to calculate the magnitude of the MUAP, noise can greatly affect the result. In this paper, we present an approach to signal reconstruction from the sEMG which employs only the bispectrum. As the cumulants and its spectrum of order above two are zero for Gaussian processes, the bispectrum will suppress Gaussian noise affecting a non-Gaussian input signal, leading to improved performance of the MUAP reconstruction. We implemented and tested our method on a simulated sEMG signal to reconstruct the MUAP.

2. BACKGROUND

The sEMG may be modelled as a filtered impulse process [2], [3]. In the digital domain, we model this as

$$y(n) = \sum_{k=0}^{N-1} h(k)e(n-k) + w(n) \tag{1}$$

where *e(n)* is a stationary, non-Gaussian white i.i.d. random process and *w(n)* is zero mean additive Gaussian noise which is independent from *e(n)*.

The input and output of the system can be expressed in terms of higher order spectra (HOS) or cumulant spectra and the frequency response of the system and, therefore, the discrete bispectrum of LTI model of Equation (1) can be expressed as [4], [5]

$$B(k, l) = \gamma H(k)H(l)H^*(k+l) \tag{2}$$

where * denotes the complex conjugate, γ is third order cumulant of $e(n)$ and $H(\circ)$ is the discrete time Fourier transform (DTFT) of the signal *h(n)*. As discussed above, the bispectrum suppresses Gaussian noise affecting a non-Gaussian input signal and, therefore, the bispectrum of *w(n)* is zero. Since the bispectrum preserves both the magnitude and phase information of the signal, Equation (2) can be written as

$$B(k, l) = |B(k, l)| \exp j\Psi(k, l) \qquad \text{where} \quad \Psi(k, l) = \angle B(k, l) \tag{3}$$

Therefore, it can be further expressed as

$$\begin{aligned} |B(k, l)| &= |\gamma||H(k)||H(l)||H^*(k, l)| \qquad &\text{. . . (a)} \\ \Psi(k, l) &= \varphi(k) + \varphi(l) - \varphi(k, l) \qquad &\text{. . . (b)} \end{aligned} \tag{4}$$

where $\varphi(\circ) = \angle H(\circ)$. From the symmetry properties of the bispectrum, the principal argument of the bispectrum is enough to estimate the bispectrum of the system if the sampling frequency is at least twice time greater than the system frequency [6] and if the whole bispectrum area, that is, $-\pi$ to π is sampled in $N = 2^{\nu} > (2Q + 1)$ equispaced FFT frequency points, the appropriate boundary of the principal domain will be $0 \le l \le k \le (2Q + 1)/2$ and $0 \le l + k \le (2Q + 1)/2$.

3. SIGNAL RECONSTRUCTION

Equation (4)(a) and (b) can be used to solve for the magnitude and phase of $H(\omega)$ respectively. Apart from the error caused by the estimation methods and noise, in many cases, the estimated phase of the bispectrum of the signal may differ by $2n(k, l)\pi$ from the true phase where $n(k, l)$ is an integer. As a result, the estimated Fourier phase from Equation (4)(b) may be incorrect [4], [7]. To overcome this problem, it is necessary to include a phase unwrapping step and for this we employ the method of [7].

3.1 Magnitude Estimation

To evaluate the Fourier magnitude of the impulse response system, we take Equation (4)(a) and, after taking logarithms on both side of Equation (4)(a), we find

$$\log|B(k, l)| = \log|\gamma| + \log|H(k)| + \log|H(l)| + \log|H^*(k + l)| \tag{5}$$

By considering all points of principal domain except for $k = 0$, we can write in matrix form as

$$\beta = A_m \eta \tag{6}$$

where

$\beta = [\log|B(1, 1)|, \log|B(2, 1)|, \log|B(2, 2)| \ldots\ldots\ldots\ldots, \log|B(N/2 - 2, 1)|]^T$,

$\eta = [\log|\gamma|, \log|H(1)|, \log|H(2)| \ldots\ldots\ldots\ldots, \log|H(N/2 - 1)|]^T$

and A_m is the matrix contains all coefficients that can be written as follows

$$A_m = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & \ldots\ldots\ldots & \ldots & 0 \\ 1 & 1 & 1 & 1 & 0 & \ldots\ldots\ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots & \ldots & \ldots \\ 1 & 1 & 0 & 0 & 0 & \ldots\ldots\ldots & 1 & 1 \end{bmatrix}$$

Here the size of A_m matrix is $(P - N/2) \times (N/2)$ where $P = (N/4 + 1)^2$. The following least squares solution gives the solution of η

$$\eta = (A_m{}^T A_m)^{-1} A_m{}^T \beta \tag{7}$$

Now to get the value of $\log|H(0)|$, the following relation can be drawn from Equation (5)

$$\eta(0) = \log|H(0)| = [\{\log|B(0, 0)| - \log|\gamma|\}/3] \tag{8}$$

Therefore, from Equation (7) and Equation (8), we obtain full set of $\eta(n)$ where $n = 0, 1, 2, \ldots\ldots, N/2$,. Since η is the logarithmic value of Fourier magnitude H [Equation (5)], we obtain the Fourier magnitude value after applying anti-logarithmic function as follows

$$H(n) = \exp[\eta(n)] \tag{9}$$

3.2 Phase Estimation

We estimate the phase based on Equation (4)(b). As shown in Equation (11) the phase of the bispectrum estimated from the data may differ by $2n(k, l)\pi$ from the true phase, where $n(k, l) \in \{-1, 0, 1\}$ [7].

$$\psi + 2n(k, l)\pi = A_\phi \phi \tag{10}$$

where A_ϕ is the coefficient matrix that comes from each point of the principal domain

$$A_\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \dots & 1 & -1 \end{bmatrix}$$

As A_ϕ is not a square matrix, least squares approaches to solving for ϕ wil require phase unwrapping [7]. We apply the method of [7] which performs unwrapping in a 2-stage process. We first solve for the principal argument of the phase along $l = 1$ by solving the equation

$$\psi = M\phi \tag{11}$$

where $\psi = [\Psi(1, 1), \Psi(2, 1), \dots\dots, \Psi(N/2 - 2, 1)]^T$, $\phi = [\varphi(1), \varphi(2), \dots\dots, \varphi(N/2 - 2)]^T$ and M is coefficient matrix of size $[(N/2 - 2) \times (N/2 - 2)]$ as below

$$M = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & \dots\dots\dots & 0 \\ 1 & 1 & -1 & 0 & 0 & \dots\dots\dots & 0 \\ 1 & 0 & 1 & -1 & 0 & \dots\dots\dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots\dots\dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots\dots\dots & 0 \\ 1 & 0 & 0 & 0 & 0 & \dots\dots 1 & -1 \end{bmatrix}$$

Now the $n(k, l)$ vector can be obtained by using A_ϕ matrix from Equation (10) as following

$$n(k, l) = \frac{1}{2\pi}(A_\phi \phi - \psi) \tag{12}$$

As the resulting k matrix does not always have integer value we round the elementsof $n(k, l)$ to the nearest integer value such that $n(k, l) \in \{-(1, 0, 1)\}$. Therefore, once we have the $n(k, l)$ value, we can easily estimate the true phase value from Equation (11) as following. A least square method has been applied to estimate the true phase value

$$\phi = (A_\phi{}^T A_\phi)^{-1} A_\phi{}^T \{\psi + 2n(k, l)\pi\} \tag{13}$$

where the last column of A_ϕ are removed to make it a full rank matrix.

3.3 MUAP Reconstruction

Since Bispectrum suppress the Gaussian noise, Equation (9) and Equation (13) carry the Fourier magnitude and Fourier phase value of MUAP respectively which are free from any noise. Inverse Discrete Fourier Transform (IDFT) gives the time domain signal of the estimated MUAP.

4. RESULTS

To test our algorithm, we simulated an sEMG signal (Figure 1) and deconvolved the MUAP from this signal. A single MUAP waveform is assumed as

$$h(t) = t[\exp(-|t|/\tau)] \tag{14}$$

We have taken 2048 samples and 50 numbers of fibers to estimate the MUAP. We consider both the noise-free case and noisy case. Figure 2 shows the real system response and estimated response for noise free case and noisy case.

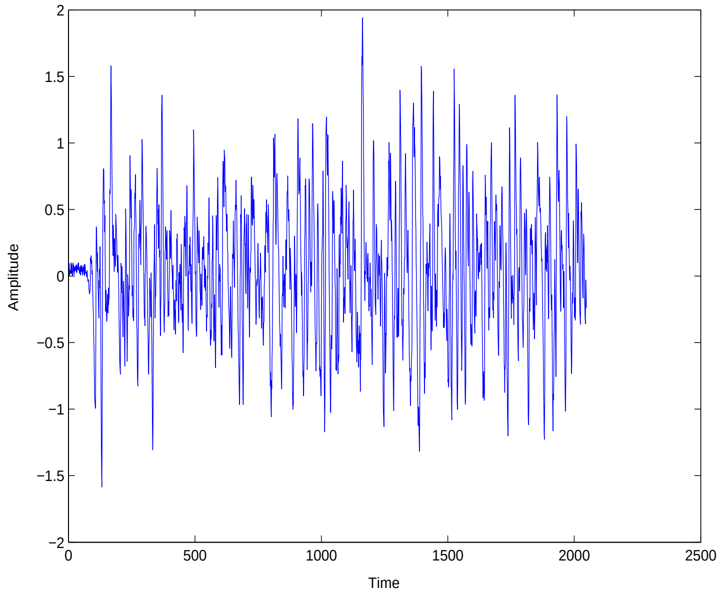

Figure 1: An artificially generated EMG signal

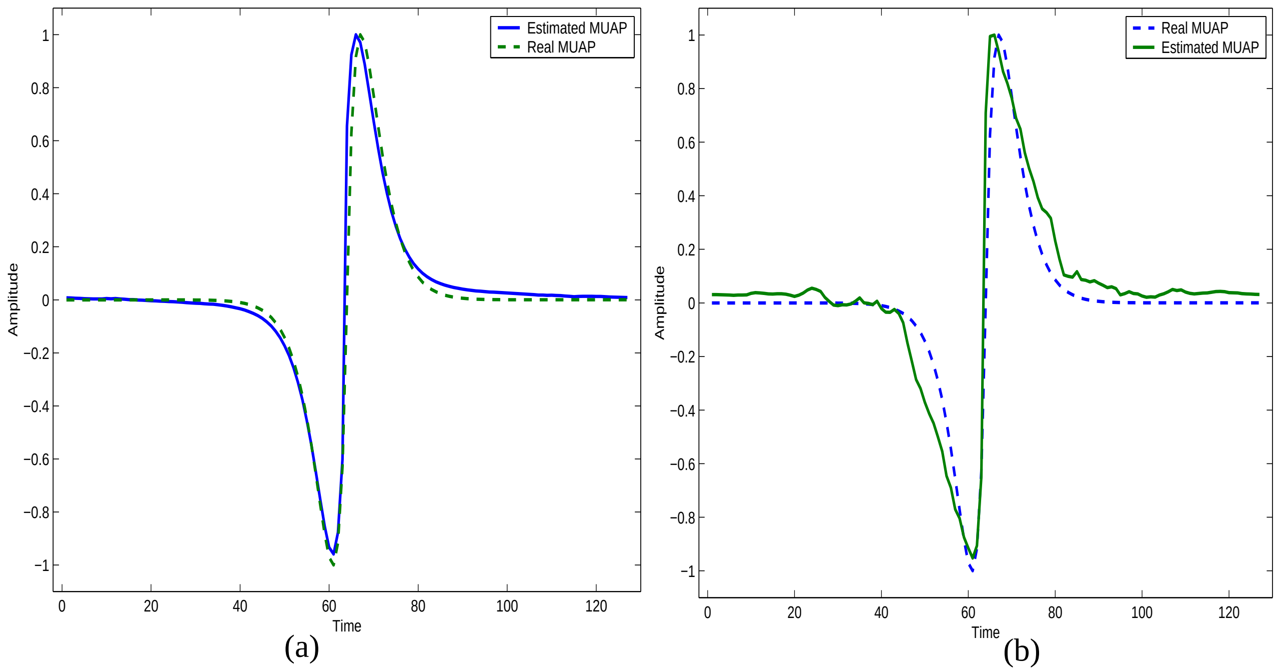

Figure 2: Real and estimated MUAP (a) noise free case and (b) noisy case

To test the performance of the proposed algorithm, we have also carefully reestablish the algorithm proposed by [2] and the comparison has been done. Figure 3 shows one comparison picture between the estimation technique by [2] and this proposed algorithm. We consider the white noise varience $\sigma = 0.1$.

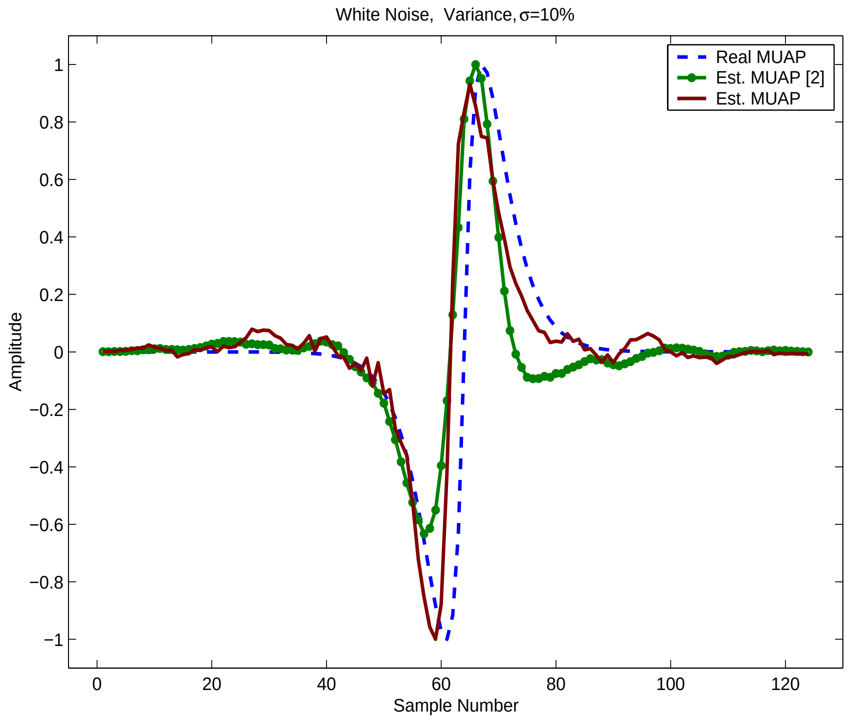

Figure 3: Comparison between proposed MUAP estimation and MUAP estimated By [2].

5. CONCLUSION

In this paper we presented a better method for reconstruction MUAP from an sEMG. Our approach estimates both the Fourier phase and magnitude from the bispectrum only, taking advantage of its Gaussian noise suppressing property. We also considered the existing phase unwrapping procedure to reconstruction of MUAP. To test this technique, we first tested this on a simulated sEMG signal for both noisy and noise-free case. An existing method for reconstructing MUAP has carefully been reestablished and compared with this proposed algorithm and found better performance.

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