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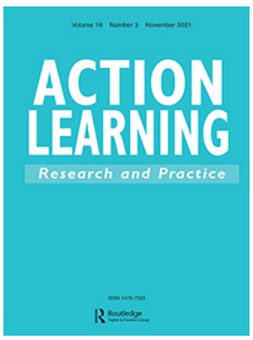

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An action learning approach to mathematics learning in the light of the cognitional theory of Bernard Lonergan

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ABSTRACT

The educational benefits of challenge- or problem-based approaches to learning are now well established. Action Research (AR) and Action Learning (AL) together provide educators with an ethic, a research methodology and a pedagogical strategy for harnessing and developing the motive power of purposeful activity for reflective enquiry in teaching and learning. However it is argued here that AR and AL implementation demands a sophisticated epistemological awareness on the part of the teacher-researcher. This paper suggests that the cognitional theory of Bernard Lonergan comprises a powerful resource particularly suited to underpin, inform and orient the practice of AL applied to the teaching of mathematics through practical problem solving. In this paper, aspects of Lonergan's thought are outlined and brought to bear on the development of an AL approach for teaching mathematics and its applications through collaborative practical problem solving. Lonergan's thought, as well as offering a theoretical framework of great clarity, when brought to bear on the development and implementation of AL strategies, has the potential to guide researchers, teachers and students in the design and implementation of such strategies and worth incorporating in practice. Also most beneficial for students as they negotiate the complex and dynamic epistemological territory that characterises AL.

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Problem-solving; mathematics learning; action research; action learning; epistemology

Introduction

Aristotle famously declared that 'all men by nature desire to know' (Aristotle, Metaph. A. 1, 980a1). Education seeks to mobilise this desire to develop the student's potential to understand, explain and, it may be, to reimagine and act in the world. But prior to all knowing and all explanation is questioning. The teacher is, therefore, concerned to elicit the student's curiosity, to awaken the desire to know such that the student's own questions motivate learning. Therefore, educational strategies such as problem-based learning and action learning (Harland 2003) often seek to stimulate enquiry using

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practical problems. A professional or personal need or game-like challenge may stimulate goal-directed purposeful enquiry on the part of students (Csikszentmihalyi 1990; Harland 2003). Using Action Learning (AL) implies that a collaborative approach to problem solving is valued. The set problem may include multiple potential solutions in either its practical or mathematical aspects or both. It is suggested in this paper that the cognitional theory of Bernard Lonergan provides a conceptual framework that is particularly suited for guiding the teacher as she negotiates the complex epistemological territory that must be traversed when applying AL to teaching mathematics through solving practical problems.

Action learning originated with the work of Revans who defined a problem-solving method characterised by taking an action and reflecting on the results, as an educational pedagogy for business development and problem-solving generally (Revans 1980, 1982). Since then the term 'action learning' has been applied to a variety of similar processes employed in a wide range of contexts (Brook, Pedler, and Burgoyne 2012; Mumford 1994; Smith and O'Neil 2003a; Smith and O'Neil 2003b). Proponents of action learning hold that optimal learning occurs when the student is guided in investigating questions related to their own concerns and goals (McGill and Beaty 1995) and where the problem-solving success builds confidence and motivates students to engage with increasingly complex challenges. A self-reinforcing cycle of enquiry, learning and personal satisfaction results (Csikszentmihalyi 1990). The relevant literature also suggests that metacognitive awareness contributes positively to problem-solving ability (Hassan and Rahman 2017).

This paper reviews some important thinkers associated with the emergence of AL and problem-solving approaches in education, with particular reference to teaching mathematics thorough practical problem solving. AL contends that the commitment to practical action is motivationally effective and epistemologically relevant for education. AL is contrasted with more traditional passive approaches. The place of metacognition (thinking about thinking) within AL is then outlined. The cognitional theory of Bernard Lonergan is described and, drawing on the work of Coghlan in particular (Coghlan 2012) Lonergan's thought is related to AL is elucidated.

The process of AL is then considered in more detail and the potential of Lonergan's thought to elucidate, structure and guide the action learning process is illustrated. Lonergan's work restores common sense or practical knowing its role and dignity alongside theoretical knowing. His recovery of insight as a central cognitional phenomenon vindicates the intuitions of educators who, concerned as they are with bringing the struggles of the student to fruition, can appreciate Lonergan's remark that 'the fruit of truth must grow on the tree of the subject' (Lonergan 1972, 71). His insistence on the need for rigorous cognitional self-knowledge (the realm of 'interiority') for truly critical thought is consistent with Revans praxeology and underpins philosophically the modern reflexive turn. This paper focusses on the use of practical problem solving for teaching mathematics. Beyond the complexities of AL in general, the use of AL in motivating maths learning raises two specific issues: potential misunderstandings that may flow from AL practitioners' use of the puzzle-problem distinction and the need to negotiate the diverse epistemological territories of theory, practice and interiority where Lonergan's thought may be a guide.

Background

This paper suggests that action learning constitutes an appropriate strategy for teaching and learning mathematics through practical problem solving.

Many students struggle to engage with and grasp mathematical concepts and reasoning when introduced to Mathematics in its pure or abstract theoretical form yet may grasp these same concepts and master the associated reasoning abilities in a practical problem-solving context (Papert 1980). Mathematical insights may accumulate to allow more complex problems to be addressed, where knowledge, skills and confidence develop and form a self-reinforcing cycle. A number of key elements are needed in a learning strategy aiming to develop learners' quantitative problem-solving skills (Lester et al. 1994). These include practical problems that learners can relate to and that elicit engagement in learners, a learning environment that facilitates group enquiry into mathematical problems and, in due course, a process that promotes overt (or reflexive) attention to the thinking processes involved in a problem-solving episode.

Revans reasoned that AL involves work on real practical problems where learning is from reflection on actions taken and progress is through questioning (Pedler, Burgoyne, and Brook 2005). Dilworth (1998) argues that action learning starts with an inquiry into a real problem so regardless of whether the problem is 'tactical or strategic … [the process of] learning is strategic' (Dilworth 1998, 36). Action learning in mathematics education can be defined as learning through student individual work on a real problem followed by reflection on this work. In most cases, this work is supported by a 'more knowledgeable other' (Abramovich, Grinshpan, and Milligan 2019, 3). This paper examines the use of AL in the form of using practical problem solving to motivate the learning of mathematics. AR is that cyclical iterative process of action and reflection on and in action. Similarly action learning is learning derived from reflection on action taken to solve a practical problem, leading to further action as a result of this learning (Zuber-Skerritt 2001).

The thesis here is that designing and implementing action learning for teaching mathematics spans a number of epistemological categories that may be in mutual tension. Collaborative enquiry necessarily involves ethically informed practical knowledge. Solving even partially open-ended problems (as distinct from puzzles) will involve the negotiation of evaluative criteria for various solutions. Implementation of mathematical procedures will involve the cognitive re-structuring of the practical problem in abstract or theoretical mathematical terms. Furthermore, the metacognitive move within AL involves reflexive knowledge. Designing and doing AL using practical problems in teaching mathematics, therefore, involves the teacher-researcher and students in mobilising, apprehending and then negotiating, in real time, the dynamic interplay between three distinct epistemological fields: theory, practice and reflexivity. This requires epistemological understanding on the part of the teacher both in the design and implementation phases and of the students as they negotiate these contrasting epistemological realms. It is proposed here that the philosophical work of Bernard Lonergan provides a conceptual framework that is uniquely suited to guide this dynamic interplay, concerned as it is with what Lonergan calls the 'realms of meaning' of theory, practice and interiority (Coghlan 2010, 2016; Lonergan 1972, 78–90.)

Practical problem-solving in mathematics education

Students can learn to formulate and solve practical problems using mathematical concepts and abstract reasoning. These abilities are exercised in one context and then applied in multiple contexts to address new problems (Conway and Sloane 2005; Oldham 2001). In recent times problem-solving has been recognised as central for mathematics education. Although mathematical problems have been a part of the mathematics curriculum for many years, in the UK the Cockcroft Report suggested that the ability to solve problems is at the heart of mathematics and problem-solving is an essential element in teaching mathematics to all ages and abilities (Cockcroft 1982). The Irish Mathematics curriculum is now explicitly project based (NCCA 2021). Developing a positive disposition toward problem-solving underpins the Junior Cycle framework (NCCA 2009) and is explicitly mentioned in the national Literacy and Numeracy strategy (DES 2011). Therefore the impetus across the entire curriculum is moving from solving puzzles to solving problems, and this is the context for the AL approach detailed in this paper. Therefore while local 'puzzles' may occur within the larger problem context, the overall problem-solving process involves practical-social, theoretical, creative, evaluative and reflexive processes.

Pólya (1965), the father of the present-day movement of mathematical problem-solving, viewed problem-solving and teaching as practical arts. His work emphasised the continuity between mathematics and problem-solving. He emphasised the learner's solution process rather than the answer. Pólya (1965) suggested a four-stage solution process of understanding the problem, identifying existing relevant experience and knowledge to create a plan (for calculating, performing mathematical operations, identifying coefficients etc.), carrying out the plan and looking back or reflecting on the experience to check for errors or loopholes. Pólya described an important role for heuristics and suggested that observing and participating in problem-solving is the basis on which heuristics are built. Cronin suggests that Pólya's approach is one instance of Lonergan's general empirical method in operation (Cronin 2001, 71).

Dewey (1913) believed that problem-solving, which he called reflective thinking, was crucial in education.:

> The need of thinking to accomplish something beyond thinking is more potent than thinking for its own sake. All people at the outset, and the majority of people probably all their lives, attain ordering of thought through ordering of action (Dewey 1926, 41)

Dewey's observation is, effectively, an argument for problem- and project-based active pedagogies since most of us are pre-disposed to be practical. In Lonergan's terms, we dwell habitually in the dramatic-practical 'pattern of experience' (Lonergan 1992, 293) and its associated common-sensical 'realm of meaning' (Lonergan 1972, 78). Lonergan treated practical or common sense intelligence at some length but he was alert to the proneness of common sense to bias and, following Aristotle, he posited the 'pure desire to know' as the normative source of authentic human enquiry. However, when applied in learning mathematics, AL is starting from the more epistemologically complex practical domain. Lonergan elaborated a theory of polymorphic consciousness that accounts for both theory and practice and allows for the possibility of a student's total absorption in enquiry for its own sake (Lonergan 1992; Walmsley 2008). While Dewey and Lonergan would differ on many fundamental issues, their common

preoccupation with the process (or method) of human learning makes both highly relevant thinkers for educationalists (Shea 1991). Similarly, Lonergan was concerned to elucidate human knowing as seeking intelligibility in contrast to the sensory extraversion that characterises animal knowing. Both Pólya and Dewey agree that the hurried transfer of domain-specific information or technique to learners is of little value (Dewey 1913; Pólya 1965). Learners need time to absorb new information, link it to existing knowledge, consider its relevance to the question at hand (in AL, relevance is a key motivation for engagement with any subject matter), and in due course, transform it by reflective thinking into forms of knowledge suitable for more general application in multiple contexts.

Action learning

Action Learning is a pedagogical approach that links the world of learning with the world of action through a reflective process within small cooperative learning groups (Pedler 2011). AL by its nature speaks to two persistent epistemological dichotomies: the theory-practice dichotomy and the subject–object dichotomy. Though motivationally powerful, an excessive focus on practical problem-solving in learning could lead to neglect of theoretical questions while concern to engage and affirm the struggling subject (i.e. the student) could obscure the need for critical thinking to reach objective conclusions.

Much higher education involves the lecturer presenting ideas to students in lectures. Many achieve the designated outcomes according to the assessment metrics of the system. However such methods may not be the most effective for all students. Action learning stresses the importance of social engagement, dialogue and physical doing in the learning process. It has a proven track record in relation to adult learning (McGill and Beaty 1995) and can be defined as "a continuous process of learning and reflection, supported by colleagues, with an intention of getting things done." Through action learning individuals learn with and from each other by working on real problems and reflecting on their own experiences' (McNiff, Lomax, and Whitehead 2003, 21). AL is therefore an approach that links the world of learning with the world of action through a reflective process within small cooperative learning groups known as 'action learning sets' (McGill and Beaty 1995). AL shifts the emphasis from the transmission of concepts to stimulating learners to raise questions in pursuit of solutions to problems. The problem stimulus is all the more effective if the learner has some choice in problem setting. Some freedom of choice is, in the authors' experience, achievable with appropriate effort on the part of the facilitator. The desire for a solution motivates learners to engage collaboratively with the problem, employing theoretical concepts as needed to help solve the problem, but the problem solution takes priority over any theory employed.

Research usually produces public outcomes, whereas the outcomes of AL are usually confined to the individual or learning group. All AR projects are AL projects, but the converse does not hold true (McGill and Beaty 1995). Research is not the primary aim of AL, so participants may do little or no ancillary data-gathering. While McGill and Beaty (1995) argue that AR can be an individual activity, it is more commonly a group activity and this is so for AL also. AL curricula may be developed for specific contexts and the presentation of Lonergan's conceptual framework here emphasises the context of teaching and

learning mathematics through collaborative practical problem solving with reflection on process.

Lonergan's cognitional theory

Bernard Lonergan, born in 1904, is recognised as one of the most significant philosophical thinkers of this century (Newsweek 1970). Lonergan's cognitional theory was unfolded at length in his keystone work, Insight, first published in 1957 (Lonergan 1992) and summarised in a later essay 'Cognitional Structure' (Lonergan 1967). The book was written 'to assist the reader in effecting a personal appropriation of the concrete dynamic structure immanent and recurrently operative in his own cognitional activities' (Lonergan 1992, 11). Lonergan held that the process by which normal adults come to know is invariant and that any attempt to deny this process will necessarily involve the sceptic in a performative contradiction (Lonergan 1992). He described a spontaneous, structured cognitional process that operates in a readily recognisable pattern It is the unfolding of a 'pure desire to know' (Lonergan 1992, 373), a desire that may, however, be stunted by bias (Lonergan 1992) or unfold freely toward its object which is reality.

Insight is that activity by which the mind grasps the intelligible connections between things that previously had appeared disparate (Colleran 2002). It comes as a breakthrough or release from the tension of enquiry. Before insight, the matter may seem endlessly perplexing, but once insight occurs, unorganised data fall into patterns where none was apparent before.

Insight often occurs suddenly when the mind is at a high pitch of creative activity (I get it!); or in a moment of relaxed passivity: 'it dawned on me'. It is a function of inner conditions rather than outer circumstances. Memory plays a role when the insight grasps the relation between present circumstances and prior knowledge. Insight pivots between the concrete and the abstract and once grasped, it passes into the habitual texture of our mind (Lonergan 1992) as a relatively permanent acquisition (Lonergan 1992). Insights accumulate and cluster to form a habitual capacity for problem-solving. Furthermore the 'insight into insight' that Lonergan calls 'self-appropriation' confers on the learner a reflexive or metacognitive understanding that facilitates future learning (Lonergan 1992; Naickamparambil 1997). This metacognitive 'insight into insight' has implications for teaching generally because appears simple to the teacher, who has already grasped the point, leaves the student in a state of puzzlement. However, the teacher can, if she shifts her attention reflexively to her own cognitive process and affective experience, recall with empathy her own former puzzlement. She can then suggest clues and divert attention away from distractions, to hasten the pupil's journey toward insight. This has particular relevance for AL where reflection on process is a key element.

Insight is abstractive (Corcoran 1975). However, Lonergan emphasised that theoretical insight is an enriching abstraction that tells us more about reality. Insight grasps intelligibility in sensible images (or sounds). It abstracts from particulars, formulating intelligibility as concepts that are strictly unimaginable. Students solving practical problems likewise move from the sensible particulars of the practical problem to the abstractions of Mathematics. The abstract character of much theory leaves us yearning for the comforting world of images and picture thinking. Thus, Euclidean geometry is taught (and

understood) by reference to diagrams, yet its theorems depend for their validity on pure abstractions: definitions of lines and points that render them strictly unimaginable because of their necessarily infinitesimal dimensions. Insights cluster and accumulate (Corcoran 1975). Lonergan's emphasis, like a teacher's, is on the event of insight in the learner's mind.

The three-fold structure of knowing

In Insight, Lonergan's focus is not the known (content), but the knowing (process). Intelligence operates in a variety of patterns (Lonergan calls them patterns of experience) including theoretical and practical or common sense patterns (Lonergan 1992). Common sense knowing occurs spontaneously, responding to the sensory experiences and practical needs of the here-and-now. 'I feel hungry: what shall I eat?' It does not seek to generalise. However, though enquiry may begin in some practical engagement, it can shift gear and pose questions that reflect a desire for explanation rather than practical problem-solving: 'what is hunger?' or 'how is it caused?' Insights of a theoretical or scientific character may then occur. It is the possibility of this shift that motivates the use of AL in the teaching of theoretical subjects like Mathematics through practical problem-solving. Following insight, intelligence moves to a third, critical level, spontaneously asking 'Is it so?' Reflective insight askes; 'What conditions would need to be satisfied to justify an affirmative judgement?' (Lonergan 1992). Lonergan's three-fold structure of experience, understanding and judging constitutes Knowing, as shown in Figure 1.

'Common sense operates from a distinctive viewpoint and pursues an ideal of its own'. It is concerned with 'the successful performance of daily tasks, the discovery of immediate

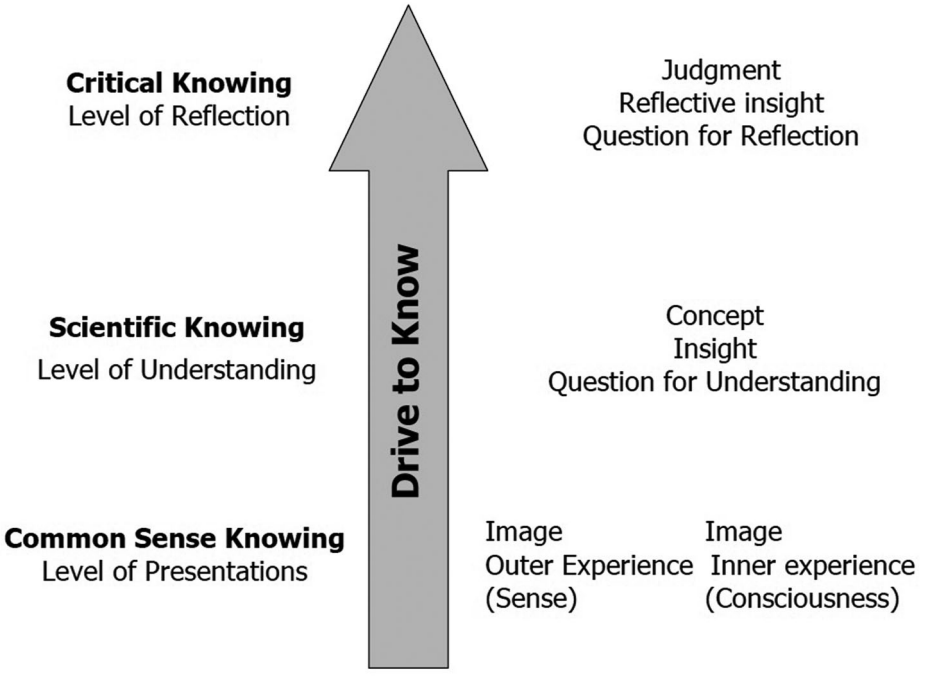

Figure 1. Lonergan's three stages of knowing (Connolly 2007).

solutions' (Lonergan 1992, 199). Common sense knowing is often content with a descriptive account. For the person of common sense, reality is taken to be 'the already out there now real', something to be encountered by sensory confrontation. Common sense may formulate its insights as proverbs yet most proverbs have a limited scope of application. To have common sense is precisely to understand the particular context, especially the social context, sufficiently to know how to proceed in that instance. Common sense may harbour a prejudice against theoretical knowing: 'What is the use of algebra?' Lonergan honours common sense as being enormously valuable and a valid manifestation of human intelligence but he warns that it is prone to bias and may be blended with common nonsense!

The theoretical or scientific orientation seeks explanations not just of how things are related to us, but how they are related to each other. Cognitional process moves from concrete to abstract, in its search for systematic understanding of the universal case. Science moves from the familiar world of sights and sounds into the bewildering world of abstract theoretical and mathematical constructs. Common sense accelerates rapidly from practical questions to insight to decision, while scientific or theoretical understanding lingers in the enquiry phase of cognitional process. Nevertheless, even scientific knowing does not inhabit an exclusively abstract world since insight pivots between concrete experience and abstract intelligibility.

Lonergan insisted that the same cognitional structure operates in scientific knowing and in the world of common sense. But in scientific knowing, proximate human needs must recede if the 'pure desire' is to unfold in a disinterested way, whereas in common sense, practical human concerns may dominate. This is the tension that must be negotiated in an AL context. Students require guidance as they shift from interpersonal negotiation to technical, theoretical and reflexive questions. Insights are preceded by questions, and questions are therefore crucial to having insights.

Also though further insights occur spontaneously, it remains to determine whether the original insight is correct or not: therefore there is a crucial distinction between questions for intelligence (what? why?) leading to direct insights and questions for judgment (Is it so?) leading to reflective insights of fact or truth. Spontaneously (or prompted by the skilled teacher), students weigh the evidence, spot the flaw for or against a hypothesis (especially when it is not their own), whether common-sensical or theoretical.

> Human knowing, then, is not experience alone, not understanding alone, not judgement alone; it is not a combination of one's experience and understanding, or of only experience and judgement, or of only understanding and judgement; finally it is not something totally apart from experience, understanding, and judgement. (Lonergan 1967, 223)

Each instance of insight seeks creative expression. The same insight may be expressed in a new way and many problems may be solved in multiple ways. As Carley says 'teachers who expect students to respond by expressing their own understanding are creating a flourishing framework for freedom of consciousness and re-creation of imagination' (Carley 2005, 21).

Having described and explained the process of knowing, Lonergan then invites each of us to recognise, understand, affirm and appropriate that universal pattern in ourselves (Lonergan 1992, 13). He advocates a shifting of attention from the contents of our cognitional operations to the operations themselves. This reflexive move brings cognitional

structure to bear on its own operations, attending to them, grasping them in their relations and affirming them as so (Lonergan 1992, 352). This process of cognitional affirmation and self-acceptance he terms self-appropriation (Lonergan 1992; Naickamparambil 1997). We move into the realm of meaning called interiority (Lonergan 1972). Coghlan shows how this first person move allows us to integrate our knowing and navigate the shifts in cognitional mode between practical or common sense and theoretical patterns that characterise the practice of AR and AL (Coghlan 2010, 2013, 2016). It is suggested therefore that advertence to the realm of interiority provides a compass to orient the AL teacher as she guides the students in navigating these shifts in cognitional mode. Coghlan asserts that there is a very close connection between Lonergan's tripartite cognitional theory and the tripartite praxeology of Revans, the founder of AL (Coghlan 2010). Revans postulates that three systems of enquiry are successively engaged in a cycle when addressing practical AL problems. System alpha is concerned to gain insights into the situation as it presents itself, to discern intelligible order in the data. System beta searches for solutions, appropriate actions that would resolve the problem. System gamma focuses on reflexive consideration of the operations and learning involved in the engagement (Coghlan 2010; Coghlan and Coughlan 2010). Coghlan relates Revans' account to Lonergan's realms of meaning thus:

> In terms of Revans' praxeology, students work to learn how to analyse situations, to engage in cycles of action and reflection while always keeping an eye to not only what they are learning but how they are learning. In this way they are learning to move through the three realms of learning, and to recognise how interiority enables them to understand how each realm has its role in their learning. (Coghlan 2010, 301)

When learners attend to, come to understand and know their own cognitional process, they have the ability to manage their critical/metacognitive knowing. They will have a fixed base in their cognitional structure from which they can approach learning in general.

Applying Lonergan's theory to action learning

AL, when applied to Mathematics teaching and learning, uses practical problems to engage common-sense intelligence collaboratively, which in turn stimulates questions for practical, theoretical and reflective intelligence as cognitional process unfolds spontaneously.

Common sense engages with the challenge and asks: 'What might some possible solutions look like?' 'Which solutions are similar, which distinct (gradual shift from common sense/description to scientific definition)'? The practical-evaluative question emerges: 'Which solution is best? Why?' Gradually the questions become more precise, more searching:

> What conditions must a solution satisfy to be acceptable (part of reflective insight)? Can any of these conditions be expressed mathematically (theoretical insight)? What would I need to know to be satisfied that these conditions are actually satisfied (reflective insight)?

At opportune moments the tutor directs the attention of students overtly to the processes of enquiry they have engaged in, the data taken into consideration, the nature of the questions raised (practical-ethical, theoretical or reflexive), the social process and questions that led to progress, the concepts employed, the move from a proposed or possible solution to an actual solution. Such a strategy, having created an active, supportive yet

challenging learning environment, stimulates intellectual activity, and then endeavours to attend overtly to the social and intellectual processes that are in play. The approach follows the principles of AL and moves through three stages, corresponding to Lonergan's three stages in cognitional process.

Stage 1 – Common sense knowing

According to the basic constructivist theory, students learn most effectively when they are given the opportunity to explore and create knowledge in pursuit of a goal that is of personal interest and concern to them (Papert 1980). Well-designed practical problems engage common-sense intelligence, for example:

> Compete in teams to build the lightest possible bridge using a fixed set of components. Compare alternative solutions. Explain why some are more efficient than others? Use concepts from vector mechanics to explain how equilibrium is maintained? What other evaluative categories apart from weight might be used?

The initial problem is simply stated in common-sense terms and is easily grasped. A meeting process may be suggested and initially guided. Ideas may be generated and explored initially through physical manipulation (sensory extraversion) and then engage theoretical enquiry in an attempt to grasp intelligible patterns, e.g. to classify and thereby place initial results of enquiry in an explanatory perspective. The common-sense or practical goal stimulates engagement yet at the same time it yields a rich harvest of abstract theoretical issues to ponder.

Stage 2 – Scientific knowing

In this stage of the AL approach, students are encouraged to work together in teams and to some extent monitor their own learning and progress. The process of dialogue is mutually stimulating as students explore and develop a variety of ideas, and discovery possible new solutions. The tutor's role is to identify and pose penetrating questions and place the proposed solutions in suggestive arrangements likely to lead the further questioning that will move inquiry from a common-sense realm to a theoretical realm. In testing out their thoughts and designs, students will propose novel solutions and uncover questions that they have never thought about before. These questions will, in turn, lead them into consideration of concepts and theorems (e.g. parallelogram law of vector addition) whose potentially practical significance was previously not apparent. For example, if building and programming a robot, students are led to think in new ways about computation, discrete math, linear equations and geometry. These concepts now become demonstrably useful ideas, allowing the student to see how and why a specific solution of their works (Papert 1980). Thus the students develop not only a better understanding of the theoretical concepts and principles but in due course, following a reflective consideration of the problem-solving process which they have just engaged in, can learn to apply the concepts in other contexts. Because the student learns through her own experimentation, she experiences a personal connection to the idea by engaging with a concrete problem and develops a positive outlook on learning (Papert 2000).

Cronin suggests that the distinction between an intellectualist approach to knowing and a conceptualist approach is the difference between an enriching

abstraction and an impoverishing abstraction. The intellectualist approach affirms that cognition is an activity that a knowing subject performs, not something that happens to them (Cronin 2001, 2017). The conceptualist is concerned with the pure abstract concept which is known but not with the human mind that knows. This distinction corresponds to the student-centered (AL) versus content-centered learning. In AL, the cycles of activity stimulate questions and lead toward insights. The questions progress from routine common-sense understanding, through scientific questioning (to a deeper understanding of principles that can elucidate similar future problems), to constructing a problem-solving method from a considerations of the realm of interiority (critical knowing.)

Stage 3 – Critical knowing

Lonergan says 'the attainment of a critical position means … . that one distinguishes between the different patterns of one's own experience' (Lonergan 1992, 293). This involves identifying the various kinds of questions that were asked at various stages in the problem-solving process, noting the sequence of suggested solutions, recalling how one suggestion brought forth another, how questions for theoretical intelligence arose (How is the force here related to the load there?) or questions for reflection (But is this a real solution? Is it stable?). Critical awareness of their own cognitional process will enable learners to manage their problem-solving more effectively because they have now affirmed the method by which they come to know: they have learned how they learn and proactively participated in the process.

What abstract ideas or concepts were evoked: vector? Component of a force? What relations were invoked: equations of equilibrium? At this stage learners examine explicitly the skills and 'best practices' or implicit heuristic procedures employed in the previous stages: were algebraic equations set up involving the unknown 'x' and solved? Did they deconstruct and separate out parts of the puzzle for individual consideration? How was this achieved? What insights were critical in enabling this approach of employing sub-procedures? What kind of reasoning and/or logic was employed in this process? How might this inform their approach to breaking up future mathematical challenges?

They may also reflect on the motivational and affective aspects of the process: what did they find engaging and why? For example, more advanced problems may evaluate the relative utility of alternative analyses, e.g. with computers (involving purely abstract matrices) or manually (using the visual medium of graphical statics). Some student may prefer graphical over purely computational approaches, occasioning the need for metacognitive and evaluative work in the group. What kinds of feelings (excitement, frustration, boredom, anxiety, satisfaction) were evoked by or during the problem-solving process? What light might this reflection throw on the student's future approaches to problem-solving? Later works of Lonergan point to feelings as apprehensions of value as his focus widens beyond the cognitional subject to examine the existential subject. While it would take us too far afield here to explore this avenue further, suffice it to say that few experienced teachers would deny the central role of the student's feelings in learning (and the teacher's in teaching) even if discernment of the wise way to proceed in this realm is difficult.

When, through reflection, students gradually learn to recognise the stages of problem-solving and action (Schön 1987), to grasp their own cognitional process in act, and recognise the various realms of meaning in performance (and their associated biases), they are equipped to take on a global perspective and plan their work, breaking the problem into parts, proposing solutions, applying mathematical analysis to parts as needed, negotiating evaluative criteria, evaluating solutions and reviewing process metacognitively. Tasks are designed to enable students grasp the functionality and power of the idea of sub-procedures, using Lonergan's theory as a guide.

As practitioners, the authors have found that Lonergan's thought is a powerful framework to make sense of our experience and guide our practice. The following brief examples are drawn from our teaching practice which explicitly requires reflection of students. One teacher reflects 'I use mathematical software to help teach about the reversibility of concepts, an area often problematic for learners, thereby connecting the concrete to abstract and assisting students in developing understanding and ultimately their action learning.' The other author says with Lonergan's help:

> "I taught them how to analyse triangulated frameworks (trusses) under gravity loads using graphical vector mechanics and simultaneous equilibrium equations. Yet when asked to analyse a similar truss in a horizontal plane resisting lateral wind loads, they are puzzled. I feel mild frustration

Then I imagine Lonergan speaking: 'The body experiences gravity in sensory terms. Their insight is incomplete, still tethered to sense experience. They need more varied examples to free them from sensory extraversion and reveal, through insight, the abstraction that engineers call a truss'. When invited to do so, students will demonstrate metacognitive interiority about problem solving. One reflects that, notwithstanding the teacher's prompting, 'I never found [brainstorming] to work great for me. I am more of a step-by-step person. After realising this about myself it made my learning process easier'."

In following the action cycles, AL functions by engaging students in collaborative problem-solving in the course of which mathematical concepts and procedures are applied and illuminated. The practical challenge motivates students to shift from the accessible everyday practical pattern of experience into the theoretical pattern where mathematical concepts can be encountered and grasped in theoretical terms (Galbraith 2013; Lave 1988; Mason 1998). However, discussion and evaluation bring students repeatedly back into the practical-social-evaluative realm. The requirement to metacognitively review process develops students' understanding of their own social and reasoning processes rather than emphasising only the 'right answer' (Lesh and Zawojewski 2007; Schoenfeld 1994). They move between the realms of meaning of common-sense and theory while the teacher (and sometimes the student) controls the shifts by adverting to the third realm of meaning: reflexive interiority (Coghlan 2010).

The authors' personal research is practice based and oriented toward problem or design-based pedagogies. We continue to develop an integrated (or 'constructionist') problem-based approach in our university teaching which has positively impacted our teaching and our students' learning with Lonergan as our guide (Cosgrove and Reilly, 2019). The experience of seeing students grasp insights that span theory, practice and interiority is the reward that motivates our investment in AL to teach Mathematics and its applications through practical problem-solving.

Conclusion

The problem-solving learner, whether that be teacher or student, invokes the innate, dynamic and self-assembling activities of attentive awareness, intelligent enquiry and reasoned critique. Lonergan's cognitional theory provides teachers and students with a sure guide to bring cognitional process into conscious awareness (Connolly, Moore and Murphy, 2008); Carley 2005; Colleran 2002). Lonergan's invitation to self-appropriation, if accepted, enables people to experience and understand for themselves their own cognitional process and to name, identify and recognise it when it recurs. This heightened awareness provides us with a critical tool to facilitate the authentic unfolding of 'the pure desire' while minimising the potential interference between common-sense and theoretical patterns of cognition.

The AL analysis presented, viewed through the lens of Lonergan's cognitional theory, reorients education away from the transmission of pre-formulated concepts and toward the engagement of the 'pure desire to know' of each student as it draws the student through the levels of cognitional process and through the three realms of meaning of theory practice and interiority. A practical problem provokes spontaneous common-sense enquiry into multiple possible solutions and analyses. With tutor guidance, enquiry moves into the realm of theory (though some students may make the leap to theory almost immediately without the stimulus of a practical problem). This approach offers a rationale to students for learning mathematical topics (Hiebert et al. 1996; Prusak, Hershkowitz, and Schwarz 2013). It is a strategy that focusses on the student's framing fruitful questions rather than always reaching right answers. Practical and higher order theoretical and reflexive enquiry, promoted by encouraging problem-solving skills, is best supported by valuing and elucidating reflexively the process of inquisitive and creative enquiry (Mason 1998).

Both authors have found in their use of AL in their teaching that Lonergan prompts them to do new things, such as using physical objects to link the senses to abstract ideas or student reflection, but it also clarifies what we are doing while in the middle of the process. AL is collaborative, democratic and concerned about real situations. An optimal action learning environment should support and provide opportunity for free choice and personal creativity in a supportive and structured atmosphere. It is the emotionally rewarding state of being optimally challenged, or being in a state of flow, that motivates rich learning (Csikszentmihalyi 1990). Having engaged our students in valuable theoretical thinking motivated by practical problems, we can then help to make our students aware of, and reflect upon, the authenticity of own thinking and learning processes thereby increasing the odds of future success (Coghlan 2008) and invite the reader to do likewise.

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