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Nonlinear analysis of a two-degree-of-freedom energy harvester

Valeria Nico, Ronan Frizzell, Jeff Punch

Abstract In recent years the use of Wireless Sensor Networks (WSNs) has increased rapidly, enabled by the development of small and ultra-low power electronics. The majority of these sensors are battery powered, and this can lead to high maintenance costs when batteries have to be replaced. A practical solution to power sensor networks comes from kinetic energy harvesting, which is the conversion of the vibrations present in the ambient into electrical energy. To overcome the problems of narrow bandwidth and high resonant frequency at small scale for conventional harvesters, a nonlinear two-degree-of-Freedom $(2DoF)$velocity-amplified vibrational energy harvesting has been developed. Electromagnetic induction was chosen as the transduction mechanism because it can be readily implemented in a device that uses velocity amplification. The harvester consists of two masses relatively oscillating one inside the other, between four sets of magnetic springs. Collisions between the two masses can occur, and they transfer momentum from the heavier to the lighter mass, increasing the velocity of the latter. Bispectral analysis was carried out on the device, which revealed the presence of quadratic phase couplings between the Fourier modes and also period doubling. Both these phenomena are associated with the use of nonlinear magnetic springs, and an understanding of these effects can help to enlarge the bandwidth of the device.

Key words: energy harvesting, $2DoF$, nonlinear, bispectral analysis

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1 Introduction

The presence of wireless sensor networks is widespread in many aspects of everyday life [13], in part associated with the concept of "smart cities", which are urban environments in which sensors monitor both environmental conditions and other activities within the city. In the past few years, great improvements in handling data in smart cities has been achieved, however the problem of powering the sensors has not been satisfactorily solved. Sensors are usually battery powered to ensure the independence of each sensor from the mains supply, but batteries have a limited lifetime and must be replaced and disposed of. Moreover, their replacement in a large network can be difficult since some of the sensors could be embedded in structures and therefore difficult to reach [7].

A possible solution for powering a large network comes from the conversion of the ambient energy present in the environment. Among all the possible sources, kinetic energy available from ambient vibrations is one of the most common forms, and it can be converted by using variable capacitors, piezoelectric or magnetostrictive materials and through electromagnetic induction. Conventional vibrational energy harvesters (VEHs) are generally based on a linear mass-spring resonator configuration for which the resonant frequency of the device must be tuned to the main frequency of the ambient vibrations. To overcome the problems of narrow bandwidth and the high resonant frequencies associated with small-scale devices, a 2-Degree-of-Freedom ($2DoF$) electromagnetic velocity-amplified VEH [11] has been developed. The device comprises two masses oscillating between two sets of nonlinear magnetic springs. Impacts between the two masses can occur, which allows momentum transfer from the heavier to the lighter mass, providing velocity amplification. The dynamic behaviour of the system, and hence the output voltage, is highly influenced by the use of magnetic springs that introduce a hardening nonlinearity in the system.

Conventional power spectra show the frequency distribution of the output signal of the harvester, without any phase information that is relevant for a nonlinear system [6]. For this reason, the nonlinear response of the device is analysed in this paper using bispectral techniques such as bicoherence, to show the increasing nonlinearity when the external acceleration is increased. Bispectral analysis was first introduced in 1963 by Hasselman for studying ocean waves [5], and since then it has been used to study fluid mechanics [14] and mechanical systems [3], quantum mechanics systems [8] and more recently in the study of electroencephalographic signals [1]. To the best of the authors' knowledge, the method only appears once in the open literature on kinetic energy harvesters [9], where it was applied to the analysis of a piezoelectric membrane.

In this paper, bispectral analysis is performed on experimental data to show the presence of quadratic phase coupling (QPC) in the Fourier modes of the output voltage of the harvester, and to reveal how the increasing excitation leads to a period-doubling. The analysis reveals evidence of the excitation of modes not present in the external acceleration, which enlarge the bandwidth of the device; these modes are associated with the magnetic springs.

Section 2 will present a brief introduction to bispectral analysis, section 3 will describe the prototype and the experimental set-up and, finally, experimental results will be provided in section 4 to show the effectiveness of bispectral analysis techniques in detecting nonlinearities and providing a greater understanding of the complex dynamics of velocity-amplified VEHs.

2 Bispectral analysis theory

The general aspects of higher order analysis techniques, in particular bispectral techniques, are presented in this section. A discrete-time real valued stationary random process $x(t)$ can be expressed by means of its Fourier transform [6]:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{-i\omega_k t} \tag{1}$$

where $X_{-k} = X_k^*$ is the Fourier coefficient at radial frequency ω_k=$2\pi k/T$, the subscript k is a frequency index, T is the record length of $x(t)$, and the asterisk indicates the complex conjugate.

By means of the expectation value, or average, operator E [·], it is possible to define the power spectrum of the process $x(t)$ [6]:

$$P_{xx}(k) = E[X_k X_k^*] \tag{2}$$

where $P_{xx}(k)$ is real valued and nonnegative ($P_{xx}(k) \geq 0$), symmetric ($P_{xx}(k) = P_{xx}(-k)$) and shows the frequency distribution of the modes forming the signal. However, the power spectrum has no phase information so it cannot be used to study nonlinear interactions. For this purpose, the concept of higher order spectral analysis was developed [6] and, in this way, it is possible to isolate phase couplings between interacting Fourier components, which is an effective method for developing a detailed understanding of the complex dynamics of nonlinear systems.

One of the most commonly used detectors for couplings is the bispectrum, defined as [6]:

$$B(k,l) = E[X_k X_l X_{k+l}^*] \tag{3}$$

The bispectrum at the frequency pair (k,l) is proportional to the spectrum of the signal at the frequencies k, l and $k+l$; for this reason, it is common to define its normalised magnitude, the bicoherence, as:

$$b^2(k,l) = \frac{| B(k,l) |^2}{E[| X_k X_l |^2] E[| X_{k+l} |^2]}. \tag{4}$$

Considering three modes with radial frequencies ω_k, ω_l and $\omega_{k+l} = \omega_k + \omega_l$, b^2 represents the fraction of power at the radial frequency sum $\omega_k + \omega_l$ that has quadratic phase coupling (QPC) between ω_k and ω_l [12]. If the three components are spontaneously excited modes, it is expected that each component has a random phase; when the statistical averaging in Eq.3 is carried out, the bispectrum, and hence the bicoherence, will vanish. On the contrary, if the three modes are quadratically phase coupled to each other ($\theta_{k+l} = \theta_k + \theta_l$), the averaging will not lead to a zero value of the bispectrum, and the bicoherence will have a value close to unity.

As an example, a discrete test signal is defined as $x(t) = y(t) + z(t) + \xi(t)$ where:

$$y(t) = cos(\omega_1 t + \theta_1) + cos(\omega_2 t + \theta_2) \tag{5}$$

$$z(t) = cos[(\omega_1 + \omega_2)t + \theta_1 + \theta_2] \tag{6}$$

in which θ_1 and θ_2 are uniformly distributed random phases in [-π;π], $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $f_1 = 30Hz$ and $f_2 = 20Hz$, and $\xi(t)$ is a Gaussian noise term [6]. Fig. 1 shows the bicoherence plot of the test signal. Since $X_{-k} = X_k^*$, the bicoherence satisfies symmetry relations and, in particular, for a discrete time series with Nyquist frequency f_N, the bicoherence is completely defined by values in a triangle of vertices at (f_1=0, f_2=0), ($f_1 = f_{N/2}$, f_2=$f_{N/2}$) and (f_1=f_N, f_2=0). Moreover it is symmetric to the bisector of the f_1-f_2 plane [12], as shown in Fig. 1. For this reason in Fig.1, only the lower part of the plot is relevant for the analysis. The presence of a peak at the pair (30Hz, 20Hz) of amplitude $b^2 = 1$ is evident, due to strong QPC between the two frequencies. In the next section bicoherence plots of the experimental data will be presented to identify the presence of QPC in the harvester response.

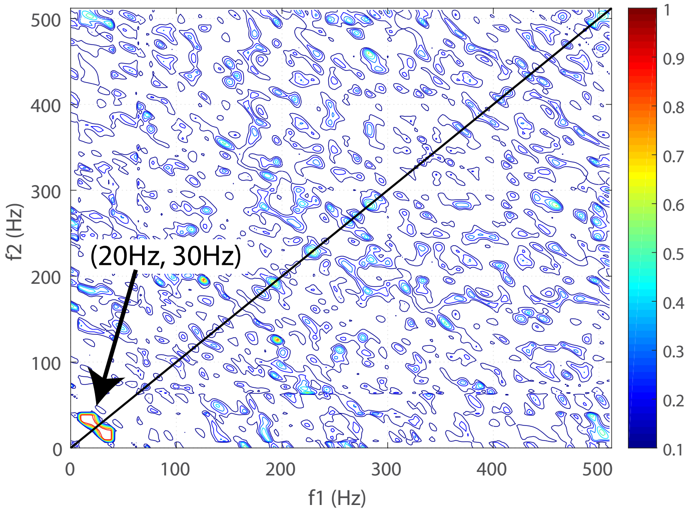
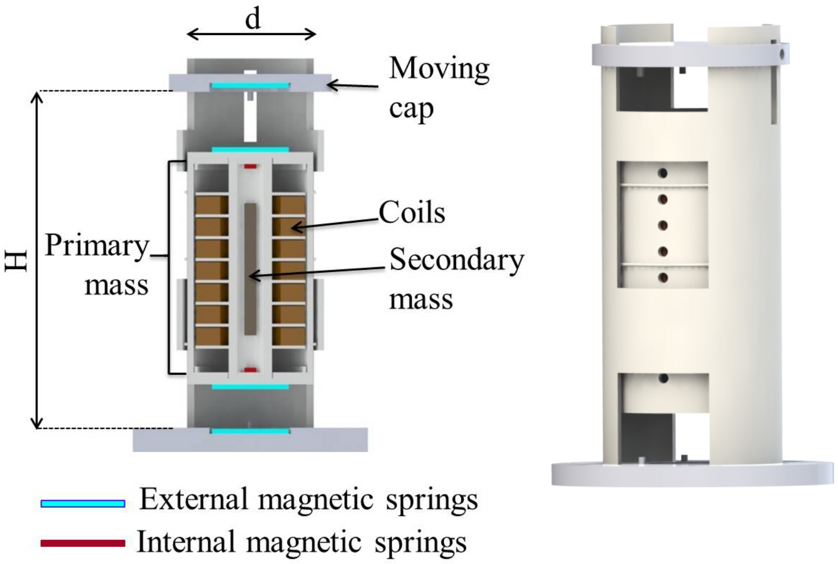


Fig. 1 Bicoherence of a test signal formed by three coupled waves.

3 Harvester description

The harvester under examination is shown in Fig. 2. It is based on a concept developed by the authors [4], and fully characterised in a paper by the same group [11]. It consists of two masses, an external larger mass (Mass 1) and an internal smaller one (Mass 2), relativity oscillating one inside the other between two sets of magnetic springs. Stoppers between the masses and between Mass 1 and the housing are present to prevent damage to the magnets. The use of magnetic springs reduces the mechanical energy losses compared to the case with mechanical springs, and it introduces a hardening nonlinearity in the system that enhances the bandwidth. The effect of the nonlinear springs under increasing levels of external excitation is increased, will be studied in detail in section 4 using bicoherence plots.



(a) Section of the prototype. (b) Assembly of the prototype.

Fig. 2 Schematic of the harvester.

Impacts between the two masses can occur that allow momentum transfer from Mass 1 to Mass 2, providing velocity amplification. Since according to Faraday's Law, the induced voltage is proportional to the relative velocity between a magnet and a coil, electromagnetic induction is chosen as the transduction mechanism because it is easily implemented in a device that exploits velocity amplification: the larger mass is made up of seven coils, while the smaller mass is made up of 5 magnets in a Halbach configuration. A detailed description of the device can be found in [11, 10, 2]

A schematic of the experimental setup is presented in Fig. 3.

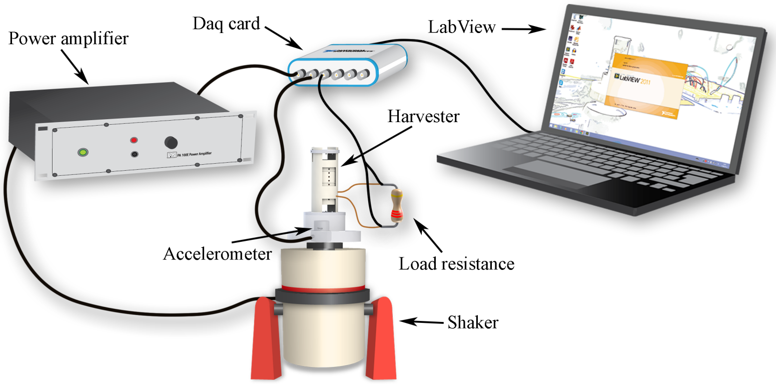
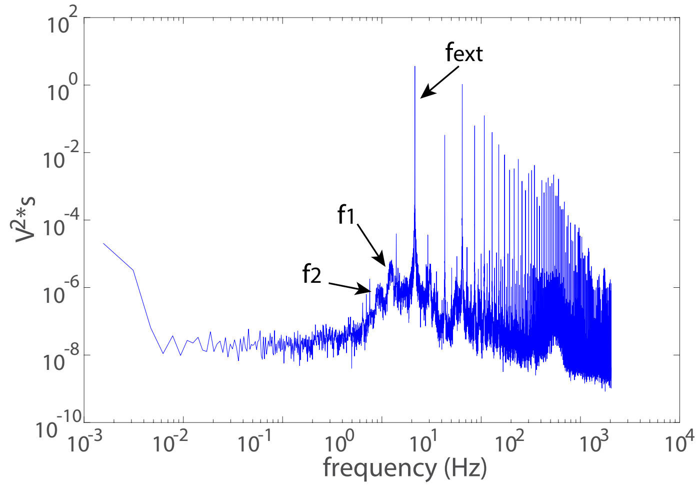


Fig. 3 Schematic of the experimental setup.

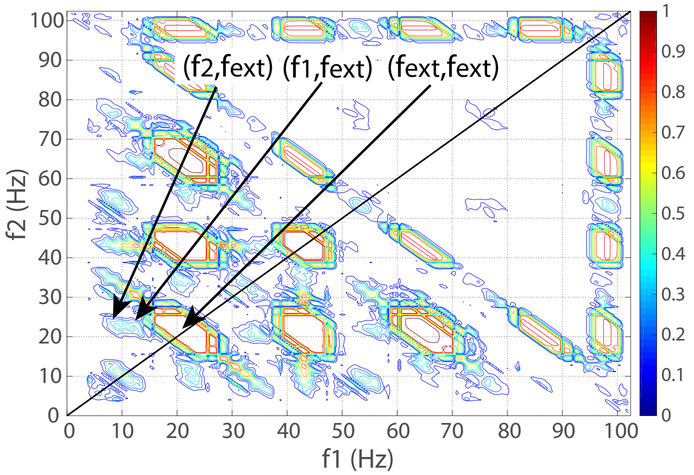
The harvester was mounted on an LSD V406 electromagnetic shaker. A PCB Piezotronic accelerometer was placed on the head of the shaker, below the harvester, to provide a feedback which was used to ensure that the shaker was given the desired level of excitation. Labview was used to supply a voltage signal to the shaker through an LDS PA100E amplifier with a variable gain control, and to acquire the signals from the accelerometer and the harvester.

4 Experimental results

In order to study the effect of acceleration on the output voltage, the system was tested at a fixed frequency $f_{ext} = 21.5Hz$ for two different acceleration (rms) levels: $a_1 = 0.4g$ and $a_2 = 0.8g$. The frequency was chosen far from the resonant frequency of the device ($f_{res} = 16Hz$) so that any possible nonlinearities were due to the increasing acceleration and not to impacts with the stoppers (which were obseved to happen at f_{res} even at very low excitation levels). The signals for the two different excitations were recorded across a load resistance of $R_L = 7k\Omega$ for 100 seconds in order to obtain a sufficient resolution in the low frequency spectrum. Figs. 4 a and b show the power spectrum and the bicoherence of the signal for the lower acceleration level.



(a) Power spectrum of the output voltage.

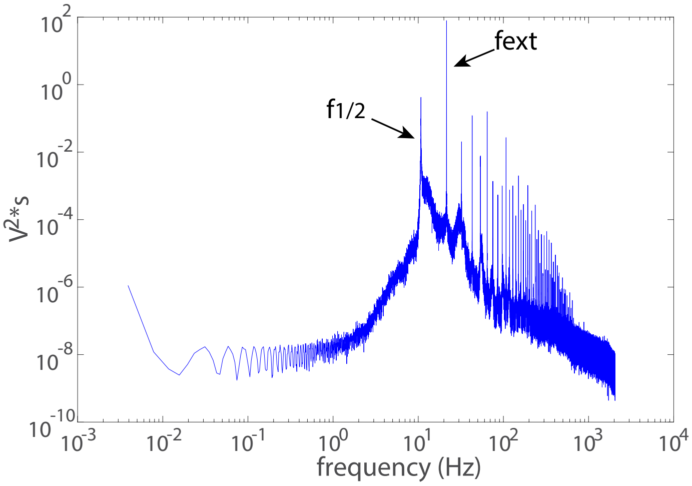


(b) Bicoherence of the output voltage.

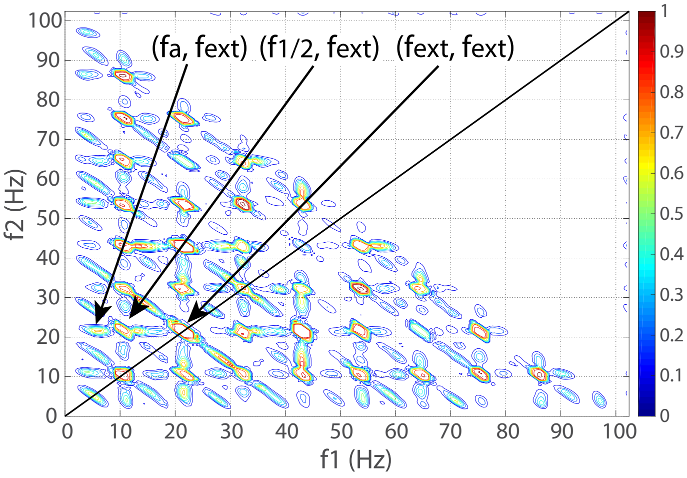
Fig. 4 Spectral analysis of the output for an excitation of $a_{rms} = 0.4g$ at 21.5Hz.

From Fig. 4a, it is evident that there is a main peak at f_{ext} but also at lower frequencies as $f_1 = 12.2Hz$ and $f_2 = 9.4Hz$ are excited. The power spectrum, however, does not have any information on the phases of the components, or on how the modes were excited. With reference to the bicoherence in Fig. 4b, due to its symmetry (section 2), only the lower part of the plot is relevant for the analysis. It is interesting to note the presence of peaks in the plot, meaning that the excitation frequency is coupled with the higher harmonics and also with the two lower frequencies (f_1 and f_2). It is important to acknowledge that $f_{ext} = f_1 + f_2$ and so f_1 and f_2 are excited through QPC, even though the coupling is not strong (b^2 = 0.5 for the pair (f_{ext},f_1) and b^2 = 0.48 for the pair (f_{ext},f_2)).

Increasing the acceleration amplitude, the larger mass experienced a stronger repulsive magnetic force and the nonlinearities in the system are expected to increase. Figs. 5 a and b show the power spectrum and the bicoherence of the signal for $a_2 = 0.8g$.



(a) Power Spectrum of the output voltage.



(b) Bicoherence of the output voltage.

Fig. 5 Spectral analysis of the output for an excitation of a_{rms} = $0.8g$ at 21.5Hz.

From the comparison of Fig. 4a and 5a, it is possible to see that the bandwidth of the device is enlarged and that a peak at $f_{1/2} = 10.7Hz = f_{ext}/2$ is present that is evidence of period-doubling. However, it does not contain information on the increase of nonlinearities and, moreover, it is not possible to tell if $f_{1/2}$ is excited independently from f_{ext}. Fig. 5b shows the bicoherence of the output voltage and the increase in the number and magnitude of peaks, compared to Fig. 4b. The driving frequency is coupled through QPC to the higher harmonics as in the lower acceleration case, and to $f_{1/2}$; this is evidence of period-doubling due to the hardening effect associated with the magnetic springs. The excitation frequency is also coupled to $f_a = 5Hz$, a coupling that was not evident in Fig. 4b. Moreover there are peaks at lower frequencies, indicating presence of QPC between $f_{1/2}$, its harmonics and f_a, and between f_a and its harmonics. The greater total number of peaks compared to Fig. 4b indicates the expected increase of nonlinearities, since the larger mass could reach positions closer to the magnets and, consequently, experience a stronger repulsive force.

5 Conclusion

Bispectral analysis was conducted on the output voltage of a nonlinear electromagnetic energy harvester to show the effect of increasing the amplitude of the external excitation. It is shown that the power spectrum carries information about the frequency content of a signal but has no information about the phase interaction between modes, while the bicoherence measures the fraction of power due to QPC. Two tests were carried out at different acceleration levels, but at the same driving frequency. It was shown that more couplings between modes due to QPC were present in the bicoherence for the higher amplitude case, evidence of increased nonlinearities in the system. In particular, the peak at the pair $(f_{1/2}\ ,\ f_{ext})$ is symptomatic of period-doubling which is a characteristic consequence of the hardening effect introduced by the magnetic springs. Such period-doubling can be exploited in order to enhance the harvested power because, by using a nonlinear potential, it could be possible to excite modes not present in the external acceleration and, hence, widen the bandwith of the response.

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